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Takamitsu Kurita

Faculty of Economics

Fukuoka University

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Center for Advanced Economic Study

Fukuoka University

(CAES)

8-19-1 Nanakuma, Jonan-ku, Fukuoka,
JAPAN 814-0180
+81-92-871-6631(Ex.2219)

Exploring the dynamics of the unemployment rate: A reduced representation of Japanese data

Takamitsu Kurita*
Faculty of Economics, Fukuoka University

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Abstract

This paper aims to obtain a parsimonious econometric model accounting for the dynamics of Japan's unemployment rate in recent years. A cointegrated vector autoregressive model including the rate of unemployment is estimated and then subjected to a series of tests for model specification. It is shown that both wage inflation and terms of trade play important roles in the underlying cointegrating relationship revealed from the data. The study then arrives at a reduced equilibrium correction model for the unemployment rate conditional on a set of weakly exogenous variables. The preferred model and its derivative are judged useful as forecasting devices.

Keywords: Unemployment rate, Cointegrated vector autoregressive model, Model reduction, Equilibrium correction model
JEL Classification Codes: C32, C51, E24

1 Introduction

This paper, using cointegrated vector autoregressive (VAR) methodology, aims to achieve a parsimonious dynamic econometric model of Japan's unemployment rate. A cointegrated VAR system containing the rate of unemployment is estimated and then subjected to a series of tests for the model reduction; various time series characteristics of economic variables in the system are revealed and discussed in the model reduction process. This introductory section reviews the related literature and the recent Japanese economy, and then presents the motivations for empirical exploration. The organisation of this paper is then described.

Unemployment may be seen as one of the most serious economic problems in almost every part of the world. Policy makers often find it necessary to reduce or moderate unemployment by means of economic policies. Understanding the dynamics of unemployment and predicting its future behaviour would be essential for the implementation

*Correspondence to: Faculty of Economics, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka 814-0180, Japan. E-mail: tkurita@fukuoka-u.ac.jp

of effective economic policies. Thus it is important to obtain a time series econometric model accounting for the dynamics of the rate of unemployment; such a model may be utilised as a quantitative device for policy simulation, out-of-sample forecasting, and so forth. In order to build a dynamic econometric model with considerable explanatory power, it is necessary to identify the underlying long-run economic relationships embedded in the data. Literature on macroeconomics, coupled with an overview of the recent Japanese economy, may guide us in the choice of such economic variables as should be jointly analysed with the rate of unemployment.

Phillips (1958) studies an empirical negative correlation between the rate of unemployment and wage inflation in the UK, which results in the so-called Phillips curve; the curve has paved the way for a great deal of empirical and theoretical research, accounting for the interactions between unemployment and inflation in a number of countries and regions. Literature on the Phillips curve is so extensive that it is next to impossible to refer to all of the primary papers here. For the investigation of dynamic relationships between unemployment and wage inflation rates, the following references, at least, should be noted: Tobin (1972); Barro and Grossman (1976); Hamada and Kurosaka (1986); Blanchard and Katz (1999); Marcellino and Mizon (2001); Bårdsen, Eitrheim, Jansen and Nymoen (2005); Russell and Banerjee (2008); Castle and Hendry (2009); and Granger and Jeon (2011).

With regard to the underlying causal relationship between unemployment and wage inflation rates, Phillips (1958) indicates the existence of a causal flow running from unemployment to wage inflation. Both of these variables are, however, determined inside a macroeconomic system; this fact gives rise to the possibility that an impact flow may also run, in some cases, from inflation to unemployment. Fisher (1926) also discusses an empirical relationship between unemployment and price inflation in US time series data, indicating the reverse causal flow, that is, a possible causal effect of inflation on unemployment. It is true that causality is very difficult to argue in the time series context; causality in the sense of Granger (1969) may not necessarily correspond to the underlying true causality. In a cointegrated VAR analysis, however, one can at least discuss the variables that are related to a pushing force in a multivariate time series system; the investigation of such a pushing force is feasible as a result of revealing the variables that are equilibrium correcting, that is, reacting to disequilibrium errors represented by a set of empirical cointegrating combinations. See Juselius (2006, Ch.5) for details of pushing and pulling forces in a cointegrated system. Which is more dominant: employment or wage adjustment in the recent Japanese economy? Are the estimated cointegrating linkages consistent with the Phillips curve or some other interpretable macro relationships? These questions will be answered by the empirical exploration in this paper.

Let us also review some of the characteristics of the Japanese economy and unemployment rate in recent years, which provide motivation for the empirical investigation in this paper. The Japanese economy had suffered from prolonged stagnation since the collapse of an asset-price bubble in the early 1990s. Prices started to decrease gradually in the late 1990s, followed by a deflationary state which continued for several years. Expansionary monetary policy adopted in the deflationary period rendered short-term interest rates effectively zero, bringing about a macroeconomic situation close to a liquidity trap. Reflecting this situation, the unemployment rate started to rise and showed a sharp increase around the end of the 1990s. The unemployment rate reached a peak in 2003 and then gradually decreased as an export-led economic recovery occurred. However, a financial crisis arising in the US triggered off a global economic slump in 2008, giving rise to a sharp increase in Japan's unemployment rate in 2009. A plot of the data of Japan's recent unemployment rate is displayed in the Appendix. Also see Yoshikawa (2007) for an overview of the recent Japanese economy. From the standpoint of labour economists and policy makers, a theory-consistent econometric model that can describe and forecast the dynamics of the unemployment rate would be very useful and informative. This paper pursues such a reliable dynamic econometric model. To the best of the author's knowledge, the present paper is the first empirical study in the literature that is successful in attaining such a parsimonious data-congruent model of the dynamics of Japan's recent unemployment rate.

Let us present a brief review of recent developments in time series econometrics. Macroeconomic time series data, in general, tend to exhibit non-stationary behaviour; therefore they should be perceived as processes integrated of order 1 (denoted as $I(1)$ henceforth) rather than stationary processes. Cointegration, introduced by Granger (1981), allows us to investigate the existence of long-run *stationary* combinations between non-stationary variables. Johansen (1988, 1996) then develops a likelihood-based cointegrated VAR analysis, which plays a key role in the recent advances of time series econometrics. See Juselius (2006) for extensive empirical research using the cointegrated VAR system. The cointegrated VAR analysis is well fitted in general-to-specific modelling methodology. See Hendry (1995) and Campos, Ericsson and Hendry (2005), *inter alia*, for details of the methodology. In the cointegrated VAR analysis, the modelling commences with the investigation of a general unrestricted VAR model, which is subsequently subjected to various tests for the model reduction and specification; empirical illustrations are provided by Hendry and Mizon (1993) and Kurita (2007), *inter alia*. Furthermore, weak exogeneity, introduced by Engle, Hendry and Richard (1983), also plays an important part in econometrics. Weak exogeneity permits us to model a partial or conditional model solely, in place of a full or joint model, in order to conduct efficient statistical inference for a set of parameters of interest. Weak exogeneity is concerned with the identification of pulling and pushing forces in a cointegrated system, thus playing an important part in the analysis of this paper. See Johansen (1992) as well as Urbain (1992) for details of weak exogeneity in a cointegrated system.

The organisation of the rest of this paper is as follows. Section 2 presents a testable hypothesis on the dynamics of Japan's unemployment rate, and Section 3 reviews a likelihood-based cointegrated VAR analysis. Section 4 conducts a comprehensive analysis of Japan's macroeconomic time series data, arriving at a parsimonious econometric model of the unemployment rate. Section 5 then examines the forecasting performance of the preferred parsimonious model. The overall summary and conclusion are provided in Section 6. The empirical analysis and graphics in this paper use *CATS in RATS* (Dennis, Hansen, Johansen and Juselius, 2005) and *PcGive* (Doornik and Hendry, 2007).

2 Unemployment dynamics: a hypothesis

This section discusses hypothetical empirical dynamics of the rate of unemployment in Japan. A model for unemployment dynamics, such as that demonstrated by Barro and Grossman (1976, Ch.5), allows us to conceive that the rate of measured unemployment reflects the underlying excess supply for labour. This reasoning may then justify introducing the following hypothetical dynamic structure of the rate of unemployment:

$$\Delta u_t = -\lambda \left(u_{t-1} - u_{t-1}^*\right) + \xi_t, \tag{1}$$

where u_t is the observed unemployment rate, u_t^* is the potential unemployment rate depending on the underlying excess supply for labour, λ is the scalar adjustment parameter satisfying $0 < \lambda < 1$, and ξ_t is a stationary error process influenced by omitted short-run dynamics. In most cases, an increase in excess supply for labour tends to be reflected in a decrease in working hours or an increase in work sharing, then leading to a rise in the number of unemployed people. Thus, (1) may be seen as a plausible representation of the underlying unemployment rate dynamics, in which the volume of λ indicates how swiftly the adjustment takes place.

If u_t is seen as a non-stationary $I(1)$ process, possibly due to hysteresis (Blanchard and Summers, 1987), equation (1) then indicates the existence of a potential stationary combination, or, $u_t - u_t^* \sim I(0)$. In order to map equation (1) to an estimable equation, it is necessary to give an explicit expression of u_t^*. For this purpose, let us introduce the following linear specification: $u_t^* = \delta' Z_t + c$, where Z_t denotes a l-dimensional vector consisting of macroeconomic variables which reflect the underlying excess supply for labour, while δ is the corresponding l-dimensional parameter vector and c is a constant term. Then, one finds

$$u_t - \delta' Z_t - c \sim I(0), \tag{2}$$

so that the study is now translated into the investigation of a cointegrating relationship embedded in the data set for $(u_t, Z_t')'$. It is therefore important, in the empirical exploration, to seek such restrictions on the estimates for δ as allow us to treat (2) as a meaningful long-run economic relationship. Using the specification for u_t^* above and taking account of a set of unspecified effects of short-run dynamics, we are able to express (1) as an equilibrium correction model (ECM) as follows:

$$\Delta u_t = -\lambda \left(u_{t-1} - \delta' Z_{t-1} - c\right) + \ s.r.d. \ + \ \upsilon_t, \tag{3}$$

where $s.r.d.$ stands for $I(0)$ short-run dynamics consisting of various first-order difference terms and υ_t is a purified innovation process distributed as independent and identical normal.

Next, let us consider a set of candidate variables for Z_t. The excess supply of labour should have a downward influence on wages and prices, thereby giving rise to the phenomenon of creeping inflation or deflation; that is, empirical inflation measures may be seen as proxies for the underlying excess supply of labour. The reasoning permits us to introduce the rates of year-on-year wage and price inflation, denoted as π_t^w and π_t^p respectively, in the set for Z_t. The choice of π_t^w and π_t^p is also deemed consistent with various specifications of the Phillips curve known in the literature. In addition, given the fact that export-led growth in the period of 2002-2007 is accompanied with a decrease in the rate of unemployment, it would be reasonable to include the ratio of export price to import price in the set for Z_t as a measure of relative price competitiveness. See Backus, Kehoe and Kydland (1994) and Romer (2001, Ch.5) for a possible role played by such a price ratio, or terms of trade, in open macroeconomic models. Let us denote p_t^{ex} and p_t^{im} as the logged export and import prices, respectively; the terms of trade, s_t, is then given by $s_t = p_t^{ex} - p_t^{im}$.

In equation (3) Z_t is treated as exogenous or given, but it is possible that Z_t may be a set of endogenous variables, thus being influenced by u_t in a large VAR system encompassing (3). Considering that the primary interest of this paper is to model the dynamics of u_t, it would be preferable for us to be justified in modelling the single-equation ECM, (3), instead of modelling the full system, in which all the variables are perceived as endogenous variables. In order to justify this approach, thus estimating the parameters in (3) without any loss of information, it is important to ensure that the required condition is statistically satisfied in the empirical study. The condition is synonymous with weak exogeneity, which is discussed below in the context of a cointegrated VAR model.

3 A cointegrated VAR model encompassing the ECM

This section introduces a cointegrated VAR(k) model encompassing the ECM, (3), then discussing weak exogeneity and out-of-sample forecasting. The argument in the previous section leads to the introduction of the following set of variables:

$$X_t = (u_t, \pi_t^w, \pi_t^p, s_t)', \tag{4}$$

which can be formulated as a cointegrated VAR(k) model developed by Johansen (1988, 1996). That is,

$$\Delta X_t = \alpha\left(\beta' X_{t-1} + \gamma\right) + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \varepsilon_t, \quad \text{for} \quad t = 1, ..., T, \tag{5}$$

where a sequence of innovations ε_t has independent and identical normal $N(0, \Omega)$ distributions conditional on $X_{-k+1}, ..., X_0$ and D_t is a q-dimensional vector of deterministic terms such as centred seasonal dummy variables. The parameters in (5) vary freely, defined as follows: $\alpha, \beta \in \mathbf{R}^{4\times r}$ for $r < 4$, $\gamma \in \mathbf{R}^{r\times 1}$, $\Gamma_i \in \mathbf{R}^{4\times 4}$, $\Phi \in \mathbf{R}^{4\times q}$, and $\Omega \in \mathbf{R}^{4\times 4}$ is a positive definite matrix. The set of vectors, α, is called adjustment vectors, while (β', γ) is referred to as cointegrating vectors.

The index r, which is called cointegrating rank, is usually unknown to researchers; thus it is often necessary to determine r based on a formal statistical analysis. A log-likelihood ratio ($\log LR$) test statistic is composed of the null hypothesis of r cointegration rank, $\mathsf{H}(r)$, against the alternative hypothesis, $\mathsf{H}(p)$. The statistic's limiting quantiles are provided by Johansen (1996, Ch.15), hence it is possible to conduct a likelihood-based asymptotic inference for the choice of r. Determining r in (5) then enables us to test various restrictions on α, β and γ in order to seek the adjustment structure and cointegrating relationships subject to economic interpretations. Cointegrating relationships, $\beta' X_{t-1}+\gamma$, which represent a set of stationary linear combinations of the variables, act as long-run equilibrium correction mechanisms in (5). It is therefore necessary, in the empirical study, to check whether or not (2) belongs to a class of estimated cointegrating relationships.

Let us also state that an orthogonal complement $\alpha_\perp$ is defined in such a way that $\alpha'_\perp \alpha = 0$ with the matrix $(\alpha, \alpha_\perp)$ being of full rank. As shown by Johansen (1996, Ch.3), a series of shocks $\alpha'_\perp \varepsilon_t$ accumulates to the underlying common stochastic trends, which accounts for the creation of non-stationary trend observed in X_t. It is, therefore, possible to treat $\alpha'_\perp X_t$ as linear combinations of such variables as associated with the underlying pushing force in the system.

Next, let the process be decomposed as $X_t = (Y'_t, Z'_t)'$ for $Y_t \in \mathbf{R}^m$, $Z_t \in \mathbf{R}^n$, $m+n = 4$ and $m \geq r$, with a view to deriving the ECM from the cointegrated VAR system as a partial or conditional data-representation. The parameters and error terms of (5) are, in the corresponding manner, expressed as

$$\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix}, \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix}, \ \Phi = \begin{pmatrix} \Phi_y \\ \Phi_z \end{pmatrix}, \ \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix}, \ \Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yz} \\ \Omega_{zy} & \Omega_{zz} \end{pmatrix}.$$

Suppose that $\alpha_z = 0$ holds, so that (5) is decomposed into a conditional model for Y_t given Z_t and a marginal model for Z_t:

$$\Delta Y_t = \omega \Delta Z_t + \alpha_y \left(\beta' X_{t-1} + \gamma\right) + \sum_{i=1}^{k-1} \widetilde{\Gamma}_{y,i} \Delta X_{t-i} + \widetilde{\Phi}_y D_t + \widetilde{\varepsilon}_{y,t}, \tag{6}$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \Phi_z D_t + \varepsilon_{z,t}, \tag{7}$$

where

$$\omega = \Omega_{yz}\Omega_{zz}^{-1}, \ \widetilde{\Gamma}_{y,i} = \Gamma_{y,i} - \omega\Gamma_{z,i}, \ \widetilde{\Phi}_y = \Phi_y - \omega\Phi_z, \ \widetilde{\varepsilon}_{y,t} = \varepsilon_{y,t} - \omega\varepsilon_{z,t},$$

and

$$\begin{pmatrix} \widetilde{\varepsilon}_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} = N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{yy.z} & 0 \\ 0 & \Omega_{zz} \end{pmatrix}\right],$$

for $\Omega_{yy.z} = \Omega_{yy} - \Omega_{yz}\Omega_{zz}^{-1}\Omega_{zy}$. Note that the marginal model, (7), is free from $\beta^{*\prime} X^*_{t-1}$. If the condition of $\alpha_z = 0$ is fulfilled, Z_t is then weakly exogenous for the parameters of the conditional model, (6), that is,

$$\alpha_y, \ \beta, \ \gamma, \ \omega, \ \widetilde{\Gamma}_{y,1}, ..., \widetilde{\Gamma}_{y,k-1}, \ \widetilde{\Phi}_y, \ \text{and } \Omega_{yy.z}. \tag{8}$$

Furthermore, if it turns out that $Y_t = u_t$ and $Z_t = \left(\pi^w_t, \pi^p_t, s_t\right)$, the parameters in (8) then correspond to those appearing in (3), thus they can be seen as a class of parameters of interest. Hence, the conditional model (6) is interpreted as the ECM given by (3). It is, of course, possible that Y_t may cause Z_t in the sense of Granger (1969) via its short-run dynamics in the marginal model (7), *i.e.* the past values of u_t are likely to affect π^w_t, π^p_t and s_t in (7). However, provided that the condition for weak exogeneity $\alpha_z = 0$ is satisfied, the parameters of interest, (8), can be estimated from equation (6) alone with no loss of information, hence there is no need for the estimation of equation (7). See Johansen (1992) for further details of a partial system and weak exogeneity.

Let us suppose that the cointegrating rank is unity, or $r = 1$, in a situation where $\alpha_z = 0$ holds for $Y_t = u_t$ and $Z_t = (\pi_t^w, \pi_t^p, s_t)$. The orthogonal complement of α is then expressed as $\alpha_\perp = (0, I_3)'$, so that the overall pushing force is simply given by $\varepsilon_{z,t}$ and its related variables exactly correspond to Z_t. Thus, in this case, all the three variables in Z_t are seen as responsible for the creation of the pushing force, which accounts for the stochastic-trending behaviour of Y_t. It is, therefore, of much interest to check whether or not the pair, $r = 1$ and $\alpha_z = 0$, is empirically valid.

In addition, if it turns out that $\Omega_{yz} = 0$ so $\omega = 0$, then the conditional model is further simplified to

$$\Delta Y_t = \alpha_y \left(\beta' X_{t-1} + \gamma\right) + \sum_{i=1}^{k-1} \Gamma_{y,i} \Delta X_{t-i} + \Phi_y D_t + \varepsilon_{y,t}, \tag{9}$$

which can then be used for one-step ahead forecasting; that is, given information at t, equation (9) generates an out-of-sample forecast at $t+1$, or $\Delta\widehat{Y}_{t+1}$. Thus, for the purpose of one-step ahead forecasting using the conditional model, it is important to check if $\omega = 0$ or not in the empirical exploration pursued in this paper.

4 Modelling the unemployment rate

This section, analysing monthly time series data, seeks an empirical model for Japan's unemployment rate in recent years. The selected sample period runs from January in 2000 to December in 2009, denoted as 2000.1 - 2009.12 henceforth. See the Appendix for details of the data and their overview. This section consists of four sub-sections. Section 4.1 examines the underlying cointegrating rank, and Section 4.2 then proceeds to identifying cointegrating relations. Section 4.3 conducts the model reduction so as to attain a parsimonious ECM for Japan's unemployment rate.

4.1 Determining the cointegrating rank

This sub-section examines the validity of an estimated unrestricted VAR model for (4), then moving on to the choice of the cointegrating rank. The unrestricted VAR model is simply seen as a statistical representation of the data, so that its estimated coefficients are not necessarily subject to meaningful economic interpretations. Identifying the cointegrating relations and conducting the model reduction then enables us to seek such interpretations. The VAR model should give a basis for a likelihood-based analysis of cointegration and its residuals hence need to satisfy the condition of normality and temporal independence, as in accordance with ε_t in (5). The lag-length of the VAR model is set three based on F-test statistics on retained regressors. The number of observations effective for estimation is, therefore, 117. A set of centred seasonal dummy variables is incorporated in the VAR model, as in line with (5), in order to capture a seasonal pattern observed in the data.

Single equation tests	u_t	π_t^w	π_t^p	s_t
Autocorr.[F_{ar}(3,90)]	1.03[0.39]	0.90[0.45]	0.99[0.40]	1.66[0.18]
ARCH [F_{arch}(3,87)]	0.90[0.45]	0.39[0.76]	0.14[0.94]	1.41[0.24]
Hetero.[F_{het}(24,68)]	0.75[0.78]	0.53[0.96]	1.03[0.45]	1.51[0.10]
Normality [χ^2_{nd}(2)]	0.61[0.74]	2.36[0.31]	2.14[0.34]	3.27[0.19]

Vector tests			
Autocorr.[F_{ar}(16,263)]	1.19 [0.27]	Hetero.[F_{het}(240,571)]	0.92 [0.77]
- [F_{ar}(48,302)]	1.14 [0.25]	Normality [χ^2_{nd}(8)]	8.58 [0.38]

Note. The figures in the square brackets are p-values.

Table 1: Residual diagnostic tests for the unrestricted VAR model

Table 1 records a battery of residual diagnostic tests for the unrestricted VAR model. Most of the test results are provided in the form $F_j(k, T-l)$, which denotes an approximate F-test against the alternative hypothesis j: kth-order serial correlation (F_{ar}: see Godfrey, 1978, Nielsen, 2006), kth-order ARCH (F_{arch}: see Engle, 1982), heteroscedasticity (F_{het}: see White, 1980). A chi-square test for normality (χ^2_{nd}: see Doornik and Hansen, 2008) is also reported. The diagnostic test statistics are all judged to be insignificant at the 5% level, indicating that the residuals fulfill the condition of independent Gaussian distribution with no temporal dependence; thus the VAR model is seen as a satisfactory representation of the data. The VAR model can be subjected to the subsequent likelihood-based analysis for cointegration and model reduction.

	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
$-2\log Q\,(\mathrm{H}(r)\,\vert\mathrm{H}(p))$	64.76[0.00]**	33.59[0.07]	15.75[0.19]	2.14[0.75]
$-2\log Q^{BC}\,(\mathrm{H}(r)\,\vert\mathrm{H}(p))$	59.21[0.02]*	29.42[0.19]	12.67[0.40]	1.81[0.81]

mod ($r=1$)	1	1	1	0.76	0.69	0.56

Note. The figures in the square brackets are p-values.
$**$ and $*$ denote significance at the 1% and 5% levels, respectively.

Table 2: log LR tests for the cointegration rank

The estimation of the well-formulated VAR model allows us to turn to the issue of choosing the cointegrating rank. Table 2 presents two types of log LR test statistics for the cointegrating rank, together with a set of eigenvalues of a companion matrix of the VAR model. A standard log LR test statistic for the cointegrating rank (see Johansen, 1988, 1996) is denoted as $-2\log Q\,(\mathsf{H}(r)\,\vert\mathsf{H}(p))$, while a Bartlett-corrected log LR test statistic, *i.e.* a log LR test statistic adjusted for the sample size (see Johansen, 2002) is denoted as $-2\log Q^{BC}(\mathsf{H}(r)|\mathsf{H}(p))$. According to the first panel of Table 2, both of the $\log LR$ tests reject the null hypotheses of $r = 0$, but do not reject the remaining hypotheses at the 5% level; hence the results of the tests are in favour of $r = 1$. The second panel provides modulus (denoted as mod) of the six largest eigenvalues of a companion matrix of the VAR model restricted with $r = 1$, or with three unit roots. No eigenvalue greater than 1.0 implies that the model does not include any explosive root. It should also be noted that all the eigenvalues apart from the imposed unit roots are distinct from unity. Judging from these outcomes, the restriction of $r = 1$ seems to be well fitted to the data. Hence, we choose $r = 1$, which paves the way for a further investigation of time series properties of the data.

4.2 Restrictions on the adjustment and cointegrating vectors

We are in a position to examine various joint restrictions on the adjustment and cointegrating vectors in the cointegrated VAR model. Table 3 reports a set of unrestricted estimates for α and (β', γ) normalised for u_t. It should be noted, according to the table, that the estimates for α corresponding to π_t^w and s_t seem to be statistically insignificant; the coefficient of s_t is, in particular, smaller than its standard error, indicating that the coefficient is very likely to be zero. Thus, as reviewed in Section 3, both π_t^w and s_t may be judged to be weakly exogenous for such parameters of interest as (β', γ), in support of the view that these are the core members of the underlying pushing force in the system. Turning to the estimates for β, one finds that a zero restriction may be placed on the coefficient for π_t^p, as the value of its standard error is relatively large. It is, therefore, conceivable that π_t^p may be excluded from the cointegrating relationship.

	u_t	π_t^w	π_t^p	s_t	1
$\widehat{\alpha}'$	-0.19 (0.04)	0.05 (0.12)	0.21 (0.09)	0.002 (0.005)	–
$\left(\widehat{\beta}', \widehat{\gamma}\right)$	1 (–)	0.45 (0.07)	-0.11 (0.11)	-2.41 (0.48)	-4.52 (0.07)

Note. The figures in the parentheses are standard errors.

Table 3: Unrestricted estimates of α and (β', γ)

The results reported in Table 3 encourage the following restrictions: (i) zero restrictions on the elements of α corresponding to π_t^w and s_t, and (ii) a zero restriction on the element of β corresponding to π_t^p. These restrictions being imposed upon α and β, Table 4 reports a set of restricted estimates, coupled with the corresponding $\log LR$ test statistic, which has an asymptotic χ^2 distribution under the null hypothesis. The expression *df* in the table stands for a degree of freedom. According to Table 4, the null hypothesis of the joint restrictions is not rejected at the 5% level, suggesting that both π_t^w and s_t are weakly exogenous for the parameters of interest, while π_t^p plays no significant role in the cointegrating space.

	u_t	π_t^w	π_t^p	s_t	1	$\log LR$
$\widehat{\alpha}'$	-0.20 (0.04)	0 $(-)$	0.20 (0.09)	0 $(-)$	$-$	$1.47[0.69]_{(df=3)}$
$\left(\widehat{\beta}', \widehat{\gamma}\right)$	1 $(-)$	0.44 (0.06)	0 $(-)$	-2.15 (0.35)	-4.48 (0.07)	

Note. The figures in the parentheses are standard errors, while the figure in the square bracket is a p-value.

Table 4: Restrictions on α and (β', γ)

Moreover, the estimate for α corresponding to π_t^p seems to be marginally significant; it may thus be possible to impose a zero restriction on the estimate as well. This additional restriction is placed on α, so that the resultant estimates and $\log LR$ test statistic are recorded in Table 5. The p-value of the $\log LR$ test statistic given in the table is greater than 0.05, hence the hypothesis is not rejected at the 5% level again. The result indicates that π_t^p may also be perceived as weakly exogenous for the parameters of interest. Given

	u_t	π_t^w	π_t^p	s_t	1	$\log LR$
$\widehat{\alpha}'$	-0.20 (0.04)	0 $(-)$	0 $(-)$	0 $(-)$	$-$	$6.77[0.15]_{(df=4)}$
$\left(\widehat{\beta}', \widehat{\gamma}\right)$	1 $(-)$	0.46 (0.07)	0 $(-)$	-2.25 (0.39)	-4.42 (0.07)	

Note. The figures in the parentheses are standard errors, while the figure in the square bracket is a p-value.

Table 5: Specification of α and (β', γ)

the estimates for α in Table 5, a set of variables associated with the pushing force may be given as

$$\widehat{\alpha}'_{\perp} X_t = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_t, \\ \pi_t^w \\ \pi_t^p \\ s_t \end{pmatrix} = \begin{pmatrix} \pi_t^w \\ \pi_t^p \\ s_t \end{pmatrix}.$$

All the variables in X_t except u_t can, thus, account for the generation of the underlying pushing force embedded in the cointegrated VAR system.

Furthermore, Figure 1(a) displays a time series plot of the restricted cointegrating linkage reported in Table 5. According to the figure, the plot exhibits no trending behaviour while its variance seems to be stable over the whole sample period; the feature of the plot indicates stationarity, in support of the validity of the restriction imposed on the cointegrating vector. Figure 1(b) then presents a recursive plot of the $\log LR$ test statistic reported in Table 5. The recursive plot uniformly stays below the 5% critical value,

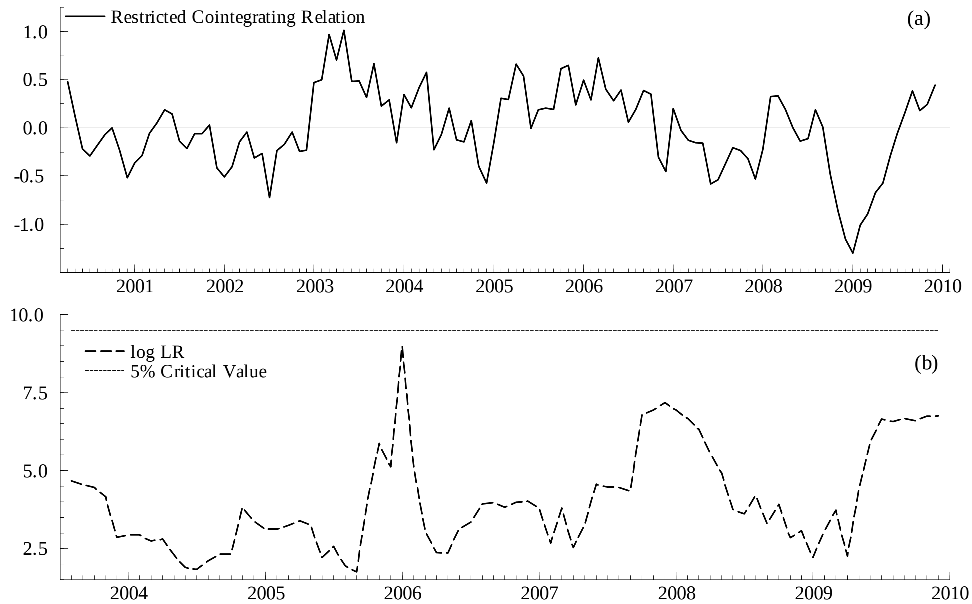


Figure 1: The restricted cointegrating relation and recursive log LR test

suggesting that the set of restrictions examined in Table 5 can be a valid representation of the time-invariant characteristics of the data.

Overall, the analysis has shown that π_t^w, π_t^p and s_t are judged to be jointly weakly exogenous for the parameters of interest such as (β', γ), while the long-run economic relationship is expressed as

$$u_t + 0.46\pi_t^w - 2.25s_t - 4.42 \sim I(0), \tag{10}$$

which is interpreted as an empirical representation of (2). Treating (10) as an equation and solving it for u_t, one finds that u_t tends to negatively synchronise with π_t^w, while it tends to positively synchronise with s_t. The sign of the coefficient for π_t^w is thus consistent with the conventional Phillips curve; π_t^w may be seen as a proxy for the underlying excess supply of labour, the existence of which can bring about an upsurge in u_t. Moreover, s_t may correspond to an empirical measure of relative price competitiveness such that an increase in s_t has a negative impact on export-led growths, thereby increasing u_t. These economic interpretations, therefore, allow us to regard (10) as an empirical representation of the meaningful long-run economic linkage for u_t.

In addition, one should note that u_t exclusively reacts to disequilibrium errors represented by (10) while neither π_t^w nor π_t^p adjusts to (10), as shown in Table 5. The finding indicates that employment adjustment is dominant in the recent Japanese economy as compared with wage adjustment, thus consistent with the phenomenon of flattened Phillips curves (see Kuttner and Robinson, 2010, *inter alia*). Let us consider the implication of this finding in the context of the recent Japanese economy. If wages have approached their lower bounds as a result of prolonged economic stagnation since the beginning of the 1990s, sluggish wage adjustment may take place in response to various shocks hitting the stagnated economy. A decline in the bargaining power of labour unions is also plausible, reflecting the long-lasting stagnation; this can result in an increase in layoffs in the face of an economic downturn. These factors may lead to a situation where shocks on the economy are more likely to be absorbed in the movements of the unemployment rate than those of wages. This conceivable situation seems to be reflected in such dominant employment or quantity adjustment as indicated in the results of Table 5. The revealed adjustment structure, moreover, may be partly explained by regulatory reforms relating to worker dispatch implemented in the sample period of interest. The adjustment and cointegrating coefficients reported in Table 5, thus, seem to be well interpretable as a reflection of Japan's prolonged stagnation after the collapse of the asset-price bubble and some regulatory shifts.

The interpretable long-run relationship, (10), promises to act as an equilibrium correction mechanism in a parsimonious ECM for u_t pursued in the next sub-section.

4.3 A parsimonious ECM for the unemployment rate

This sub-section, based on the empirical results obtained above, seeks a parsimonious ECM for Japan's unemployment rate, which corresponds to equation (3) in Section 2. First, the data are mapped to the $I(0)$ space by differencing and using the long-run relationship given by (10). A single-equation ECM is estimated conditional on all of the first-differenced weakly exogenous variables, that is, $\Delta\pi_t^w$, $\Delta\pi_t^p$ and Δs_t. This model is seen as an empirical expression of the conditional model, (6), discussed in Section 3. Insignificant regressors are then removed from the model step by step; it is found that $\Delta\pi_t^w$, $\Delta\pi_t^p$ and Δs_t are all insignificant, together with its past values, and are therefore deleted from the model.

As a consequence of the model reduction, the author arrives at the following reduced ECM for u_t:

$$\begin{aligned} \widehat{\Delta u_t} \;=\; & -\underset{(0.03)}{0.17}\left(u_{t-1} + 0.46\pi_{t-1}^w - 2.25s_{t-1} - 4.42\right) - \underset{(0.09)}{0.19}\,\Delta u_{t-1} - \underset{(0.09)}{0.17}\,\Delta u_{t-2} \\ & + \textit{seasonal dummy variables}, \\ & \widehat{\sigma} = 0.14,\ F_{ar}(3{,}100) = 1.72[0.17],\ \chi_{nd}^2(2) = 0.45[0.80], \\ & F_{arch}(3{,}97) = 0.16[0.92],\ F_{het}(17{,}85) = 0.44[0.97], \end{aligned} \tag{11}$$

where $\widehat{\sigma}$ is the standard error of the regression. None of the diagnostic tests is significant at the 5% level, therefore suggesting that the parsimonious ECM above is a satisfactory representation of the data. Figure 2(a) displays the actual and fitted values, while Figure 2(b) records the scaled residuals. Furthermore, the residuals' correlogram is displayed in Figure 2(c), while the residuals' density function is presented in Figure 2(d). In line with the diagnostic tests above, none of the graphs provides evidence against the statistical validity of the model. The equilibrium correction term, as expected, is highly significant in (11), indicating the existence of a stable adjustment towards the long-run equilibrium.

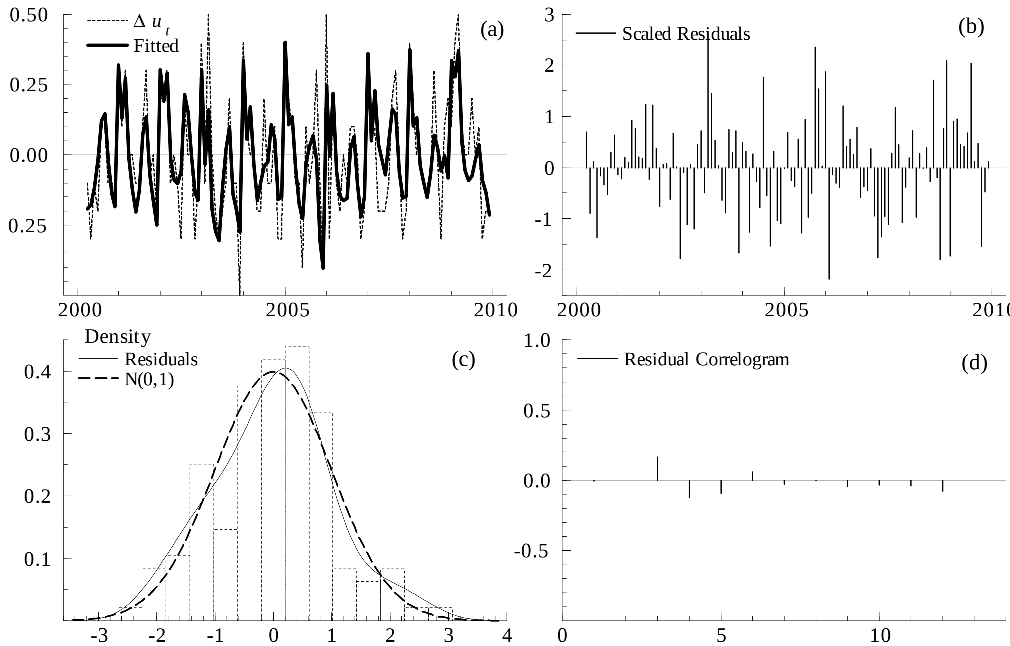


Figure 2: Actual and fitted values, scaled residuals, residual density and correlogram

The overall evidence thus allows us to conclude that the parsimonious ECM, (11), is seen as a data-congruent representation.

It should also be noted that none of the contemporaneous regressors turns out to be significant (11), so that the ECM corresponds to (9) in Section 3 and can be employed for the purpose of one-step ahead forecasting. The final section examines the forecasting performance of the preferred ECM.

5 Forecasting the unemployment rate

Finally, this section investigates the forecasting performance of the preferred ECM above. Let us first discuss the implication of the ECM in terms of forecasting, based on Clements and Hendry (1999, Ch.3). If (11) coincides with the data generation process or at least contains causally-relevant information, the model is then capable of generating better forecasts than non-causal models, based on the criterion of mean square forecast errors. If, however, structural shifts have occurred in its mean-parameters of (11), the subsequent forecasts will amount to a series of serious systematic mis-forecasts. Let us consider this phenomenon in the context of the conditional ECM, (9), discussed in Section 3. Suppose that the mean parameter γ shifts to γ^*; the mechanism existing in the model is to move back to the old equilibrium γ, which is now distinct from the new equilibrium γ^*. That is, the model reacts to the imposed equilibrium, instead of correcting errors from the true equilibrium. This yields a battery of systematic forecast errors, and it can thus happen that a simple non-causal model outperforms the ECM in out-of-sample forecast comparisons.

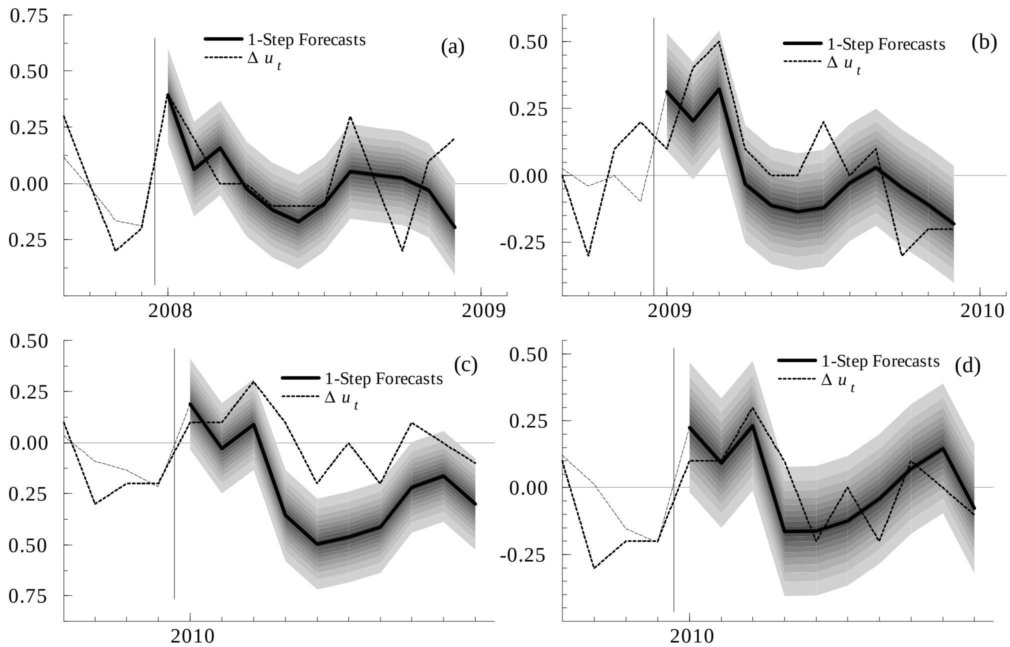


Figure 3: Sequences of one-step ahead forecasts

Figure 3 (a), (b), and (c) record the sequences of one-step ahead forecasts of the preferred ECM for various forecasting horizons. With regard to Figure 3 (a) and (b), the ECM is estimated using the data up to 2007.12 and 2008.12, respectively, while for Figure 3(c) the ECM is estimated using the full-sample data. Figure 3 (a) and (b) demonstrate that most of the forecasts tend to lie in the confidence bands; thus the model's overall forecasting performance up to the end of 2009 can be seen as fairly satisfactory, although we observe the influences of the global depression caused by the US financial crisis from the end of 2008 onwards.

According to in Figure 3(c), however, the model has generated a series of systematic mis-forecasts after 2010.2. The ECM may suffer from a shift in the mean parameter at the beginning of 2010, reflecting the effect of the long-lasting global recession. The ECM subjected to such a structural shift, as discussed above, is prone to give rise to a battery of mis-forecasts. As demonstrated by Hendry (2006), differencing the ECM can be an effective remedy for the problem; such forecasts as derived from a differenced ECM, or a DECM, are less seriously affected by the deterministic shift than those of the original ECM. The ECM, (11), is thus replaced by the regression of Δu_t on Δu_{t-1}, $\Delta^2 u_{t-1}$, $\Delta^2 u_{t-2}$, $\Delta u_{t-1} + 0.46\Delta\pi^w_{t-1} - 2.25\Delta s_{t-1}$ and a set of seasonal dummy variables; this newly-introduced regression model can be perceived as a DECM derived from (11). The DECM, estimated using the full-sample data, is then employed to yield a sequence of one-step forecasts for the same forecasting horizon as Figure 3(c). Figure 3(d) records the

DECM-based forecasts, clearly showing that its forecasting performance is better than that of the original ECM reported in Figure 3(c).

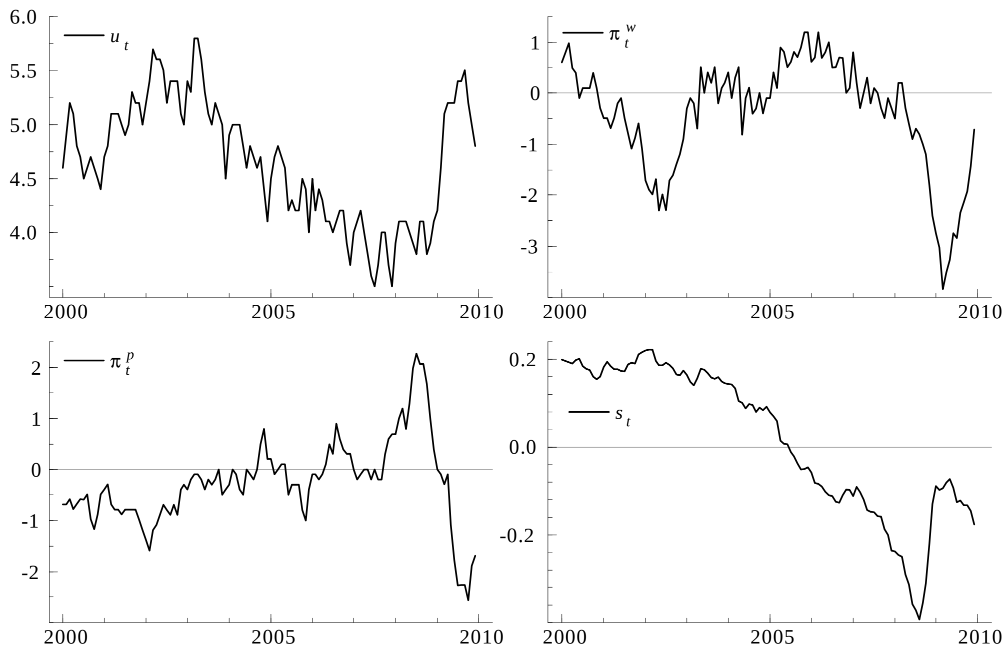
Overall, the study has demonstrated that the ECM, (11), is useful for the purpose of out-of-sample forecasting and its derivative, the DECM, can be employed as a reliable forecasting device in the presence of a deterministic break in the data.

6 Summary and Conclusion

This paper pursues a reduced econometric model explaining the dynamics of Japan's recent unemployment rate. A cointegrated VAR model consisting of the rate of unemployment and other macroeconomic variables is estimated and then subjected to a battery of tests for the model reduction. Various time series properties, such as weak exogeneity, are explored in the model reduction process. The study also demonstrates that both wage inflation and the terms of trade play an important part in a cointegrating relationship estimated from the data. A parsimonious ECM is then estimated centering on the rate of unemployment; the preferred ECM is seen as a data-congruent representation and is also judged to be useful as an out-of-sample forecasting device, augmented with its derivative model, the DECM. The empirical evidence shown in this paper sheds useful light on a deeper understanding of the recent Japanese economy. The evidence also promises to be quantitative information useful for the development of future economic policies in Japan.

Appendix:

(Data overview)



(Data definitions and sources)

u_t = the rate of total unemployment, <1>,
π_t^w = the rate of year-on-year inflation based on the log of a monthly wage index (establishments with 30 employees or more), <2>,
π_t^p = the rate of year-on-year inflation based on the log of a general consumer price index, <3>,
s_t = the difference between the log of an export price index and the log of an import price index, <4>.

<1> Labour Force Survey; The website of Ministry of Internal Affairs and Communication.
<2> Monthly Labour Survey; The website of Ministry of Health, Labour and Welfare.
<3> Consumer Price Index; The website of Ministry of Internal Affairs and Communication.
<4> Corporate Goods Price Index; The website of Bank of Japan.

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