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Cointegration and Multiplicative Seasonality: Application to CPI Inflation in Japan

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Abstract

This note aims to conduct an empirical analysis of CPI inflation in Japan using a cointegrated vector autoregressive model with partial short-run dynamics. It is demonstrated that the model is able to describe short-run dynamics subject to multiplicative seasonality better than the standard vector autoregression.

KEY WORDS: Cointegration, Vector Autoregressive Models, Partial Short-Run Dynamics, Multiplicative Seasonality.

1 Introduction

This note aims to perform an empirical analysis of CPI inflation in Japan using a cointegrated vector autoregressive (VAR) model with partial short-run dynamics. The model belongs to a class of the cointegrated VAR models with adjusted short-run dynamics, introduced by Kurita and Nielsen (2007). The model is able to describe complicated short-run dynamics associated with seasonality in a more flexible way than the standard VAR model. Yet the likelihood analysis is based on the conventional reduced rank regression as in Johansen (1988, 1996), and the use of the standard asymptotic tables is therefore justified.

First, consider a vector autoregression of order k and dimension p for a time series, $X_{-k+1}, ..., X_0, X_1, ..., X_T$, given by the equation

$$\triangle X_t = (\Pi, \Pi_c) \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \triangle X_{t-i} + \varepsilon_t, \quad \text{for } t = 1, ..., T, \tag{1}$$

where the innovations $\varepsilon_1, ..., \varepsilon_T$ have independent and identical normal $N(0, \Omega)$ distributions, and the starting values $X_{-k+1}, ..., X_0$ are conditioned upon. The parameters Π, Γ_i, $\Omega \in \mathbf{R}^{p \times p}$ and $\Pi_c \in \mathbf{R}^p$ vary freely and Ω is positive definite. Following Johansen (1992a) and Juselius (2006, Section 4.2), the autoregressive model can then be reparameterised as

$$\triangle^2 X_t = (\Pi, \Pi_c) \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} - \Gamma \triangle X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \triangle^2 X_{t-i} + \varepsilon_t, \tag{2}$$

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where $\Gamma = I - \sum_{i=2}^{k-1} \Gamma_i$ and $\Psi_i = -\sum_{j=i+1}^{k-1} \Gamma_j$. Note that model (2) is equivalent to (1). There is a one-one mapping between the parameters of the two models, and the likelihood functions are equivalent since the innovation ε_t is not affected by the reparameterisation. The hypothesis of reduced cointegrating rank is then given by

$$\mathsf{H(r)}: \quad rank\,(\Pi, \Pi_c) \leq r \text{ or } (\Pi, \Pi_c) = \alpha(\beta', \gamma'),$$

where $\alpha, \beta \in \mathbf{R}^{p \times r}$ and $\gamma' \in \mathbf{R}^r$. According to the Granger-Johansen representation theorem (see Johansen, 1996, Theorem 4.2), the cointegrated relation $\beta' X_{t-1}$ determines the *long-run dynamics* of the model. Kurita and Nielsen (2007) refer to α and Γ as the parameters for *medium-run dynamics* as they describe how the process adjusts to changes in $\beta' X_{t-1}$ and $\triangle X_{t-1}$, respectively. The parameters $\Psi_1, ...\Psi_{k-2}$ are defined as those for the *short-run dynamics* of the model in that they are irrelevant to the evolution of the common stochastic trends.

We are in a position to consider a class of vector autoregressive models with adjusted short-run dynamics. The class of the models is based on (2) under $\mathsf{H(r)}$ and can generally be expressed as follows:

$$\triangle^2 X_t = \alpha(\beta', \gamma') \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} - \Gamma \triangle X_{t-1} + \Phi V_t + \varepsilon_t, \tag{3}$$

where $V_t \in \mathbf{R}^q$ is a set of short-run dynamics with parameters $\Phi \in \mathbf{R}^{p \times q}$. Specifying ΦV_t in (3) generates a class of dynamics-adjusted models. Kurita and Nielsen (2007) prove that the likelihood analysis of the class of the models is based on the conventional reduced rank regression as in Johansen (1988, 1996), and thus the standard asymptotic tables can be used.

This note adopts a member of the class, *a cointegrated VAR model with partial short-run dynamics*. In the model ΦV_t is defined so as to capture complicated short-run dynamics, possibly associated with seasonality or some other time series properties. Decompose $X_t = \left(X'_{1,t}, X'_{2,t}\right)'$ and specify ΦV_t as

$$\Phi V_t = \sum_{i=1}^{l-2} \Psi_i \triangle^2 X_{t-i} + \sum_{s=l+m-1}^{k-2} \Psi_s^{(block)} \triangle^2 X_{t-s}, \quad \text{for } 0 \leq m, \tag{4}$$

where $\Psi_s^{(block)}$ represents a matrix with zero elements in the block columns corresponding to $X_{2,t}$, that is,

$$\Psi_s^{(block)} = \begin{pmatrix} \Psi_{s,1} & 0 \\ \Psi_{s,2} & 0 \end{pmatrix}. \tag{5}$$

Note that ΦV_t consists of the second-order differenced terms with different lag lengths, so that $\triangle^2 X_{t-l-m+1}$ does not have to be consecutive to $\triangle^2 X_{t-l+2}$. Also note that (5) can be defined in such a way that zero elements in the block columns correspond to $X_{1,t}$ rather than $X_{2,t}$. The adjusted short-run dynamics term (4) allows us to analyse models with complicated lag structure; for instance, a model for time series subject to multiplicative seasonal effects, as demonstrated below.

In Section 2 an empirical analysis of CPI inflation in Japan is conducted using the cointegrated VAR model with partial short-run dynamics. An overall conclusion is provided in Section 3. All the empirical analyses and graphics in this note use *OxMetrics* / *PcGive* (Doornik and Hendry, 2006) and *PcGets* (Krolzig and Hendry, 2001).

2 Empirical Analysis

This section performs an empirical analysis using CPI inflation and an interest rate differential in Japan. The empirical study demonstrates that a cointegrated VAR model with partial short-run dynamics is useful in addressing residual autocorrelation caused by multiplicative seasonality.

It is often pointed out that a differential between the long and short term interest rates is related to inflation and economic growth (see Ichiue, 2004, *inter alia*). Thus, cointegration analysis is expected to reveal a long-run relationship between the series of annual inflation and an interest rate differential. The annual inflation (π_t) is based on the year-on-year change of the Japanese consumer price index (CPI) in percentage points. The interest rate differential (rd_t) is calculated by taking a difference between the money market yield (the over-night call money rate, percent per annum) and the 10-year government bond yield (percent per annum) in Japan. The series are monthly data and the sample period runs from March 1986 to March 1997. All the data are taken from *International Financial Statistics* (International Monetary Fund).

Both of the series are displayed in Figure 1(a), and their differences are presented in Figure 1(b). In Figure 1(a) their comovement is evident, suggesting the existence of a cointegrating relation between the two variables. Figure 1(b) shows that the sample means of the differenced series appear to be zero, so it seems reasonable to choose the specification of a VAR model with a restricted constant.

Starting with the estimation of a general unrestricted model with lag length 5, F tetsts for model reduction suggest that the lag length could be reduced from 5 to 2. However, the residual correlogram for the VAR(2) model shows some evidence of serial correlation. Figure 1(c) presents the residual correlogram for the π_t equation together with confidence intervals reported by *PcGets*, and residual autocorrelations corresponding to lag lengths 10 and 12 appear to be significant, and the autocorrelation for the 12th lag seems to be too strong to be disregarded. The 12th-order serial autocorrelation test in the VAR(2) model is indeed significant in the equation for π_t: $\mathrm{F}_{ar}(12,116) = 2.12[0.02]^*$, where $\mathrm{F}_{ar}(\mathrm{k},\cdot)$ is a test for kth order serial correlation reported as an F statistic (see Godfrey, 1978; Nielsen, 2007). If the 12th-order autocorrelation test is performed by excluding lagged residuals from order 1 to 11, then the test is highly significant ($\mathrm{F}_{ar}(1,127) = 16.46[0.0001]^{**}$). Judging from the lag orders, the residuals of the inflation equation seem to be subject to some seasonal effects, although the inflation is measured by the year-on-year change of the CPI and considered to be free from seasonality.

These findings indicate that the annual inflation could be described by a class of multiplicative seasonal models (see Box, Jenkins and Reinsel, 1994, Ch.9). Let us consider a univariate example as follows:

$$x_t - x_{t-12} = y_t \quad \text{and} \quad y_t = y_{t-1} + u_t,$$

where

$$u_t = \phi_1 u_{t-10} - \phi_2 u_{t-12} + \varepsilon_t, \quad for \quad \phi_1 > 0 \text{ and } \phi_2 > 0, \tag{6}$$

and the roots of the characteristic polynomial for (6) are assumed to lie outside the unit circle. Note that the variable y_t may correspond to π_t in the empirical model. Also note that the minus sign is put in front of ϕ_2, since the correlogram for the 12th lag takes a negative value in Figure 1(c). The use of the lag operator L leads to the following

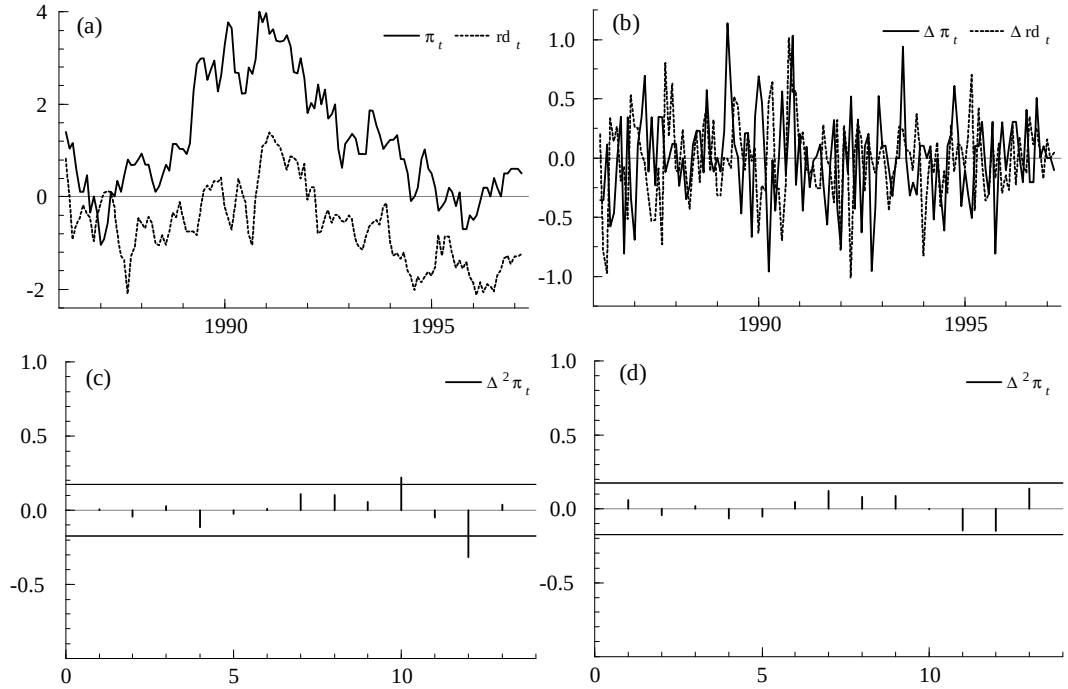

Figure 1: Japanese Inflation, Interest Rate Differential and Residual Correlogram

multiplicative seasonal model:

$$(1-L)\left(1-\phi_1 L^{10}+\phi_2 L^{12}\right) y_t = \varepsilon_t. \tag{7}$$

Multiplying out and using the difference operator $\triangle$, (7) is expressed as

$$\triangle y_t = \phi_1 \triangle y_{t-10} - \phi_2 \triangle y_{t-12} + \varepsilon_t, \tag{8}$$

which suggests that the lagged values y_{t-13} and y_{t-11}, in addition to y_{t-12} and y_{t-10}, contain useful information for the description of y_t. If it turns out that $\phi_1 = \phi_2 = \phi$, then (8) can be written as

$$\triangle^2 y_t = -\triangle y_{t-1} + \phi \triangle^2 y_{t-10} + \phi \triangle^2 y_{t-11} + \varepsilon_t, \tag{9}$$

which can be nested in a model with partially reduced short-run dynamics given by (4).

Thus we define $X_t = (\pi_t, rd_t)'$ and specify a dynamics-adjusted model as follows:

$$\triangle^2 X_t = (\Pi, \Pi_c)\begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} - \Gamma \triangle X_{t-1} + \Psi_{10}^{(block)} \triangle^2 X_{t-10} + \Psi_{11}^{(block)} \triangle^2 X_{t-11} + \varepsilon_t \tag{10}$$

where

$$\Psi_s^{(block)} = \begin{pmatrix} \Psi_{s,1} & 0 \\ \Psi_{s,2} & 0 \end{pmatrix} \quad \text{for } s = 10, 11.$$

Model (10) is a VAR(2) model with partial short-run dynamics $\triangle^2 \pi_{t-10}$ and $\triangle^2 \pi_{t-11}$ and referred to as DAVAR(2) in this section. The residual correlogram for the π_t equation in

(10) is presented in Figure 1(d), in which no evidence of residual autocorrelation is found. Table 1 presents a battery of diagnostic test statistics including the autocorrelation tests. In the table, $\mathrm{F}_{arch}(\mathrm{k},\cdot)$ is a test for kth order ARCH (see Engle, 1982), while $\chi^2_{nd}(\cdot)$ is a test for residual normality (see Doornik and Hansen, 1994). The overall test results indicate that no problem is found in the residuals of the model. DAVAR(2) is therefore considered to be a satisfactory approximation to the underlying data generating mechanism subject to multiplicative seasonality.

	$\mathrm{F}_{ar}(1,125)$	$\mathrm{F}_{ar}(12,114)$	$\mathrm{F}_{arch}(1,124)$	$\mathrm{F}_{arch}(12,102)$	$\chi^2_{nd}(2)$
π_t	3.54[0.06]	1.12[0.35]	0.48[0.49]	0.67[0.77]	0.69[0.71]
rd_t	0.88[0.35]	0.90[0.54]	1.76[0.19]	1.33[0.21]	0.60[0.74]
	$\mathrm{F}_{ar}(4,246)$	$\mathrm{F}_{ar}(48,202)$	$\chi^2_{nd}(4)$		
System	1.12[0.35]	1.07[0.36]	1.27[0.87]		

Note. Figures in brackets are p-values.

Table 1: Mis-Specification Tests for DAVAR(2)

Next, $I(1)$ cointegration analysis is performed using DAVAR(2). For the sake of comparison, we also introduce a conventional VAR model with a number of lagged values, which aims to capture the seasonality remaining in the residuals. The starting model is chosen to be a VAR(15) model, and model reduction continues until the first rejection is observed. Reduction from a VAR(13) model to a VAR(12) model is rejected with the statistic $\mathrm{F}(4,210) = 4.108[0.003]^{**}$, which suggests that π_{t-13} is highly significant in the system. This finding is consistent with the feature of model (8). As expected, autocorrelation tests indicate that the VAR(13) system is also free from serial correlation in the residuals. It is true that the VAR(13) model is a rather large system, but this type of model specification tends to be adopted in practice if the residual autocorrelation cannot be removed by changing variables or sample periods.

Table 2 presents $\log LR$ tests for cointegrating rank of the two autocorrelation-free models. The tests provide evidence of one cointegration rank in DAVAR(2), whereas no cointegration is found in VAR(13). A number of insignificant or slightly significant lagged terms (*i.e.* π_{t-i} and rd_{t-i}, $i = 3, ..., 9$) are included in VAR(13) and the model is considered to be extremely over-specified. It seems that a number of nuisance parameters have affected the test statistic, thereby leading to the finding of no cointegration. In contrast, DAVAR(2) supports the rank of one, consistent with the conjecture based on Figure 1(a), and also addresses the autocorrelation problem stemming from unadjusted

	$DAVAR(2)$	$VAR(13)$
$-2\log Q\left(H(0)\vert H(2)\right)$	$24.55[0.01]^{*}$	9.90[0.65]
$-2\log Q\left(H(1)\vert H(2)\right)$	3.13[0.57]	2.42[0.70]

Note. Figures in brackets are p-values.

Table 2: Choice of Cointegrating Rank in the Two Autocorrelation-Free Models

seasonality.

We now focus on DAVAR(2) and the cointegrating vector is normalised with respect to the interest rate differential, and the inflation and restricted constant terms are judged to be significant in the space β. Both series are then mapped to $I(0)$ space by differencing and using the normalised cointegrating relation, which is interpreted as an equilibrium correction term in the $I(0)$ representation. The model reduction leads to the following parsimonious representation:

$$\begin{aligned}\triangle^2\widehat{\pi}_t &= \underset{(0.08)}{-1.02} \triangle \pi_{t-1} + \underset{(\mathbf{0.06})}{\mathbf{0.26}} \triangle^2 \pi_{t-10} + \underset{(\mathbf{0.06})}{\mathbf{0.27}} \triangle^2 \pi_{t-11}, \\ \triangle^2\widehat{rd}_t &= \underset{(0.08)}{-0.87} \triangle rd_{t-1} - \underset{(0.05)}{0.22} ecm_{t-1},\end{aligned}$$

where

$$ecm_t = rd_t - 0.56\pi_t + 1.35.$$

As expected, the adjusted short-run dynamics are highly significant in the equation for $\triangle^2\pi_t$ (presented in bold), playing a crucial role in removing the seasonal effects found in the residuals. It turns out that the equilibrium correction term and interest rate differentials are all insignificant in the $\triangle^2\pi_t$ equation; thus, the annual inflation is judged to be strongly exogenous for the parameters of interest in this small econometric system (see Engle, Hendry and Richard, 1983, for strong exogeneity). The estimated variance-covariance matrix has diagonal structure, so that a contemporaneous regressor $\triangle^2\pi_t$ in the equation for $\triangle^2 rd_t$ is insignificant and therefore excluded from the equation.

Note that the equation for $\triangle^2\pi_t$ appears to be of the same structure as (9). The restriction of -1 on the coefficient for $\triangle\pi_{t-1}$, together with the restriction of the same coefficient for $\triangle^2\pi_{t-10}$ and $\triangle^2\pi_{t-11}$, are accepted with the test statistic $\chi^2(2) = 0.13[0.94]$.

3 Conclusion

This note has conducted an empirical analysis of CPI inflation in Japan using a cointegrated VAR model with partial short-run dynamics. The model belongs to a class of the cointegrated VAR models with adjusted short-run dynamics, explored by Kurita and Nielsen (2007). It has been demonstrated that the model is able to describe short-run dynamics subject to multiplicative seasonality in a more flexible way than a standard VAR model. The empirical study provides an encouraging result in terms of the applicability of the proposed dynamics-adjusted model to econometric analysis.

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