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# A simulation-based analysis of parameter-stability tests using conditional cointegration models

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## Abstract

This note conducts a simulation study of parameter-stability tests using conditional cointegrated vector autoregressive (CVAR) models. Monte Carlo experiments show that, in small samples, the stability tests based on conditional CVAR models under the assumption of weak exogeneity are more powerful than those based on a joint CVAR model; the reverse is observed, however, when the assumption does not hold. The overall assessment of the experiments leads to a practical procedure for testing the constancy of cointegrating parameters.

JEL classification: C32, C52.

Keywords: Parameter stability, Conditional tests, Cointegrated vector autoregressive models, Weak exogeneity, Monte Carlo experiments.

## 1 Introduction

This note performs a simulation-based analysis of parameter-constancy tests using conditional cointegrated vector autoregressive (CVAR) models. Various Monte Carlo experiments are conducted with a view to shedding light on the power performance of the tests in a small-sample context. The introductory section briefly reviews the related literature and motivates the Monte Carlo study.

It is known that economic time series data are prone to exhibit non-stationary behaviour and should often be perceived as processes integrated of order 1 (denoted as  $I(1)$  henceforth) rather than stationary processes. The concept of cointegration, pioneered by Granger (1981), thus plays a key role in the analysis of economic time series data. Johansen (1988, 1996) then develops an econometric methodology for a CVAR model, which now gains great popularity in the field of modelling economic time series. See Hendry and Mizon (1993), Juselius (2006), and Kurita (2007), *inter alia*, for empirical illustrations. In the CVAR analysis, the stability of parameters is required in order to validate standard asymptotic inferences for cointegration. It is therefore important, in empirical analyses,

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to verify that a set of estimated coefficients, those for cointegrating parameters in particular, are time-invariant over a sample period for estimation. Hansen and Johansen (1999) introduce various test statistics for parameter constancy using the technique of recursively estimating a CVAR model; their tests for the stability of cointegrating vectors, based on Nyblom (1989), belong to a class of quasi Lagrange multiplier (*LM*) tests. See also Seo (1998) for various other likelihood-based tests for parameter-constancy in a CVAR model.

In econometric analysis in general, it is a common practice to construct a relatively small model conditional on some explanatory or exogenous variables. Such a model may be called a conditional model, in contrast to a standard joint model in which all variables are treated as endogenous rather than exogenous. Considering that a macro economy should, in principle, be treated as a system in which variables are more or less interdependent, one finds it important to clarify a condition for the valid analysis of such a conditional model. In the context of econometric modelling, the condition is synonymous with weak exogeneity for parameters of interest, a concept related to efficient statistical inferences, introduced by Engle, Hendry and Richard (1983). The fulfilment of weak exogeneity, hence, allows us to estimate a conditional CVAR model alone, instead of a joint CVAR model, for the purpose of making statistical inferences with no loss of information. See Johansen (1992) and Urbain (1992) for details on weak exogeneity in a CVAR model. Parameter constancy, as discussed above, is also viewed as a requirement for standard inferences using a conditional CVAR model.

Provided that modelling a small model conditional on weakly exogenous variables is of interest in econometric investigation, two types of quasi *LM* tests for parameter constancy are conceivable in the CVAR framework: one is based on a standard joint model, while the other on a conditional model. It is then important to answer the question of which test can be more reliable, in finite samples, in detecting variations in cointegrating parameters. Recalling that the additional pre-knowledge of weakly exogenous variables is required to justify conditional-system modelling, one may conjecture that the test based on a conditional model will be more powerful in small samples than that based on a joint model. This note, thus, conducts various Monte Carlo experiments to see whether or not the conjecture holds true. A simulation-based analysis can be informative for this type of comparison in a small-sample context. To the best of the author's knowledge, this note is the first study in literature that investigates the small-sample power performance of parameter constancy tests using both conditional and joint CVAR models.

The organisation of the rest of this note is as follows. Section 2 briefly reviews a quasi *LM* test for stability of cointegrating vectors and a CVAR model conditional on weakly exogenous variables. Section 3 conducts a Monte Carlo study to investigate differences in performance of constancy tests based on the conditional and joint models. Section 4 gives concluding remarks. All the numerical analyses and graphics in this note use *Ox* (Doornik, 2007) and *OxMetrics/PcGive* (Doornik and Hendry, 2007). This note employs the following notational conventions. For a certain matrix  $\xi$  with full column rank, its orthogonal complement  $\xi_{\perp}$  is defined such that  $\xi'_{\perp}\xi = 0$  with the matrix  $(\xi, \xi_{\perp})$  being of full rank;  $\hat{\theta}$  denotes a full-sample maximum likelihood estimator of a certain parameter  $\theta$ ; a symbol  $\xrightarrow{w}$  signifies weak convergence.

## 2 Testing parameter stability in cointegrated models

Let us consider a CVAR( $k$ ) model for a  $p$ -dimensional time series  $X_t$  as follows:

$$\Delta X_t = \alpha (\beta' X_{t-1} + \gamma) + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \quad \text{for } t = 1, \dots, T, \quad (1)$$

where a sequence of innovations  $\varepsilon_t$  has independent and identical normal  $N(0, \Omega)$  distributions (*i.i.d.* normal) conditional on  $X_{-k+1}, \dots, X_0$ , and all parameters vary freely, being specified as follows:  $\alpha, \beta \in \mathbf{R}^{p \times r}$  for  $r \leq p$ ,  $\gamma \in \mathbf{R}^{r \times 1}$ ,  $\Gamma_i \in \mathbf{R}^{p \times p}$  and  $\Omega \in \mathbf{R}^{p \times p}$  is a positive definite symmetric matrix. The index  $r$  denotes cointegrating rank, while  $\beta$  is called cointegrating parameters and  $\alpha$  is referred to as adjustment parameters. Cointegrating relationships,  $\beta' X_{t-1} + \gamma$ , representing a set of stationary linear combinations of the variables, work as long-run equilibrium correction mechanisms in equation (1). Let us denote  $\beta^{*'} = (\beta', \gamma)$  and  $X_{t-1}^{*'} = (X_{t-1}', 1)$  for future reference. See Johansen (1996) for further details of CVAR models. Equation (1) is referred to as a *joint* model in this note.

As described in the introduction, parameter stability is usually seen as a requirement for standard asymptotic inferences. This note focuses on a supremum test for the constancy of cointegrating parameters, one of the quasi *LM* tests suggested by Hansen and Johansen (1999). With regard to this supremum test, Brüggeman, Donati and Warne (2003) show that the test statistic, if formulated directly from a score function, tends to perform better in small samples than the original version of the test by Hansen and Johansen (1999). This note, thus, employs the small-sample modification suggested by Brüggeman, *et al.* (2003). Let us then define  $H_t = (\Delta X_{t-1}', \dots, \Delta X_{t-k+1}')'$  and introduce notational conventions as follows:  $R_{0t}$  denotes residuals from regression of  $\Delta X_t$  on  $H_t$ , while  $R_{1t}$  represents residuals from regression of  $X_{t-1}^{*'}$  on  $H_t$ . A sequence of sample product moments of  $R_{0t}$  and  $R_{1t}$  is then given by

$$\begin{pmatrix} S_{00}^{(t)} & S_{01}^{(t)} \\ S_{10}^{(t)} & S_{11}^{(t)} \end{pmatrix} = \frac{1}{t} \sum_{i=1}^t \begin{pmatrix} R_{0i} \\ R_{1i} \end{pmatrix} \begin{pmatrix} R_{0i} \\ R_{1i} \end{pmatrix}', \quad \text{for } t = 1, \dots, T. \quad (2)$$

The test statistic for the stability of cointegrating parameters comprises

$$Q_T^{(t)} = \left( \frac{t}{T} \right)^2 \text{tr} \left\{ (V^{(T)})^{-1} S^{(t)'} (M^{(T)})^{-1} S^{(t)} \right\}, \quad \text{for } t = 1, \dots, T, \quad (3)$$

where

$$\begin{aligned} V^{(T)} &= \hat{\alpha}' (\hat{\Omega})^{-1} \hat{\alpha}, \\ S^{(t)} &= c_{\perp}' \left( S_{01}^{(t)} - \hat{\alpha} \hat{\beta}' S_{11}^{(t)} \right)' (\hat{\Omega})^{-1} \hat{\alpha}, \\ M^{(T)} &= T^{-1} c_{\perp}' S_{11}^{(T)} c_{\perp}, \end{aligned}$$

for

$$c_{\perp} = \begin{pmatrix} \hat{\beta}_{\perp} & 0 \\ 0 & 1 \end{pmatrix}.$$

Note that  $S^{(t)}$  corresponds to a score function based on the Gaussian distribution. Hansen and Johansen (1999) show that, under the null of parameter stability, the supremum of  $Q_T^{(t)}$  has the following asymptotic distribution:

$$\sup_{1 \leq t \leq T} Q_T^{(t)} \xrightarrow{w} \sup_{s \in [0,1]} \text{tr} \{ S^*(s)' J(1)^{-1} S^*(s) \}, \quad (4)$$

where

$$S^*(s) = S(s) - J(s) J(1)^{-1} S(1),$$

for

$$J(s) = \int_0^s \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix} \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix}' du \quad \text{and} \quad S(s) = \int_0^s \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix} (dB_{2,u})'. \quad (5)$$

The processes  $B_{1,u}$  and  $B_{2,u}$  are mutually independent standard Brownian motions of dimension  $p-r$  and  $r$ , respectively. The supremum test, (4), is seen as a quasi *LM* test statistic in the context of Nyblom (1989).

Next, let us introduce a conditional CVAR model and consider the supremum test based upon it. The process  $X_t$  above is decomposed as  $X_t = (Y_t', Z_t')'$  for  $Y_t \in \mathbf{R}^q$ ,  $Z_t \in \mathbf{R}^{p-q}$  and  $q \geq r$ . The set of parameters and error terms are conformably expressed as

$$\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix}, \quad \text{and} \quad \Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yz} \\ \Omega_{zy} & \Omega_{zz} \end{pmatrix}.$$

Suppose that  $\alpha_z = 0$  holds; equation (1) is then decomposed into a *conditional* CVAR model for  $Y_t$  given  $Z_t$  and a marginal model for  $Z_t$  in the following way:

$$\Delta Y_t = \omega \Delta Z_t + \alpha_y \beta^{*'} X_{t-1}^* + \sum_{i=1}^{k-1} \tilde{\Gamma}_{y,i} \Delta X_{t-i} + \tilde{\varepsilon}_{y,t}, \quad (6)$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \varepsilon_{z,t}, \quad (7)$$

where

$$\omega = \Omega_{yz} \Omega_{zz}^{-1}, \quad \tilde{\Gamma}_{y,i} = \Gamma_{y,i} - \omega \Gamma_{z,i}, \quad \tilde{\varepsilon}_{y,t} = \varepsilon_{y,t} - \omega \varepsilon_{z,t},$$

and

$$\begin{pmatrix} \tilde{\varepsilon}_{y,t} \\ \varepsilon_{z,t} \end{pmatrix} = N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{yy.z} & 0 \\ 0 & \Omega_{zz} \end{pmatrix} \right],$$

for  $\Omega_{yy.z} = \Omega_{yy} - \Omega_{yz} \Omega_{zz}^{-1} \Omega_{zy}$ . Under the condition  $\alpha_z = 0$ , the variable  $Z_t$  is said to be weakly exogenous for the following parameters of the conditional model:

$$\alpha_y, \beta^*, \omega, \tilde{\Gamma}_{y,i}, \tilde{\mu}_y, \tilde{\Phi}_y \text{ and } \Omega_{yy.z}. \quad (8)$$

Note that, as a result of  $\alpha_z = 0$ , the marginal model (7) does not contain the term for the long-run relation,  $\beta^{*'} X_{t-1}^*$ . The fulfilment of  $\alpha_z = 0$  indicates that the conditional CVAR model (6) can be solely estimated without any loss of information, with no need for the estimation of the marginal model (7). Hence, if  $\alpha_z = 0$  holds and the parameters of interest are found in (8), one can then focus on estimating and analysing the conditional

CVAR model. The primary parameter of interest is, in most cases, deemed to be  $\beta^*$ , thus focusing on the analysis of the conditional model often makes sense in empirical research. The condition  $\alpha_z = 0$  is referred to as a weak exogeneity condition in this note. See Johansen (1992) as well as Harbo, Johansen, Nielsen and Rahbek (1998) for details of conditional CVAR models.

Under the weak exogeneity condition, the supremum test introduced above can also be constructed from the conditional CVAR model (6). In order to describe the supremum test in this context, it is necessary to redefine the notation employed above as follows:  $H_t = (\Delta Z'_t, \Delta X'_{t-1}, \dots, \Delta X'_{t-k+1})'$ , while  $R_{0t}$  and  $R_{1t}$  denote residuals from regression of  $\Delta Y_t$  on  $H_t$  and residuals from regression of  $X_{t-1}^*$  on  $H_t$ , respectively. Using these residuals, one is able to calculate the sequence of sample product moments, (2), then constructing the test statistic, (3), so that its supremum can be examined. In this note, the supremum test based on the conditional CVAR model is referred to as a conditional stability (CS) test, while that based on the joint CVAR model is called a joint stability (JS) test. It can be shown that, under the null of parameter constancy with the condition  $\alpha_z = 0$  satisfied, the CS test has the same asymptotic distribution as does the JS test. There should be differences, however, between the two test statistics in their small sample performance; differences in small-sample power properties are of particular interest from the viewpoint of applied economic research. The rest of this note, thus, conducts various Monte Carlo experiments to inspect finite-sample relative power of these stability tests.

### 3 Monte Carlo experiments

This section performs Monte Carlo analyses using both conditional and joint CVAR models. A data generation process (DGP) under study is given as follows:

$$\begin{pmatrix} \Delta X_{1,t} \\ \Delta X_{2,t} \\ \Delta X_{3,t} \\ \Delta X_{4,t} \end{pmatrix} = \begin{pmatrix} -0.2 \\ 0.2 \\ \delta \\ 0 \end{pmatrix} \begin{pmatrix} 1 + (\zeta/T) \cdot 1_{(t \leq mT)} \\ -1 - (\eta/T) \cdot 1_{(t \leq mT)} \\ -1 - (\kappa/T) \cdot 1_{(t \leq mT)} \\ 0.5 + (\lambda/T) \cdot 1_{(t \leq mT)} \\ -0.5 \end{pmatrix}' \begin{pmatrix} X_{1,t-1} \\ X_{2,t-1} \\ X_{3,t-1} \\ X_{4,t-1} \\ 1 \end{pmatrix} \\ + \begin{pmatrix} 0.2 & 0.1 & 0.1 & -0.1 \\ 0 & 0.2 & -0.1 & 0.1 \\ 0 & 0 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0.2 \end{pmatrix} \begin{pmatrix} \Delta X_{1,t-1} \\ \Delta X_{2,t-1} \\ \Delta X_{3,t-1} \\ \Delta X_{4,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{pmatrix},$$

where  $(\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, \varepsilon_{4,t})'$  has a pseudo mean-zero *i.i.d.* normal with variance  $\Omega$ , which is a positive definite symmetric matrix such that each diagonal element is unity while each off-diagonal element is a half. The index  $m$  denotes a break point in cointegrating parameters; such choices as  $m = 0.1$  and  $m = 0.5$  are employed in the simulation experiments. Four local parameters in the cointegrating space, that is,  $\zeta$ ,  $\eta$ ,  $\kappa$  and  $\lambda$ , are allowed to vary so that we can explore power properties of the stability tests under various local alternatives. The parameter in the adjustment space  $\delta$  takes 0 as a benchmark case, while  $\delta = 0.2$  is adopted when inspecting a case where  $X_{3,t}$  is not weakly exogenous with respect to the cointegrating parameters. Since the analysis centres on the finite-sample performance of

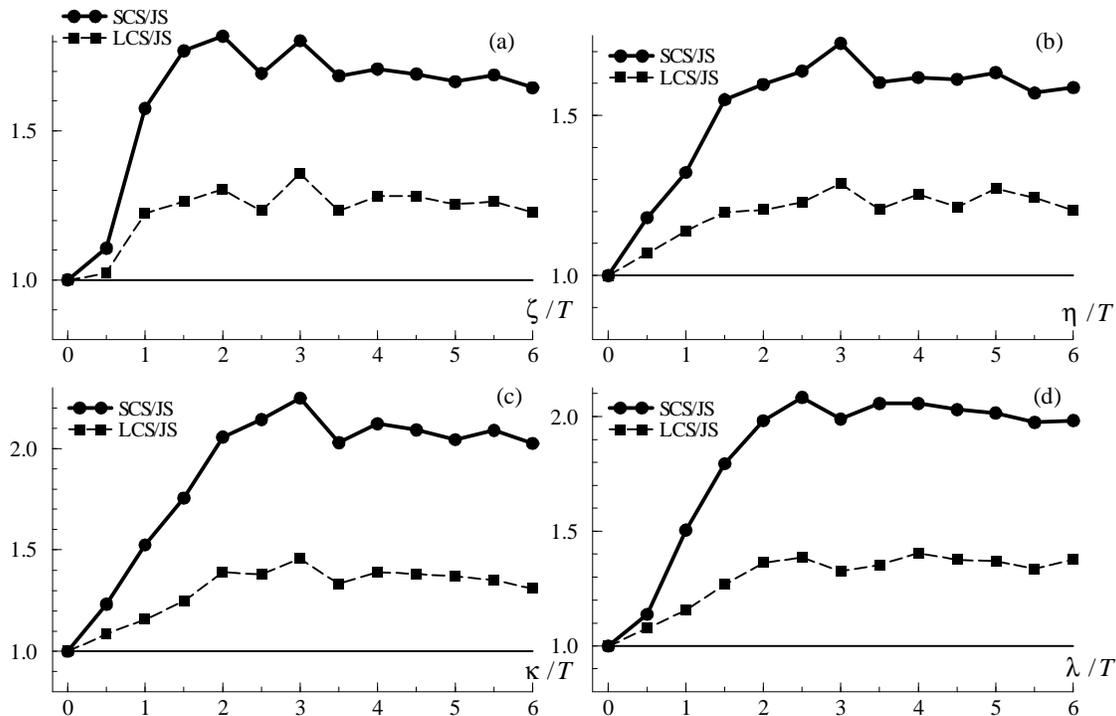


Figure 1: The ratios of size-adjusted rejection frequencies:  $\delta = 0$  and  $m = 0.1$

the test statistics, the number of observations is chosen for 50 or  $T = 50$ . The number of Monte Carlo replications, in contrast, needs to be large enough so it is given by 10,000.

Let us note that two sorts of conditional models are employed in the simulation study: one is a CVAR model for  $Y_t = (X_{1,t}, X_{2,t})'$  given  $Z_t = (X_{3,t}, X_{4,t})'$ , while the other is a CVAR model for  $Y_t = (X_{1,t}, X_{2,t}, X_{3,t})'$  given  $Z_t = X_{4,t}$ . In this section, the CS test based on the former CVAR model is referred to as a small CS test, whereas that based on the latter is called a large CS test. The JS test rests on an overall model for  $X_t = (X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t})'$ . Note that, if  $\delta = 0.2$  holds in the DGP given above thus  $X_{3,t}$  is not weakly exogenous for the cointegrating parameters, the analysis of the small conditional model is not efficient from the viewpoint of full-information statistical inferences. It is, thus, interesting to see the impact of violation of the weak exogeneity condition on the power performance of the small CS test. The JS, large and small CS test statistics are calculated using the artificial data obtained from the DGP above; each test statistic is then compared with the 90% quantile of a simulated limiting distribution for (4), so that rejection frequencies of each test statistic are recorded for parameter values under local alternatives. A group of size-adjusted rejection frequencies can, therefore, be viewed as a finite-sample local power function of each stability test in question. A comparative study is then performed using a number of ratios of local power functions derived for the three types of stability tests.

Figure 1 reports (i) the ratios of size-adjusted rejection frequencies of the small CS tests to those of the JS test (denoted as  $SCS/JS$ ) and (ii) the ratios of size-adjusted

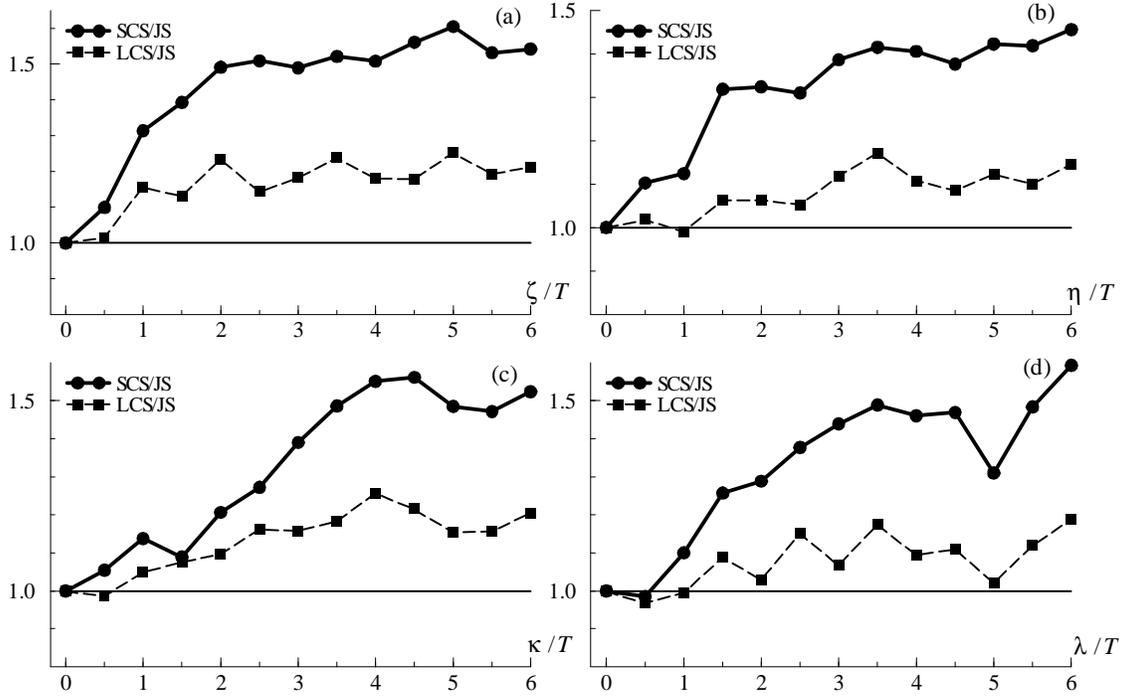


Figure 2: The ratios of size-adjusted rejection frequencies:  $\delta = 0$  and  $m = 0.5$

rejection frequencies of the large CS tests to those of the JS test (denoted as  $LCS/JS$ ), when  $\delta = 0$  and  $m = 0.1$  are selected for the DGP. That is,  $X_{3,t}$  is weakly exogenous for the cointegrating parameters and the break point lies in the verge of the whole sample period. If there is any difference in power properties between the CS and the JS tests, the ratios should then be distinct from unity when the local parameter values vary. The ratios may thus be seen as numerical measures of relative power performance of the stability tests. It turns out that all the ratios tend to be greater than unity with an increase in the local parameter values, as shown in Figure 1 (a), (b), (c) and (d). The evidence indicates that both of the CS test statistics are more powerful than the JS test statistic. Furthermore, the simulation results show that the power of the small CS test is greater than that of the large CS test. The revealed power properties of the test statistics come up to our expectations; the pre-knowledge of weak exogeneity helps us to find variations in the cointegrating parameters. In addition, the task of finding such parameter instability becomes easier to accomplish as a result of making the best use of the information on weakly exogenous variables.

Next, Figure 2 presents the two types of ratios defined above,  $SCS/JS$  and  $LCS/JS$ , with  $\delta = 0$  and  $m = 0.5$  used for the generation of the data; that is, the break point is now given in the middle of the sample period. The simulation results in the figure look fairly similar to those in Figure 1, thus allowing us to draw the same conclusion as above, regardless of the locations of break points.

Let us then turn to a case where  $\delta$  is distinct from 0, that is,  $X_{3,t}$  is not judged to

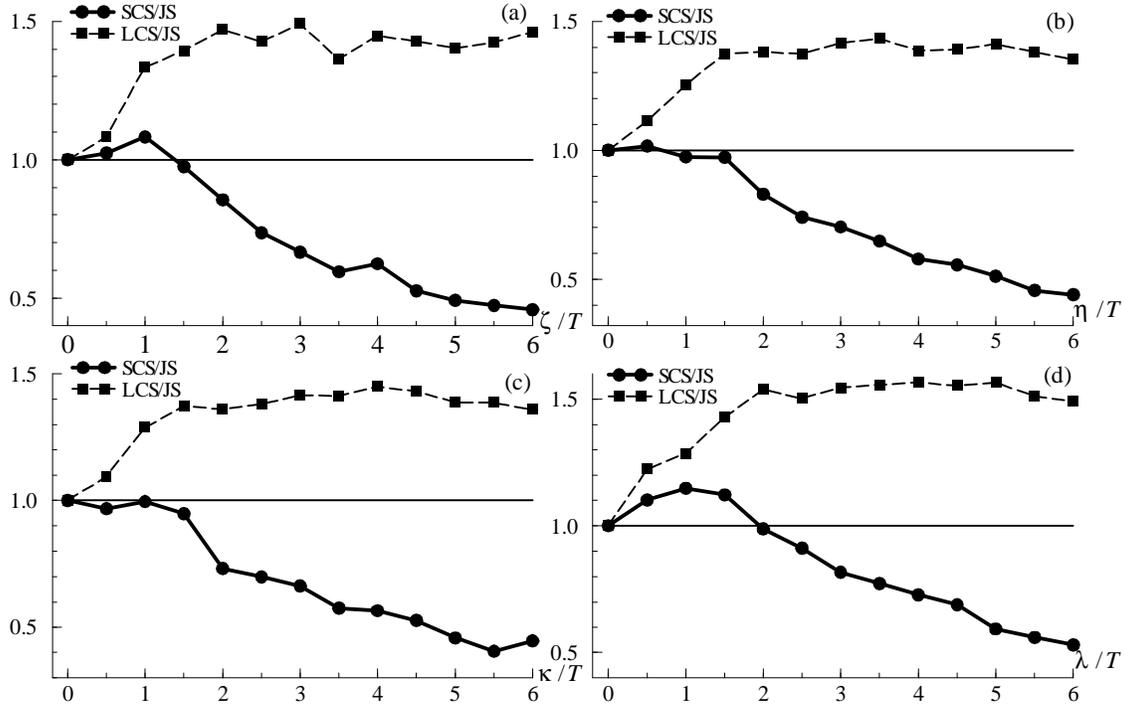


Figure 3: The ratios of size-adjusted rejection frequencies:  $\delta = 0.2$  and  $m = 0.1$

be a weakly exogenous variable with respect to the cointegrating parameters. Figure 3 presents the ratios based on the CS and JS tests, using the data generated from the process with  $\delta = 0.2$  and  $m = 0.1$ . According to the figure, the ratios of size-adjusted rejection frequencies of the large CS tests to those of the JS tests ( $LCS/JS$ ) show a tendency to be greater than unity as the local parameter values increase, in line with Figure 1. It is thus indicated that the large CS test is more powerful than the JS test. However, the ratios using the small CS tests and the JS tests ( $SCS/JS$ ) are apt to be smaller than unity with an increase in the values of local parameters; this observation indicates that the JS test is more powerful than the small CS test, in contrast to the implication of  $LCS/JS$ . Such a difference in power properties, as demonstrated in the figure, should be due to the fact that  $\delta = 0.2$  renders  $X_{3,t}$  a non-weakly exogenous variable for the cointegrating parameters, so that the small conditional model is not estimated without any loss of information. The results shown in Figure 3 indicate that the condition for weak exogeneity plays a critical role in validating conditional tests for parameter stability.

Finally, Figure 4 reports the ratios,  $LCS/JS$  and  $SCS/JS$ , when choosing  $\delta = 0.2$  and  $m = 0.5$  for the DGP above. In Figure 4 as well as Figure 3, the advantage of the large CS tests over the small CS tests is clearly depicted, irrespective of the places where the parameter shifts take place.

Overall, the Monte Carlo experiments provide us with much evidence for the use of the CS tests, under the fulfilment of the parametric condition for weak exogeneity. It is demonstrated, at the same time, that the CS tests can lead to misleading inferences

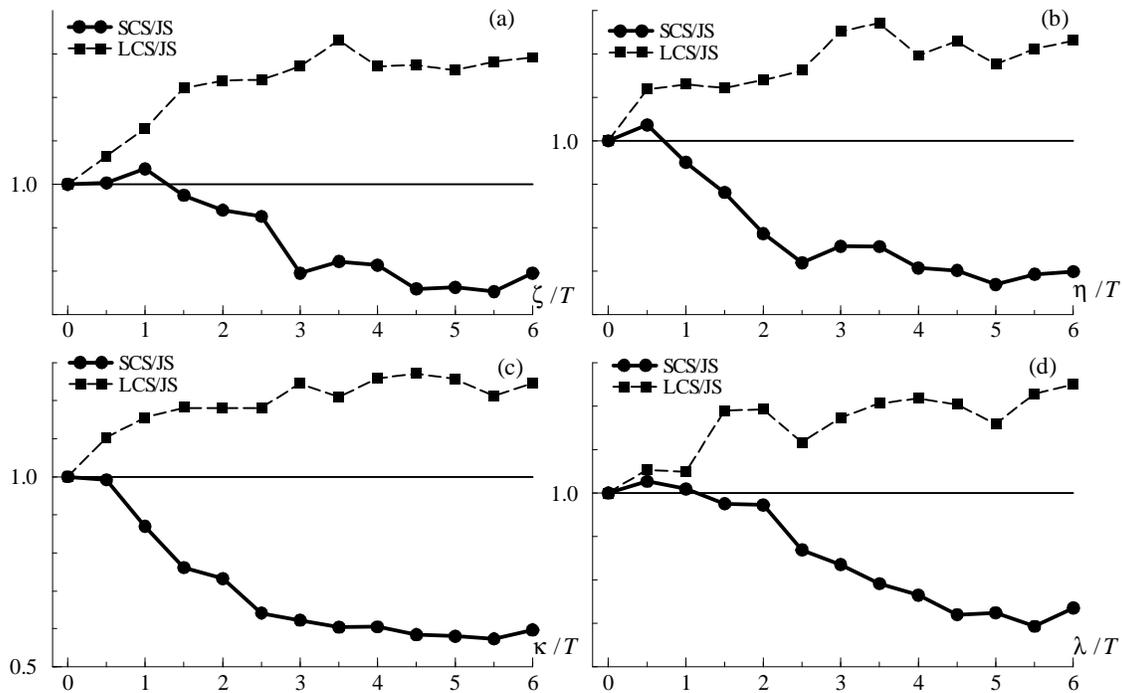


Figure 4: The ratios of size-adjusted rejection frequencies:  $\delta = 0.2$  and  $m = 0.5$

when the condition fails to be satisfied. It is practically important, given these simulation results, to formulate a workable procedure for model evaluation in the analysis of economic time series. Let us recall, first of all, that detecting parameter instability over time is not an easy task in practice. It is thus necessary to make use of as much information as possible, in order to reveal some evidence for time-varying parameters. If one is able to conjecture, based on economic theory and reasoning, that some variables could be weakly exogenous with respect to cointegrating parameters, it is then advisable to estimate a conditional CVAR model given such possibly exogenous variables as well as an overall joint CVAR model, then conducting a comparative analysis of the constancy of cointegrating parameters using both of the models. If the analysis of both CVAR models leads to an identical inferential result, this may be viewed as consolidated statistical evidence for or against the constancy of cointegrating parameters. If, however, there is a discrepancy between statistical decisions based on the two models, one then finds it necessary to perform a detailed analysis of the joint CVAR model; it is of much importance to check whether the conjecture of weak exogeneity, in fact, holds true in the joint CVAR analysis. Depending on the results of such a detailed analysis of the joint CVAR model and, in addition, taking the Monte Carlo results above into account, one can proceed to the overall model evaluation, including the final judgement about the constancy of cointegrating parameters. The analysis of both conditional and joint CVAR models, hence, allows us to draw useful quantitative information from the data and arrive at a convincing conclusion on model evaluation.

## 4 Concluding remarks

This note conducts various Monte Carlo experiments to investigate small-sample power properties of parameter-stability tests in the CVAR framework. The experiments focus on a comparative analysis of conditional CVAR models and a joint CVAR model. According to the Monte Carlo study, the CS test statistics under the assumption of weak exogeneity tend to be more powerful than the JS test statistics. The study demonstrates, at the same time, that the CS tests can lead to misleading inferences when the assumption fails to be satisfied; that is, the weak exogeneity condition plays a critical role in small-sample conditional inferences on the constancy of cointegrating parameters. The experiments shed light on the usefulness of conditional parameter-stability tests and thus allow us to formulate a practical procedure for evaluating time series models.

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