Common Stochastic Trends and Long-Run Price Leadership in the US Gasoline Market

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Abstract

This paper investigates long-run price leadership in the US gasoline market. Cointegration and common stochastic trends are reviewed, and the concept of long-run price leadership is discussed in the framework of cointegrated vector autoregression. Weekly data for gasoline prices in three areas of the US (New York Harbor, Los Angeles and Gulf Coast) are then analyzed using a cointegrated vector autoregressive model. It is demonstrated that the price set in Gulf Coast plays the role of long-run leadership in the gasoline market, influencing the price determination in the remaining locations.

KEY WORDS: Common Trends, Cointegration, Weak Exogeneity, Long-Run Price Leadership.

1 Introduction

The objective of this paper is to investigate long-run price leadership in the US gasoline market using a cointegrated vector autoregressive (VAR) model. The introductory section briefly explains cointegration, common stochastic trends and long-run price leadership, then proceeding to a description of an empirical finding in the gasoline data.

Economic time series data often exhibit trending behavior and need to be treated as integrated processes rather than stationary. Cointegration introduced by Granger (1983) therefore plays an important role in time series econometrics. A cointegrated vector autoregressive (VAR) model explored by Johansen (1988, 1996) has become one of the major econometric tools for economists (see Juselius, 2006; Kurita, 2007, inter alia, for empirical research using cointegrated VAR models). The presence of cointegration in a VAR system implies that common stochastic trends are incorporated in the system. Cointegration and common stochastic trends are therefore regarded as two sides of the

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same coin (see Johansen, 1996, Chapter 3). The presence of \( r \) cointegrating relations in an \( n \)-dimensional \( I(1) \) cointegrated VAR system (for \( r < n \)) means that the system is driven by \( n - r \) common stochastic trends.

The idea of long-run price leadership, which is demonstrated by Hunter and Burke (2007), is closely related to the presence of a single common trend in a cointegrated VAR system. Consider a group of prices interacting each other, which are well fitted in a cointegrated VAR system. The long-run price leadership corresponds to a situation where one of the prices works as a long-run driving force in the price determination. The source of the driving force is a single common stochastic trend, which is generated from accumulated innovations for the leading price. Thus, the adjustment space for the leading price in the VAR system is subject to zero restrictions such that the equation for the price does not include any equilibrium correction terms. The zero restrictions on the adjustment space imply that the leading price is weakly exogenous for a set of parameters of interest (see Engle, Hendry and Richard, 1983; Section 2 below for weak exogeneity).

This paper analyses weekly data for gasoline prices in three areas of the US: New York Harbor, Los Angeles and US Gulf Coast. A trivariate cointegration analysis demonstrates that the price set in Gulf Coast plays the role of long-run leadership in the gasoline market, affecting the price determination in the other two areas in the US. The finding of this paper gives weight to the validity of Hunter and Burke (2007)’s approach to the description of long-run price determination. Similar types of data are analyzed by Hendry and Juselius (2001), Kurita and Nielsen (2007), but both of them use a bivariate system, focusing on objectives different from that pursued in this paper.

The organization of this paper is as follows. Section 2 reviews common stochastic trends and weak exogeneity in a cointegrated VAR system, and Section 3 then explains the idea of long-run price leadership and discusses its relation to stochastic trends and weak exogeneity. In Section 4 weekly data for US gasoline prices are analyzed in order to investigate whether long-run price leadership exists in the gasoline market. All the empirical analysis and graphics in this paper use \textit{OxMetrics / PcGive} (Doornik and Hendry, 2006).

2 Common Stochastic Trends and Weak Exogeneity

This section presents the definition of common stochastic trends and the condition for weak exogeneity in the framework of an \( I(1) \) cointegrated VAR model, based on Johansen (1996). Let us consider a general unrestricted VAR(\( k \)) model for an \( n \)-dimensional time series, \( X_{k+1}, \ldots, X_T \), which is given by

\[
\Delta X_t = (\Pi, \Pi_c) \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \quad \text{for } t = 1, \ldots, T, \tag{1}
\]

where the innovations \( \varepsilon_1, \ldots, \varepsilon_T \) have independent and identical normal \( N(0, \Omega) \) distributions conditional on the starting values \( X_{-k+1}, \ldots, X_0 \). The parameters \( \Pi, \Gamma_i, \Omega \in \mathbb{R}^{n \times n} \) and \( \Pi_c \in \mathbb{R}^n \) vary freely, and \( \Omega \) is positive definite. Note that the intercept is placed in a possible cointegration space so as to avoid the generation of linear trend in \( X_t \).
In order to conduct \( I(1) \) cointegration analysis based on (1), three regularity conditions need to be introduced. The first condition is that the characteristic roots obey the equation \( |A(z)| = 0 \), where

\[
A(z) = (1 - z) I_p - 
\Pi z - \sum_{i=1}^{k-1} \Gamma_j (1 - z) z^i,
\]

and the roots satisfy \( |z| > 1 \) or \( z = 1 \). This condition ensures that the process is neither explosive nor seasonally cointegrated. The second condition is given by

\[
\text{rank} \left( \Pi, \Pi_e \right) \leq r \quad \text{or} \quad (\Pi, \Pi_c) = (\beta', \gamma'),
\]

(2)

where \( \alpha, \beta \in \mathbb{R}^{n \times r} \) and \( \gamma' \in \mathbb{R}^r \) for \( r < n \). The space spanned by \( \alpha \) is referred to as the adjustment space, while the space spanned by \( \beta \) is called the cointegration space. Condition (2) implies that there are at least \( n - r \) common stochastic trends and cointegration arises when \( r \geq 1 \). The third condition is

\[
\text{rank} \left( \alpha_\perp' \Gamma \beta_\perp \right) = n - r
\]

where \( \Gamma = I_n - \sum_{i=1}^{k-1} \Gamma_i \), and \( \alpha_\perp, \beta_\perp \in \mathbb{R}^{n \times n - r} \) are orthogonal complements such that \( \alpha_\perp \alpha_\perp = 0 \) and \( \beta_\perp \beta_\perp = 0 \) with \( (\alpha, \alpha_\perp) \) and \( (\beta, \beta_\perp) \) being of full rank. The final condition prevents the process from being \( I(2) \) or of higher order. If these three conditions are satisfied, then the solution of (1) has the Granger-Johansen representation as follows:

\[
X_t = C \sum_{i=1}^{t} \varepsilon_i + y_t + \tau_c + A,
\]

where \( C = \beta_\perp (\alpha_\perp' \Gamma \beta_\perp)^{-1} \alpha_\perp' \) and the process \( y_t \) can be given a mean-zero, stationary initial distribution. The parameter \( \tau_c \) satisfies \( \tau_c = (CT - I) \beta' \gamma' \) and \( A \) depends on the initial values such that \( \beta' \tau_c = \gamma' \) and \( \beta' A = 0 \).

The common stochastic trends embedded in \( X_t \) are almost captured by expanding the process in the direction of \( \alpha_\perp' \Gamma \), that is,

\[
\alpha_\perp' \Gamma X_t = \alpha_\perp' \sum_{i=1}^{t} \varepsilon_i + \alpha_\perp' \Gamma y_t + \alpha_\perp' \Gamma \tau_c + \alpha_\perp' \Gamma A,
\]

due to \( \alpha_\perp' \Gamma C = \alpha_\perp' \). The term \( \alpha_\perp' \sum_{i=1}^{t} \varepsilon_i \) represents the \( n - r \) dimensional common stochastic trends, which dominate other terms on the right hand side of the equation. The parameter \( \alpha_\perp \) simply represents the null space of \( \alpha \), so specifying \( \alpha \) implies how the stochastic trends are incorporated in \( X_t \).

Let us move on to the condition for weak exogeneity, which corresponds to a set of zero restrictions on the elements of \( \alpha \). Let the process be decomposed as \( X_t = (Y_t', Z_t')' \), where \( Y_t \) and \( Z_t \) are \( m \) and \( n - m \) dimensional vectors \((n > m)\), respectively. The set of parameters and innovations are correspondingly expressed as

\[
\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} \quad \text{and} \quad \Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yz} \\ \Omega_{zy} & \Omega_{zz} \end{pmatrix}.
\]
Model (1) is then decomposed into a conditional model for $Y_t$ and a marginal model for $Z_t$, that is,

$$
\Delta Y_t = \omega \Delta Z_t + (\alpha_y - \omega \alpha_z) \beta^* X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_{y,i} \Delta X_{t-i} + \tilde{\mu}_y + \tilde{\Phi}_y D_t + \tilde{\varepsilon}_{y,t},
$$

$$
\Delta Z_t = \alpha_z \beta^* X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \mu_z + \Phi_z D_t + \varepsilon_{z,t},
$$

where

$$
\omega = \Omega_{yz} \Omega_{zz}^{-1}, \quad \beta^* = (\beta', \gamma')', \quad X_{t-1}^* = (X_{t-1}', 1)'),
$$

$$
\Gamma_{y,i} = \Gamma_{y,i} - \omega \Gamma_{z,i}, \quad \tilde{\varepsilon}_{y,t} = \varepsilon_{y,t} - \omega \varepsilon_{z,t},
$$

and

$$
\begin{pmatrix}
\tilde{\varepsilon}_{y,t} \\
\varepsilon_{z,t}
\end{pmatrix}
= N \left[
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\Omega_{yy,z} & 0 \\
0 & \Omega_{zz}
\end{pmatrix}
\right].
$$

for $\Omega_{yy,z} = \Omega_{yy} - \omega \Omega_{zy}$. If $\alpha_z = 0$, then the conditional and marginal models are simplified to

$$
\Delta Y_t = \omega \Delta Z_t + \alpha_y \beta^* X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_{y,i} \Delta X_{t-i} + \tilde{\mu}_y + \tilde{\Phi}_y D_t + \tilde{\varepsilon}_{y,t},
$$

(3)

$$
\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \mu_z + \Phi_z D_t + \varepsilon_{z,t},
$$

(4)

and $Z_t$ is then said to be weakly exogenous with respect to a set of parameters of interest,

$$
\alpha_y, \quad \beta^*, \quad \omega, \quad \Gamma_{y,i} \quad \text{and} \quad \Omega_{yy,z}.
$$

(5)

Note that cointegrating relations $\beta^* X_{t-1}^*$ are not embedded in the marginal model (4). If the condition for weak exogeneity, $\alpha_z = 0$, is satisfied, the parameters (5) can then be estimated from the conditional model (3) without loss of information, with no need for the estimation of the marginal model (4). See Engle et al. (1983), Johansen (1992) and Urbain (1992) for further details of weak exogeneity.

Finally, let us consider the case where $r = m$ and $\text{rank}(\Pi, \Pi_c) \leq n - 1$ such that $r = m = n - 1$. The condition for weak exogeneity then corresponds to

$$
\alpha = \begin{pmatrix}
\alpha_y \\
0_{1 \times (n-1)}
\end{pmatrix} \quad \text{and} \quad \alpha_\perp = \begin{pmatrix}
0_{(n-1) \times 1} \\
1
\end{pmatrix}.
$$

(6)

Thus $\alpha_\perp \sum_{i=1}^{t} \varepsilon_i$ is given by

$$
\alpha_\perp \sum_{i=1}^{t} \varepsilon_i = \begin{pmatrix}
0_{1 \times (n-1)} & 1
\end{pmatrix} \sum_{i=1}^{t} \varepsilon_i = \sum_{i=1}^{t} \varepsilon_{z,i}.
$$

(7)

That is, the common stochastic trend is driven by accumulated innovations for the single variable $Z_t$, instead of those for the combination of $Y_t$ and $Z_t$. In this case, $Z_t$ is considered to be solely responsible for the creation of the common stochastic trend or non-stationary behavior in $X_t$. In other words, the set of variables $Y_t$ follows the single variable $Z_t$ in the long-run. This has a significant implication for the understanding of non-competitive price-setting behavior, which is discussed in the next section.
3 Long-Run Price Leadership

Following Hunter and Burke (2007), this section introduces the idea of a long-run price target and discusses its relation to long-run price leadership. Let us define \( X_t \) in the above VAR model as a vector of prices set by firms or sectors, \( X_t = (p_{1,t}, \ldots, p_{n,t})' \), such that all the prices adjust to some equilibrium price target. Suppose that the underlying price target is a weighted average of all the prices. Two \( n \)-dimensional vectors \( a = (a_1, \cdots, a_n)' \) and \( i = (1, \cdots, 1)' \) are then introduced so that the price target can be given by \( p_t^* = a'X_t \) for \( a'i = 1 \). Assuming the presence of cointegration stemming from price adjustment, a discrepancy between the individual price and the target price can be free from non-stationary behavior, i.e.

\[
p_{jt} - p_t^* - E(p_{jt} - p_t^*) \sim I(0), \quad \text{for} \quad j = 1, \ldots, n, \tag{8}
\]

and there are \( n - 1 \) identified cointegrating relationships. Relationship (8) apart form the expectation can also be expressed as follows:

\[
p_{jt} - p_t^* = p_{jt} - a'iX_t = a'(ii_j' - I_n)X_t = a'S_jX_t
\]

where \( i_j \) is an \( n \)-dimensional vector taking 1 for the \( j \)-th entry and 0 otherwise, and \( S_j = (ii_j' - I_n) \). The \( n \times n \) matrix \( S_j \) consists of 0, 1, and \(-1\), and its \( j \)-th row all comprises zeros. Let \( s_{g,h}^j \) be the \((g, h)\) entry of \( S_j \), and \( \sum_{h=1}^n s_{g,h}^j = 0 \) holds by definition, indicating long-run price homogeneity imposed on (8).

The presence of long-run price leadership in (8) corresponds to the case where there is a single weakly exogenous variable with \( a = \alpha_\perp \). Suppose that the weakly exogenous variable is given by the \( n \)-th price in \( X_t \) such that (6) holds with regard to \( \alpha \) and \( \alpha_\perp \). The long-run price target is then given by \( p_{n,t} \), i.e.

\[
p_t^* = \alpha_\perp'X_t = \begin{pmatrix} 0_{1 \times (n-1)} & 1 \end{pmatrix}X_t = p_{n,t}.
\]

Thus the \( n \)-th price plays the role of long-run leadership in the market, being followed by the remaining prices in \( X_t \) in the long-run. The source of the long-run price leadership is accumulated innovations for the single price \( p_{n,t} \), rather than those for the combination of \( p_{1,t}, \ldots, p_{n,t} \), as shown in (7). It follows that \( n - 1 \) cointegrating relations are given by

\[
\beta'X_t = \begin{pmatrix} 1 & 0 & \cdots & \cdots & -1 \\ 0 & 1 & 0 & \cdots & -1 \\ 0 & 0 & \cdots & \cdots & : \\ 0 & 0 & \cdots & 1 & -1 \end{pmatrix} \begin{pmatrix} p_{1,t} \\ p_{2,t} \\ \vdots \\ p_{n,t} \end{pmatrix} = \begin{pmatrix} p_{1,t} - p_{n,t} \\ p_{2,t} - p_{n,t} \\ \vdots \\ p_{n-1,t} - p_{n,t} \end{pmatrix}. \tag{9}
\]

Thus the test for long-run price leadership in \( X_t \) is expressed as a joint test for (6) and (9). The joint test corresponds to a set of restrictions on \( \alpha \) and \( \beta \), and it is known
that the log-likelihood ratio test for such restrictions has a conventional asymptotic $\chi^2$ distribution (see Johansen, 1996, Chapters 7 and 8). Using a cointegrated VAR model, the next section explores whether the long-run price leadership exists or not in the US gasoline market.

4 Long-Run Behavior of Gasoline Prices in the US

This section analyses weekly data for gasoline prices in three areas of the US: New York Harbor, Los Angeles and Gulf Coast. A trivariate cointegration analysis aims to examine whether price leadership exists in the US gasoline market.

Firstly an overview of the data is presented in order to get a glimpse of their time series properties. Figure 1 displays various time series plots of $p_{t}^{NY}$, $p_{t}^{LA}$ and $p_{t}^{GC}$, which are the logs of weekly spot prices for conventional regular gasoline (cents per gallon) in New York Harbor, Los Angeles and Gulf Coast, respectively. The sample period runs from the second week in October 1990 to the final week in December 2004. The number of observations is therefore 743. All the data are taken from the webpage of energy information administration in the US (http://www.eia.doe.gov/).

![Figure 1: An Overview of Gasoline Price Data](image)

In Figure 1 (a) all the series wander around in a similar way, exhibiting random-walk type behavior with no mean-reversion. Thus they should be treated as cointegrated
processes. The similarity observed in their behavior also indicates the presence of price leadership in the gasoline market. Petrochemical is one of the dominant industries in the area of Gulf Coast. Thus it is likely that the price set in Gulf Coast plays the role of long-run leadership in the gasoline market, affecting the price determination in the remaining areas, New York Harbor and Los Angeles.

Figure 1 (b) presents the scatter plots of $p_{NY}^t$ and $p_{GC}^t$, and as expected, there is a clear linear relationship between $p_{NY}^t$ and $p_{GC}^t$. Similarly, Figure 1 (c) displays the scatter plots of $p_{LA}^t$ and $p_{GC}^t$. Both of the series are also highly correlated, although the degree of correlation seems to be weaker than that of $p_{NY}^t$ and $p_{GC}^t$. Figure 1 (d) presents two linear combinations of variables in question, $p_{NY}^t - p_{GC}^t$ and $p_{LA}^t - p_{GC}^t$. In line with Figures 1 (b) and (d), both of the linear combinations exhibit stationary behavior, suggesting the presence of two cointegrated relations in the data.

![Figure 2: Residual Correlograms and Density Functions](image)

Next we move on to an empirical analysis using the gasoline data. The analysis starts with the estimation of a general unrestricted VAR(2) model for $X_t$ defined as follows:

$$X_t = \left( p_{NY}^t, p_{LA}^t, p_{GC}^t \right)' .$$

Residual correlograms and density functions for the VAR(2) model are displayed in Figure 2. The correlograms in Figures 2 (a), (c) and (e) present no evidence for serial correlation in the residuals. Furthermore, Figures 2 (b) and (f) indicate that the assumption of
normality is fairly satisfied with regard to the residuals for $p^N_Y$ and $p^G_C$. However, there is clear evidence for excess kurtosis in the residual density for $p^L_A$ in Figure 2 (d). The presence of excess kurtosis is probably related to autoregressive conditional heteroscedasticity (ARCH) effects in the residuals, as is often the case with market data. With regard to cointegration analysis, simulation studies show that Johansen’s maximum likelihood procedure tends to be affected by the presence of residual autocorrelation; in contrast, it is known that Johansen’s procedure is neither sensitive to excess kurtosis nor to ARCH effects in the residuals (see Reimers, 1992; Cheung and Lai, 1993; Gonzalo, 1994; Gonzalo and Pitarakis, 1998; Rahbek, Hansen and Dennis, 2002). Thus one is justified in proceeding to cointegration analysis with no evidence for residual autocorrelation, in spite of the excess kurtosis in the residual density for $p^L_A$.

<table>
<thead>
<tr>
<th></th>
<th>$r = 0$</th>
<th>$r \leq 1$</th>
<th>$r \leq 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2 \log Q (H (r)</td>
<td>H (p))$</td>
<td>165.89 [0.00]**</td>
<td>78.71 [0.00]**</td>
</tr>
<tr>
<td>$\text{mod (unrestricted)}$</td>
<td>0.98</td>
<td>0.81</td>
<td>0.76</td>
</tr>
<tr>
<td>$\text{mod (r = 1)}$</td>
<td>1.00</td>
<td>0.81</td>
<td>0.76</td>
</tr>
</tbody>
</table>

*Note.* ** denotes significance at the 1% level.

Table 1: Determination of the Cointegrating Rank

Table 1 presents the log-likelihood ratio ($\log LR$) test statistics for cointegrating rank, in addition to the modulus of the six largest roots of the companion matrix. The log $LR$ test in the first panel rejects the null of $r = 0$ and $r = 1$, hence supporting $r = 2$. The second panel provides two types of modulus (denoted mod) of the six largest eigenvalues of the companion matrix, unrestricted and restricted with $r = 2$. These are the reciprocal values of the roots of $A(z)$ discussed in the last section. No eigenvalue over 1.0 suggests that the model does not include any explosive root, and in the restricted case all the eigenvalues apart from the first one appear to be distinct from a unit root. The overall outcome suggests that the regularity conditions given in the previous section are fulfilled, supporting the validity of $I(1)$ cointegration analysis.

The choice of the cointegrating rank ($r = 2$) allows us to test restrictions on the estimates for $\alpha$ and $\beta$. The hypothesis of interest is whether the price set in Gulf Coast plays the role of long-run price leadership. Thus the validity of the restrictions (6) and (9) needs to be investigated. The joint restrictions are imposed to find the restricted estimates $\tilde{\alpha}$ and $\tilde{\beta}$:

$$X_t = \begin{pmatrix} p^N_Y \\ p^L_A \\ p^G_C \\ p^L_A \\ p^N_Y \\ p^G_C \end{pmatrix} : \quad \tilde{\alpha} = \begin{pmatrix} -0.20 \\ 0 \\ -0.16 \\ -1 \end{pmatrix}_{(0.02)} \quad \text{and} \quad \tilde{\beta}^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ -0.11 & -0.03 \end{pmatrix}_{(0.01)(0.004)},$$

where the figures in parentheses are standard errors. Note that $\gamma$ in $\beta^*$ is not subject to any restrictions. The log $LR$ test statistic for the joint restrictions is 9.71 [0.14], in which
the figure in a square bracket is a p-value according to $\chi^2 (6)$. The joint hypothesis is not rejected at the 0.05 level, leading to the conclusion that the price set in Gulf Coast drives the prices in New York Harbor and Los Angeles in the long-run.

5 Conclusion

This paper explored long-run price leadership in the US gasoline market. Cointegration and common stochastic trends were reviewed, and the concept of long-run price leadership was discussed in the framework of cointegrated vector autoregression. Weekly data for gasoline prices in three areas of the US (New York Harbor, Los Angeles and Gulf Coast) were then analyzed using a cointegrated vector autoregressive model. The analysis led to the finding that the price set in Gulf Coast plays the role of long-run leadership in the gasoline market, affecting the price determination in the remaining areas.

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