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A VON NEUMANN POLLUTION MODEL

Dedicated to Professor Dr Heinz Kurz on His 60th Birthday

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ABSTRACT

An input-output model a la von Neumann with recycling of resources and abatement of pollutants is presented. We allow for joint production, durable capital goods, and heterogeneous labour. In addition, the model covers household garbage so far neglected in the literature. The existence of an equilibrium is proved, and the effect of classification of garbage on the rates of growth and profit is discussed. A trade-off relation between the uniform growth rate and the regulation level of tolerable limits of pollutants is also suggested.

1. INTRODUCTION

Several authors, including Leontief himself, have presented generalizations of Leontief model to include pollutants: Ayres and Kneese (1969), Arrous (1994), Lager (1998), Leontief (1970, 1973), Luptacik and Böhm (1994, 1999) and Steenge (2004). Lager (1998) gives a Leontief model with alternative processes in each industry, after discussing a fatal flaw of the model with no choice of techniques. (See Fujita (1991, 2005) for the conditions of nonnegative solutions in a Leontief pollution model.) Still more serious shortcomings of the models so far proposed are: (i) there is no process of recycling resources, and (ii) kitchen refuse is completely neglected. In this paper, we represent a von Neumann model which includes recycling processes as well as garbage through consumption. As a von Neumann model, it naturally allows for joint production, thus durable capital goods. Heterogeneity of labour force is also incorporated. Our extension retains one important idea, that is, externalities in production are

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internalized by setting negative prices on pollutants, while household garbage is, as a matter of course, left out of cost calculation.

Many economists consider Leontief pollution models are useful simply because they are much used. This stereotyped judgement can be dangerous especially when we note that the input coefficients represent *status quo*, ignoring completely those processes which may be viable under different prices. The results based on Leontief type models can be valid only for a short period. For example, in Leontief pollution models, changes in regulatory tolerable limits of pollutants do not affect (shadow) prices. Lacking in recycling processes is another source of inadequate estimates. Hence, at least in theoretical consideration, we had better allow for alternative processes as well as joint production. In practical applications, it is certainly necessary to resort to some eclectic methods since no government today compiles data to construct a von Neumann type model in a proper way. This makes it possible to take into consideration an array of invented processes, which, unprofitable at present, may become viable under a set of policies.

In the next section, we explain our model, among others the practical implication of negative prices, which has not been clear even in Leontief pollution models so far presented. In Section 3, the existence of an equilibrium is proved. Section 4 gives two examples of possible applications of our model to waste problems. Finally Section 5 contains some remarks. One of them is concerned with social costs of environmental pollution; another remark with the trade-off relationships between the environmental conservation and the growth rate, and one more with the calculation of equilibrium prices.

2. A VON NEUMANN MODEL WITH BADS AND RECYCLING PROCESSES

Our model is a variant of von Neumann models as generalized by Morishima (1964, 1969), without getting into a super long-run analysis conducted by Morishima (1969), Salvadori (1980, 1988), Franke (1985), Bidard and Franke (1987), and Bidard and Hosoda (1987). There exist m normal goods and h types of waste, some of which may be recyclable with the remaining being either pollutants or simply garbage from production processes or through consumption. We call the latter simply "bads". There are n normal production processes which produce normal goods by means of normal goods with possible discharge of bads. Some of these normal production processes may use some bads as input either intentionally or unintentionally. Then there are k recycling or disposal processes which uses bads and goods as inputs to produce some goods, probably of lower quality, or simply to abate pollutants. Since we are dealing with a von Neumann model, goods of lower quality are, if necessary, to be regarded as distinct from those of higher quality. We assume there exist t types of labour force. Thus, the production, recycling or disposal processes are given as:

$$B \equiv \begin{pmatrix} B_{11}, & B_{12} \\ B_{21}, & B_{22} \end{pmatrix}, \ A \equiv \begin{pmatrix} A_{11}, & A_{12} \\ A_{21}, & A_{22} \end{pmatrix} \ \text{and} \ L \equiv (L_1, L_2).$$

B_{11}: the $m \times n$ nonnegative matrix of "goods" output coefficients in normal production processes;

B_{21}: the $h \times n$ nonnegative matrix of "bads" input coefficients in normal production processes;

B_{12}: the $m \times k$ nonnegative matrix of "goods" output coefficients in recycling processes;

B_{22}: the $h \times k$ nonnegative matrix of "bads" input coefficients in recycling or disposal processes;

A_{11}: the $m \times n$ nonnegative matrix of "goods" input coefficients in normal production processes;

A_{21}: the $h \times n$ nonnegative matrix of "bads" output coefficients in normal production processes;

A_{12}: the $m \times k$ nonnegative matrix of "goods" input coefficients in recycling or disposal processes;

A_{22}: the $h \times k$ nonnegative matrix of "bads" output coefficients in recycling or disposal processes;

L_1: the $t \times n$ nonnegative matrix of labour input coefficients in normal production processes;

L_2: the $t \times k$ nonnegative matrix of labour input coefficients in recycling or disposal processes.

(In the traditional Leontief pollution model, $B_{12} = 0$, $B_{21} = 0$, and in each column of B_{11} and B_{22} there is only one positive entry with the remaining elements being all zero. The requirement that $B_{12} = 0$ implies the absence of recycling processes.) The positive entries of B_{21} mean that normal processes may also abate some pollutants, thus implying a sort of joint production. In fact, when using a von Neumann model, we need not distinguish two types of processes, normal and non-normal. In this paper, however, these are set apart because it is easier to understand the existence of recycling processes which are assumed away in Leontief models thus far proposed.

As is stated in Section 1, the distinction between goods and bads is quite arbitrary, depending the decision of the authority or the government. Thus, bads are substances which are designated by the authority, and when they appear among outputs of a process, the owner of process has to pay some money to the authority, while they are included in the inputs of a process, the owner can get paid by the authority. It is natural to put a toxic gas which damages the health of general public in the list bads. Then, it is not so clear whether carbon dioxide is good or bad. In our model, it is a matter of decision made by the authority.

To consider the price side, we use the following symbols.

p_G: the m-row vector of goods prices;
p_B: the h-row vector of bads prices;
$p \equiv (p_G, p_B)$: the $(m+h)$-row vector of all prices;

Now we assume that the real wage rate of each type of labour force is given: $\omega_1, \omega_2, \ldots, \omega_t$, and we put $\omega \equiv (\omega_1, \omega_2, \ldots, \omega_t)$. These are defined using an m column vector of common standard consumption basket c.

$$\omega_j \equiv \frac{w_j}{p_G \cdot c},$$

where w_j is the money wage rate of labour force of type j. It is assumed that $\omega_j > 0$ for each j. The reader should not misunderstand that workers of different types really consume the same fixed basket, c, at different levels. This vector c is a given column m-vector, and we call those goods whose $c_i > 0$ *consumption goods*. The definition of real wages is a mere matter of statistical procedure adopted by most countries. It is not difficult to allow for different consumption baskets in relation to different types of labour. We have, however, chosen a simpler way. The actual consumption basket per unit of labour force of type j depends upon p and w_j, and written as $d_j(p, w_j)$. Consuming this basket, $d_j(p, w_j)$, yields a h-column vector $q_j(p, w_j)$ of bads.

The price equations or inequalities are described as:

$$p \cdot B \leq (1+r) \cdot p \cdot A^* \equiv (1+r) \cdot p \cdot (A + c^* \cdot \omega \cdot L),$$

where r is the uniform rate of profit, c^* is the $(m+h)$-column vector formed by adding h entries of 0 at the bottom, i.e., $c^* \equiv (c, 0, \ldots, 0)'$ with a prime indicating transposition. It is to be noted that in the cost calculation of each process, the bads emission by spending those profits as well as wages earned by the process on consumption is naturally out of consideration.

Some words are in order about the profits (or margins) accruing from bads inputs and outputs. In the above price equations, the bads inputs are paid by the authority after the production, while bads outputs must be paid to the authority before production, which may not be so awkward in the real situation. Abatement of pollutants is rewarded after it has been carried out, and outputting bads may not be possible before they are reported to the authority and the suitable payments are made. Here we are not dealing with a tradable permit system. On the contrary, each production process should be carried out as reported to the authority. We assume, however, that the cost of administration is negligible.

Next, we consider the quantity side. The symbol g is the uniform rate of growth, the $(m+h) \times t$ matrix D has in its j-th column the actual consumption basket by one unit of employed labour force of type j, i.e., $d_j(p, w_j)$, appended by a h-column vector $q_j(p, w_j)$ of bads which comes out when $d_j(p, w_j)$ is consumed. The $(m+h)$-column vector e means the consumption from the profits and the resulting emission of bads. This e depends on p, ω and the $(n+k)$-column vector of activity levels of processes, x, thus in more detail, e is written as $e(p,\omega,x)$. We assume that e is homogeneous in each element of the vector x. One more $(m+h)$-column vector y stands for the export or import of normal goods and the tolerable limits of bads in their annual flows, which are either decomposed/absorbed by the surrounding environment or sent to foreign countries to be disposed of. It should be noted that we assume y is also a function of x, and again homogeneous of degree one in each element of x, which is different from what is postulated in the Leontief models appearing in the literature. The vector y is after all a flow, and it may not be so unnatural to regard y as dependent on x in a context of a growing or shrinking economy. One more point about y is that each entry can be negative or positive. In the case of goods, a negative entry implies that the commodity should be imported, while a positive entry in the bads means that its excess disposal service can be exported. Now we can write down the quantity inequalities as:

$$B \cdot x \geq (1+g) \cdot (A \cdot x + D \cdot L \cdot x + e(x) + y(x)).$$

We attached only x to e and y just in order to emphasize their dependence on x in a homogeneous manner.

Dually to the case of price side, some words are necessary about the growth factor multiplied upon bads outputs. As is explained in the above for the price equations, bads inputs should be reported to the authority before production in the stage of planning, while bads inputs are thought to be disposed of at the end of production period. Then the output equations are not so problematic either.

We define an equilibrium as a four-tuple (x,p,r,g) such that

$$\begin{cases} p \cdot B \leq (1+r) \cdot p \cdot A^*, & (1) \\ B \cdot x \geq (1+g) \cdot (A \cdot x + D \cdot L \cdot x + e(x) + y(x)), & (2) \\ p \cdot B \cdot x = (1+r) \cdot p \cdot A^* \cdot x, & (3) \\ p \cdot B \cdot x = (1+g) \cdot (p \cdot A \cdot x + p \cdot D \cdot L \cdot x + p \cdot e(x) + p \cdot y(x)), & (4) \\ p \cdot B \cdot x > 0, \text{ and thus } (1+r) > 0 \text{ and } (1+g) > 0. & (5) \end{cases}$$

Eqs.(3) and (4) are requiring the rule of profitability and that of free goods respectively as named by Morishima (1969). It is reminded again that the real wage rate vector ω is given and fixed.

3. A PROOF OF EXISTENCE OF AN EQUILIBRIUM

We can prove the existence of an equilibrium in a way similar to that in von

Neumann (1945-46). First, consider two simplexes,

$$S^{m+h} \equiv \{p \,|\, p \in R_{+}^{m+h}, \sum_{i=1}^{m+h} p_i = 1\} \text{ and } S^{n+k} \equiv \{x \,|\, x \in R_{+}^{n+k}, \sum_{j=1}^{n+k} x_j = 1\}.$$

We assume that the relevant functions, $d_j(p^*, x^*), q_j(p^*, x^*)$, and $e(p, \omega, x)$, are all continuous on the set $Z \equiv S^{m+h} \times S^{n+k}$.

Take an arbitrary $z \equiv (p, x) \in Z$, and consider the following map $T((p, x))$ from Z into itself. For a given pair of (p, x), we can determine $r^{\circ}(z)$ so that the inequality (1) holds with at least one strict equality holding. (When this $r^{\circ}(z)$ is greater than $\overline{r}$, or cannot be finite, i.e.e, when the LHS of (1) is positive while the RHS is zero, we put $r^{\circ}(z) = \overline{r}$, where $\overline{r}$ is a sufficiently large positive scalar, and more precise explanation is given below.) In the same way, we can find out $g^{\circ}(z)$ so that the inequality (2) holds with at least one strict equality holding. When a strict inequality holds for the j-th process in (1), we transform its activity level as

$$T : x_j \to (\frac{1}{1 + \delta_j}) \cdot x_j,$$

where

$$\delta_j \equiv ((1 + r) \cdot p \cdot A^* - p \cdot B)_j \, ,$$

that is, the 'losses' made by operating the j-th process at a unit level under the price vector p. There should be no worry about x_j becoming negative because the above adjustment is made on the proportional scale. On the other hand, when a strict inequality holds for the i-th commodity in (2), we transform its price as

$$T : p_i \to (\frac{1}{1 + \epsilon_i}) \cdot p_i,$$

where

$$\epsilon_i \equiv (B \cdot x - (1 + g) \cdot (A \cdot x + D \cdot L \cdot x + e(x) + y(x)))_i,$$

that is, the 'excess' supply of commodity i by the activity vector x.

When some elements of x or p are really decreased in the above manner, in order to keep the vectors, x and p, in the respective simplexes, we increase, by a uniform size, their elements which correspond to the strict equalities on the other side, i.e., to change activity levels based on the price side, and price levels upon the quantity side. It is to be noted that there is at least one strict equality on both sides, which is clear by the way $r^\circ(z)$ and $g^\circ(z)$ are defined. More precisely in the case of the adjustment of x_{j*} for which process j^* an equality holds in (1),

$$T : x_{j*} \to x_{j*} + \frac{\sum_j (1 - 1/(1+\delta_j)) \cdot x_j}{\alpha},$$

where α is the number of equalities in (1) and the summation $\sum_j$ is over those indexes of processes for which a strict inequality is observed in (1).

The above transformation T of z is a continuous one from Z to Z. Then thanks to Brouwer fixed point theorem, there exists at least one fixed point (p^*, x^*) such that

$$(p^*, x^*) = T(\,(p^*, x^*)).$$

By the way of construction it is evident that this fixed point satisfies the equilibrium conditions (1)-(4) above.

Now we make three more basic assumptions, which are set out here so that these assumptions get more natural and less restrictive than those made in von Neumann models proposed thus far in order to realize the equilibrium property (5).

Assumption A1. Every row of B has at least one positive entry.

Assumption A2. Let $F \equiv \{z \in Z \,|\, \text{all prices of consumption goods are zero}\}$. There is a neighbourhood Ψ of F, in which, when we determine $g^\circ(z)$ above, at least one constraint of consumption goods is binding, while at least one constraint for non-consumption goods or bads is not binding.

Assumption A3. There exists at least one consumption commodity in our model, and each process uses at least one type of labour.

In short, A1 means that every commodity can be produced by at least one process, therefore $r^\circ(z)$ above is positive for any z. (The positive scalar $\overline{r}$ above is defined as the maximum value $r^\circ(z)$ in the set $Z - \Psi$, where $r^\circ(z)$ is finite because $p \cdot A^*$ in (1) is a strictly positive vector.) Our assumption A2 says there is no fixed point in Ψ or near F. In other words, at a fixed point, at least one consumption commodity has a positive price. And so by A3, $p^* \cdot c^* \cdot \omega \cdot L \cdot x^* > 0$ at a fixed point. This in turn implies $p^* \cdot A^* \cdot x^* > 0$. Hence the requirement (5) in an equilibrium follows. This ends our proof of the existence of an equilibrium. □

When the reader wishes to set the vector $y(x)$ as a parameter $\overline{y}$, it can be handled by defining $y(x) \equiv \overline{y} \cdot u \cdot x$, where u is the row $(n+k)$-vector whose entries all unity. After finding out a fixed point as in the above proof, we change the scale of x so that $u \cdot x = 1$ follows. This is, however, a snapshot of our growing or contracting economy.

4. TWO POSSIBLE APPLICATIONS

In this section, we suggest two simple examples of an application of our model. First an application to the effect of household garbage classifying system is discussed. Then, one more suggestion is made concerning a possible application to the effect of charging for disposal of household garbage. These two examples are related to the choice of techniques, the lack of which in most Leontief models is criticized in Lager (1998).

It is often said about household dust that "when mixed up, rubbish, but when sorted out, resources". In our model, mixed up garbage to be disposed of by burning out is regarded as one independent commodity with a negative price. And classifying or separating various types of mixed up garbage into useful as well as useless items can be regarded as independent processes, when this sort of activities are made by a firm. Now let us suppose that people are persuaded or educated to perform these activities within their house so that the costs on firms of classification are alleviated. Then, a wider range of processes become profitable, making some processes so far unused more profitable than those in use because of changes in prices, thus inviting more entries to the related industries.

In our model, consumption based on wages of one unit of labour type j produces $q_j(p, w_j)$ of bads. Thus, we can take into consideration two cases equally: one in which people have to pay for the disposal of those bads, and the other where people need not. In the former case, the quantity $q_j(p, w_j)$ of bads will be decreased if the charge for disposal gets higher. The charge may be related to the prices of bads p_B, and collected by the local governments, then used as subsidies to those who operate the relevant processes. This again enlarges the set of viable processes which abate pollutants. Such a causal flow can be analyzed using our model, process by process and commodity by commodity.

5. SOME REMARKS

Our model might not seem to have anything to do with social costs. Indeed it does. Suppose that polluted exhausted gas causes a kind of asthma in local people, i.e., a process produces polluted gas as well as people suffering from asthma, and that a set of medical services, as a process, treat a person with asthma, and cure the disease of a patient. When falling ill and recovery by medical treatment is repeated among people, the whole medical costs should be included in the social costs of this asthma caused by polluted gas. (Part of lost income of patients may also be added depending on respective situations.) In a medical process, an output in B_{22} is a particular asthma on patients, not patients themselves, and the price calculated for asthma on a patient does embody all the medical costs incurred. Next suppose that a new process is introduced to remove all the polluted gas and asthma no longer attacks on local people. In the new equilibrium, the price of removed gas represents the social cost of producing polluted gas. In reality, however, it is often impossible for people, once caught by a deadly illness caused by a pollutant, to recover to an original healthy state. Then, the medical costs which are required to drive patients merely out of bed represent a gross underestimate of social costs.

The reader may think that our model still carries a basic weak point of linear models. That is, when $y(x)$ has negative entries, they are multiplied by the growth factor annually, and will soon surpass the tolerable limits. On the contrary, our model can be used to check the possibility of realizing nonnegative vector $y(x)$ with alternative processes taken into consideration. One problem, however, is that we implicitly assume that labour force of each type is abundantly available, in other words, chronic unemployment for workers of any type.

In Fujimoto (1974), the trade-off relationship between the uniform growth rate and the level of tolerable limits of bads is presented for a Leontief model with pollutants. In our framework, if the inequality (2) has an optimality described by

$$\max_{x} \; g \quad \text{subject to } B \cdot x \geq (1+g) \cdot (A \cdot x + D \cdot L \cdot x + e(x) + \overline{y}),$$

with $\overline{y}$ as a parameter vector at an equilibrium, then, as is done in Fujimoto (1975), a severer regulation by the authority of pollutants expressed by smaller(in terms of modulus) negative values in the bads of $\overline{y}$ will bring down the growth rate g. Our conjecture is that we may be able to prove the existence of an equilibrium using this maximization problem, and obtain an easier step to the trade-off relationship.

Global stability of the price system based on the full-cost principle was also proved in Fujimoto (1974) for a model of Leontief type. It is, however, not easy to formulate, using prices only, how prices are adjusted when joint products are allowed for. And yet, in models where a generalized nonsubstitution theorem (see Fujimoto et al.(2003)) is valid, the stability can be established in the same way as in Fujimoto (1974). After all, prices and outputs are adjusted simultaneously, and we do not know beforehand which goods will be free and which processes will be making losses and thus out of use. (Because of this, one cannot use the Branin-Smale global Newton method (Branin (1972) and Smale (1976)), or one of its homotopy variants, to find out an equilibrium numerically, and to show its path stability. Besides, those algorithms are not, unfortunately, real adjustment processes.) In practice, it is worth while to examine whether the transformation explained in Section 3 brings prices and outputs near to equilibrium levels when it is applied repeatedly with an adjustment factor a such that $0 < a \leq 1$, i.e.,

$$T : x_j \to (1-a)x_j + a(\frac{1}{1+\delta_j}) \cdot x_j \;\text{ and }\; T : p_i \to (1-a)p_i + a(\frac{1}{1+\epsilon_i}) \cdot p_i$$

with necessary modifications on those prices and outputs increased. A possibly still more stable averaging process is presented in Fujimoto (1995).

It is easy to generalize our model so that each coefficient in B and A can depend on output composition x. All we have to assume is that each coefficient is continuous and homogeneous of degree zero with respect to x. Our existence proof in Section 3 is valid as it is now.

Eicher and Pethig (2005) have presented a grand ecosystem model of environment, and our model here can be an improvement of the part of their production sphere. Here in this note, we have merely given an existence proof. The model, however, may be a starting point for various extensions and theoretical analyses thereof, as we have given examples, albeit simple and intuitive, in Section 4.

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