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Abstract

I consider a two-goods and one-factor general equilibrium model consisting of n firms and m households. The number of firms is a policy variable. This paper analyzes effects of an increase in the number of firms by comparative static analysis. To the best of my knowledge, this is the first study which analytically examines effects of new entry on economic welfare in a general equilibrium model with fixed endowments under perfect competition. Allowing new entry reduces each firm's profits in the market but increases the market supply. It raises household's utility and the real GDP.

I. Introduction

The main objective of this paper is to examine whether new entry raises the national income in a general equilibrium model under perfect competition. Under fixed resource endowments and full employment setting a new entry into one market inevitably deprives other industries of resources. The new entry moves resources from one industry to the other and then it may increase output in one industry while it may reduce outputs of the other industry under a full employment economy.

The traditional partial equilibrium analysis assumes away this reduction of outputs in the other industries. According to the traditional partial equilibrium analysis, new entry shifts the market supply curve rightward, increases the quantity of output, and decreases the equilibrium price so that the economy has more of the output and attains higher social welfare in the long run. The partial equilibrium analysis neglects the welfare loss caused by a reduction of outputs in the other industries.

This paper mainly intends to confirm that the main conclusion of the traditional partial equilibrium analysis holds in a general equilibrium setting. In addition this paper finds that new entry increases the real national income or the real GDP.

The paper adopts a general equilibrium model discussed in Arrow and Hahn (1971) as a two-goods economy. Arrow and Hahn (1971) show that there exists a unique equilibrium in two-goods economy. There are various studies which discuss effects of free entry on markets under imperfect competition. Mankiw and Whinston (1986) find "excessive entry" or inefficiency of free entry. Hopenhayn (1992) considers a competitive equilibrium model where the supply of input is unlimited and the supply curve of input is horizontal. He studies effects of raising entry cost on firms. Hopenhayn and Rogerson (1993) consider a one-good and one-factor general equilibrium model where there exists only one industry in the economy. Their calibration results show that a tax on job destruction at the firm level has a negative impact on total employment and economic welfare. So far no studies analytically examine effects of new entry on economic welfare and the real GDP in a general equilibrium setting with fixed endowments under perfect competition, to the best of my knowledge.

We consider an economy where all firms make positive profits but no entry is allowed. Then we examine what will happen when we allow an additional entry in one of the markets. Here new entry is exogenously determined and we consider the number of firms as a policy variable denoted as n_a. In such circumstances this paper analyzes effects of a marginal increase in the number of firms in one market on two output markets and economic welfare using comparative static analysis.

In order to discuss these issues, this paper is organized as follows. Section II describes a general equilibrium model of two goods and one factor. In section III we take the total differentiation of the model of non-linear equations system and obtain a system of linear differential equations. Then we solve the system of linear equations and reports the results of comparative static analysis. Section IV summarizes the main results and draw conclusions.

II. The Model

The model is based on two goods economy discussed in Arrow and Hahn (1971). We consider a general equilibrium model of two goods and one factor consisting of n firms and m households. All of markets in the economy are perfectly competitive and then each economic entity is a price taker. Each firm is assumed to maximize its profits and each household is assumed to maximize its utility level subject to the budget constraint. For simplicity we assume that all firms have the same size in each of industries and all households have the equal income level from given labor hours and dividend income.

A. Firms

Each firm is perfectly competitive and employs labor as a factor of production. The production function assumed to be strictly concave and the law of diminishing returns holds. This assumption reflects the facts that we cannot consider all kinds of inputs such as environmental conditions, quality of inputs, accessibility of raw materials and so forth. And then the marginal cost curve of each firm is upward sloping and each firm makes positive profits at any price level*. Positive profits play a role of an incentive for a firm to enter into a market. There are two goods in the economy and these are denoted as the good A and the good B.

1. A Firm Producing the Good A

A firm producing the good A has a production function given as

$$q_a = q_a(l_a) \tag{1}$$

where q_a is the quantity of the good A produced by each firm,

l_a is the amount of labor employed by each firm to produce the good A.

By the law of diminishing returns, it holds that

* If entry cost, fixed cost or sunk cost exists, then the average cost curve becomes U-shaped and reaches the minimum at the some market price. In this case the main conclusion of this paper will apply only when the market price is above that level. However, no entry will occur in such a case that the market price is less than or equal to that minimum price level.

$$\frac{\partial^2 q_a}{\partial l_a^2} < 0.$$

The total quantity of the good A in the market as a whole is given as

$$Q_a = n_a\, q_a \tag{2}$$

where n_a represents the number of firms producing the good A and it is assumed that all firms in the market of the good A are equal in size.

The profit of each firm producing the good A is defined as

$$\pi_a = p_a\, q_a - w\, l_a \tag{3}$$

where π_a is the profit of each firm producing the good A, p_a is the price of the good A, and w is the wage rate.

Each firm employs labor hours to maximize its profit. The first order condition for profit maximization is given as

$\frac{\partial \pi_a}{\partial l_a} = 0$ or

$$p_a \frac{\partial q_a}{\partial l_a} = w \tag{4}$$

where the value of marginal product of labor is equal to the wage rate.

The total profits of the market as a whole is given as

$$\Pi_a = n_a\, \pi_a \tag{5}$$

where Π_a is the total profits of the market for the good A.

2. A Firm Producing the Good B

The production function of a firm producing the good B is given as

$$q_b = q_b(l_b) \tag{6}$$

where q_b is the amount of the good B produced by each firm, l_b is the amount of labor employed by each firm to produce the good B.

The law of diminishing returns is assumed to hold and it implies that

$$\frac{\partial^2 q_b}{\partial l_b^2} < 0$$

The total output of the good B in the market as a whole is given as

$$Q_b = n_b\, q_b \tag{7}$$

where n_b is the number of firms producing the good B and it is assumed that all firms in the market are equal in size.

The profit of each firm producing the good B is given as

$$\pi_b = p_b\, q_b - w\, l_b \tag{8}$$

where π_b is the profit of each firm in the market and p_b is the price of the good B.

Each firm employs labor hours to maximize its profit. The first order condition for profit maximization is given as

$\frac{\partial \pi_b}{\partial l_b} = 0$

or

$$p_b \frac{\partial q_b}{\partial l_b} = w \tag{9}$$

where the value of marginal product of labor is equal to the wage rate.

The total profits of all firms in the market as a whole is given as

$$\Pi_b = n_b\, \pi_b \tag{10}$$

where Π_b is the total profits of the market for the good B.

B. Households

Each household is assumed to be identical and purchase the good A and the good B so that it maximizes its utility level subject to its budget constraint under given labor hours. The utility function is assumed to be monotonically increasing in quantities of both goods and strictly concave*.

The utility function of each household is given as

$$u = u(q_{aj}, q_{bj}) \tag{11}$$

where u is the utility level, q_{aj} is the quantity of the good A, and q_{bj} is the quantity of the good B demanded by *j*-th household.

For simplicity each household is assumed to have the same labor hours and the same dividend income and therefore it has the same income level. The budget constraint of each household is given as

$$p_a\, q_{aj} + p_b\, q_{bj} = w\bar{l} + \pi_j \tag{12}$$

where $\bar{l}$ is the initial endowment of labor hours, π_j is the dividend or profit income held by each household.

The first order condition for the utility maximization subject to the budget constraint is given by

$$\frac{\partial u}{\partial q_{qj}} \bigg/ \frac{\partial u}{\partial q_{bj}} = \frac{p_a}{p_b} \tag{13}$$

*As is discussed later, this assumption is mitigated in part C of section 3 and the inequality (41) where we assume that the utility function is strictly quasi-concave. This mitigated assumption is adopted when we examine effects of new entry on the household's utility level and the real GDP.

where the left hand side of the equation is the marginal rate of substitution expressed by the ratio of the marginal utility between the good A and the good B.

C. Market Equilibrium Conditions

The economy consists of two goods markets and one input market. The goods markets are markets for the good A and the good B. The input market is the labor market. These three markets are assumed to be in equilibrium.

The market equilibrium condition for the good A is given as

$$n_a q_a = m q_{aj} \tag{14}$$

where m represents the number of households in the economy. The left hand side of the equation is the quantity of the good A supplied by firms and the right hand side is the quantity of the good A demanded by *j*-th household in the economy.

Similarly the market equilibrium condition for the good B is given as

$$n_b q_b = m q_{bj} \tag{15}$$

where the left hand side is the quantity of the good B supplied by firms and the right hand side is the quantity of the good B demanded by *j*-th household.

The equilibrium condition for the labor market is given by

$$n_a l_a + n_b l_b = m\bar{l} \tag{16}$$

where the left hand side is the amount of labor demanded by firms and the right hand side is the initial endowments of labor hours in the economy as a whole. The right hand side of the equation also means total amounts of labor hours supplied by m households.

The total number of firms in the economy as a whole is $n_a + n_b$, namely the sum of the numbers of firms in the two goods markets.

The dividend income comes from firm's profits. And total profits are assumed to be shared among all households equally. Then it holds that

$$n_a \pi_a + n_b \pi_b = m \pi_j . \tag{17}$$

III. Comparative Statics: Effects of New Entry

The model is a system of seventeen simultaneous equations with seventeen unknowns: q_a, Q_a, π_a, l_a, Π_a, q_b, Q_b, π_b, l_b, Π_b, q_{aj}, q_{bj}, π_j, u, w, p_a, and p_b.

From the Walras' law one of three market equilibrium conditions is not independent from the rest of the market clearing equations. Then the equilibrium condition for the market of the good B given as Equation (15) is removed from the system of equations.

The demand and supply functions in the markets are homogenous of degree zero in prices and profits. This means that one of unknowns of prices and profits is indeterminate. So the price of the good B is assumed to be constant as a numeraire.

Therefore the model becomes a system of sixteen simultaneous equations with sixteen unknowns. It is assumed that there exists a set of unique solutions for the system. Arrow and Hahn (1971) have proved the existence of unique equilibrium of the two goods economy under more general settings than this model.

A. A System of Simultaneous Differential Equations

Now the model is a system of sixteen equations with sixteen endogenous variables and four exogenous variables given as n_a, n_b, $\bar{l}$, and m.

By taking the total differentiation of the system of equations given Equations (1) through (17) except Equation (15), we get

$$dq_a = \frac{\partial q_a}{\partial l_a} dl_a \tag{18}$$

$$dQ_a = n_a\, dq_a + q_a\, dn_a \tag{19}$$

$$d\pi_a = p_a\, dq_a + q_a\, dp_a - w\, dl_a - l_a\, dw \tag{20}$$

$$p_a \frac{\partial^2 q_a}{\partial l_a^2} dl_a + \frac{\partial q_a}{\partial p_a} dp_a = dw \tag{21}$$

$$d\Pi_a = n_a\, d\pi_a + \pi_a\, dn_a \tag{22}$$

$$dq_b = \frac{\partial q_b}{\partial l_b} dl_b \tag{23}$$

$$dQ_b = n_b\, dq_b + q_b\, dn_b \tag{24}$$

$$d\pi_b = p_b\, dq_b + q_b\, dp_b - w\, l_b - l_b\, dw \tag{25}$$

$$p_b \frac{\partial q_b}{\partial dl_b} dl_b = dw \tag{26}$$

$$d\Pi_b = n_b\, d\pi_b + \pi_b\, dn_b \tag{27}$$

$$du = \frac{\partial u}{\partial q_{aj}} dq_{aj} + \frac{\partial u}{\partial q_{bj}} dq_{bj} \tag{28}$$

$$p_a\, dq_{aj} + q_{aj}\, dp_a + p_b\, dq_{bj} = w\, d\bar{l} + \bar{l}\, dw + d\pi_j \tag{29}$$

$$p_b \frac{\partial^2 u}{\partial q_{aj}^2} dq_{aj} + p_b \frac{\partial^2 u}{\partial q_{bj} \partial q_{aj}} dq_{bj} + \frac{\partial u}{\partial q_{aj}} dp_b = p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} dq_{aj} + p_a \frac{\partial u^2}{\partial q_{bj}^2} dq_{bj} + \frac{\partial u}{\partial q_{bj}} dp_a \tag{30}$$

$$n_a\, dq_a + q_a\, dn_a = m\, dq_{aj} + q_{aj}\, dm \tag{31}$$

$$n_a\,dl_a + l_a\,dn_a + n_b\,dl_b + l_b dn_b = m\,d\bar{l} + \bar{l}\,dm \tag{32}$$

$$n_a\,d\pi_a + \pi_a\,dn_a + n_b\,d\pi_b + \pi_b\,dn_b = m\,d\pi_j + \pi_j\,dm \tag{33}$$

It is assumed that the number of firms producing the good B, the initial endowments of labor hours, and the number of households are constant. Then we have $dn_b = 0$, $d\bar{l} = 0$, and $dm = 0$. This simultaneous equations system is rewritten as

$$A\mathbf{x} = \mathbf{b} \tag{34}$$

where

$$\mathbf{x}' = (dq_a, dQ_a, d\pi_a, dl_a, d\Pi_a, dq_b, dQ_b, d\pi_b, d\Pi_b, dq_{aj}, dq_{bj}, d\pi_j, dw, dp_a, du) \ ,$$

$$\mathbf{b}' = (0, q_a dn_a, 0, 0, \pi_a dn_a, 0,0,0,0,0,0,0,0, -q_a dn_a, -l_a dn_a, -\pi_a dn_a),$$

$$A = \begin{bmatrix}
1 & 0 & 0 & a_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & 0 & 1 & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{314} & a_{315} & 0 \\
0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & a_{415} & 0 \\
0 & 0 & a_{53} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{69} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{76} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{86} & 0 & 1 & a_{89} & 0 & 0 & 0 & 0 & a_{814} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{99} & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{108} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1111} & a_{1112} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1211} & a_{1212} & -1 & a_{1214} & a_{1215} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1311} & a_{1312} & 0 & 0 & a_{1315} & 0 \\
a_{141} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1411} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{154} & 0 & 0 & 0 & 0 & a_{159} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{162} & 0 & 0 & 0 & 0 & a_{168} & 0 & 0 & 0 & 0 & a_{1613} & 0 & 0 & 0
\end{bmatrix}$$

where $a_{14} = -\frac{\partial q_a}{\partial l_a}$, $a_{21} = -n_a$, $a_{31} = -p_a$, $a_{34} = w$, $a_{314} = l_a$, $a_{315} = -q_a$,

$a_{44} = -p_a \frac{\partial^2 q_a}{\partial l_a^2}$, $a_{53} = a_{21} = -n_a$, $a_{69} = -\frac{\partial a_b}{\partial l_b}$, $n_{76} = -n_b$, $n_{86} = -p_b$,

$a_{89} = a_{34} = w$, $a_{814} = l_b$, $a_{99} = p_b \frac{\partial^2 q_b}{\partial l_b^2}$, $a_{108} = a_{76} = -n_b$, $a_{1111} = -\frac{\partial u}{\partial q_{aj}}$,

$a_{1112} = -\frac{\partial u}{\partial q_{bj}}$, $a_{1211} = p_a$, $a_{1212} = p_b$, $a_{1214} = -\bar{l}$, $a_{1215} = q_{aj}$,

$a_{1311} = p_b \frac{\partial^2 u}{\partial q_{aj}^2} - p_a \frac{\partial^2 u}{\partial q_{bj} \partial q_{aj}}$, $a_{1312} = p_b \frac{\partial^2 u}{\partial q_{bj} \partial q_{aj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}$, $a_{1315} = a_{1112} = -\frac{\partial u}{\partial q_{bj}}$,

$a_{141} = -a_{21} = n_a$, $a_{1411} = -m$, $a_{154} = -a_{21} = n_a$, $a_{159} = -a_{76} = n_b$,

$a_{163} = -a_{21} = n_b$, $n_{168} = -a_{76} = n_b$, and $a_{1613} = a_{1411} = -m$.

New entry in the good A market is considered as an increase in the number of firms producing the good A, n_a. Therefore effects of new entry are examined by solving this system of equations for endogenous variables. The details of calculations is reported in the Appendix.

B. *Results of Comparative Static Analysis*

In Equation (34) the determinant of the matrix A is calculated as

$$|A| = -\left(\frac{\partial^2 q_b}{\partial l_b^2}\right) n_a p_b^2 \frac{\partial u}{\partial q_{bj}} m - \left(\frac{\partial^2 q_a}{\partial l_a^2}\right) n_b p_a p_b \frac{\partial u}{\partial q_{bj}} m$$

$$+ n_a n_b p_b \left(\frac{\partial q_a}{\partial l_a}\right)^2 \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}}\right) + n_a n_b p_b \frac{\partial q_a}{\partial l_a} \frac{\partial q_b}{\partial l_b} \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) > 0$$

where $\frac{\partial^2 q_a}{\partial l_a^2} < 0$, $\frac{\partial^2 q_b}{\partial l_b^2} < 0$, $\frac{\partial^2 u}{\partial q_{aj}^2} < 0$, $\frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} > 0$, and $\frac{\partial^2 u}{\partial q_{bj}^2} < 0$.

Solving this simultaneous equations system, I obtained effects of an increase in the number of firms producing the good A on several endogenous variables and the results are shown in the Appendix. This section presents only the results which have unambiguous sign conditions.

As for an effect on individual firm’s output level we get

$$\frac{dq_a}{dn_a}|A| = -(p_a q_a - \pi_a) n_b \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) \left(\frac{\partial q_a}{\partial l_a} \right)^2 m$$

$$- q_a p_b n_b \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) \left(\frac{\partial q_a}{\partial l_a} \right)^2 m + \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) l_a p_b^2 m \frac{\partial u}{\partial q_{bj}} \frac{\partial q_a}{\partial l_a} m .$$

Since $|A| > 0$, $p_a q_a - \pi_a = w l_a > 0$ and $\left(\frac{\partial^2 q_b}{\partial l_b^2} \right) < 0$, we get

$$\frac{dq_a}{dn_a} < 0 \tag{35}$$

This inequality means that an increase in the number of firms in the good A market reduces the quantity of the good A supplied by each individual firm in the good A market.

As for an effect on the market supply we get

$$\frac{dQ_a}{dn_a}|A| = -n_b \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) q_a p_a p_b \left(\frac{\partial u}{\partial q_{bj}} \right) m^2 - n_a \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) \frac{p_b^2}{p_a} \pi_a \left(\frac{\partial u}{\partial q_{bj}} \right) m^2$$

$$+ \pi_a n_a n_b \left(\frac{\partial q_a}{\partial l_a} \right)^2 \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) m \quad .$$

Since $q_a - \left(\frac{\partial q_a}{\partial l_a} \right) l_a = \frac{1}{p_a} \pi_a > 0$, we get

$$\frac{dQ_a}{dn_a} > 0 \tag{36}$$

This inequality means that an increase in the number of firms in the good A market raises the total quantity of the good A in the market as a whole.

As for an effect on profits of individual firm we get

$$\frac{d\pi_a}{dn_a}|A| = n_a \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) \frac{p_b^2}{p_a} \pi_a q_a \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) m$$

$$+ \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) q_a n_b p_a p_b q_a \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) m - \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) p_a p_b^2 l_a^2 \frac{\partial u}{\partial q_{bj}} m^2$$

$$+ \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) n_b p_a p_b l_a q_a \frac{\partial q_b}{\partial l_b} \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) m \quad .$$

Then we get

$$\frac{d\pi_a}{dn_a} < 0 \tag{37}$$

This inequality means that an increase in the number of firms in the good A market reduces the profit of each individual firm in the good A market.

As for an effect on the individual demand for labor we get

$$\frac{dl_a}{dn_a}|A| = \left(\frac{\partial^2 q_b}{\partial l_b^2}\right) l_a p_b^2 \frac{\partial u}{\partial q_{bj}} m^2 - n_b w l_a \frac{\partial q_a}{\partial l_a}\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) m$$

$$- q_a \frac{\partial q_a}{\partial l_a} n_b p_b \left(- p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) m \quad .$$

Then we get

$$\frac{dl_a}{dn_a} < 0 \tag{38}$$

This inequality means that an increase in the number of firms in the good A market reduces the labor hours demanded by each individual firm in the good A market.

As for an effect on individual demand for the good A we get

$$\frac{dq_{aj}}{dn_a}|A| = -n_a \left(\frac{\partial^2 q_b}{\partial l_b^2}\right) \frac{p_b^2}{p_a} \pi_a \frac{\partial u}{\partial q_{bj}} m - n_b \left(\frac{\partial^2 q_a}{\partial l_a^2}\right) q_a p_a p_b \frac{\partial u}{\partial q_{bj}} m$$

$$+ n_a n_b \frac{p_b}{p_a} \pi_a \frac{\partial q_a}{\partial l_a} \frac{\partial q_b}{\partial l_b} \left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right).$$

Then we get

$$\frac{dq_{aj}}{dn_a} > 0. \tag{39}$$

This inequality means that an increase in the number of firms in the good A market raises the quantity of the good A demanded by each individual household.

As for an effect on the price of good A we get

$$\frac{dp_a}{dn_a}|A| = n_a \left(\frac{\partial^2 q_b}{\partial l_b^2}\right) \frac{p_b^2}{p_a} \pi_a \left(- p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) m$$

$$+n_b\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)p_a p_b m\left\{q_a\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)+\frac{\partial q_b}{\partial l_b}l_a\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)\right\} .$$

Then we get

$$\frac{dp_a}{dn_a}<0. \tag{40}$$

This inequality means that an increase in the number of firms in the good A market reduces the price of the good A.

As for an effect on the household utility level we get

$$\frac{du}{dn_a}|A|$$

$$=\left\{\frac{\partial u}{\partial q_{bj}}\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)+\frac{\partial u}{\partial q_{aj}}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)\right\}\pi_a^2\left(\frac{\partial q_a}{\partial l_a}\right)^2 n_b$$

$$-n_b\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)p_a\pi_a m\frac{\partial u}{\partial q_{bj}}\frac{\partial u}{\partial q_{bj}}-n_a\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{p_b^2}{p_a}\pi_a m\frac{\partial u}{\partial q_{aj}}\frac{\partial u}{\partial q_{bj}} .$$

A sufficient condition for the constrained utility maximization is assumed to hold and this means that the determinant of the bordered Hessian is positive. This implies that

$$\left(\frac{\partial u}{\partial q_{bj}}\right)\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)+\left(\frac{\partial u}{\partial q_{aj}}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)>0 .$$

Then we get

$$\frac{du}{dn_a}>0. \tag{41}$$

This inequality means that an increase in the number of firms in the good A market raises the utility level of each individual household. It is notable that this sign condition does not require the utility function to be strictly concave. Here the utility function only needs to be strictly quasi-concave.

C. *Effect of New Entry on the Real GDP*

This section shows that new entry into the good A market or an increase in the number of firms in the good A market raises the real total income of the economy or the real GDP.

The real GDP, Y, is the value of total outputs evaluated at the base year prices. Then an effect of new entry into the good A market or an increase in the number of firms in the market, n_a, is expressed as

$$\frac{dY}{dn_a} = p_a \frac{dQ_a}{dn_a} + p_b \frac{dQ_b}{dn_a} \tag{42}$$

where Y is the real GDP evaluated at the base year (constant) prices, p_a and p_b.

Solving equation (34) for dQ_a / dn_a and dQ_b / dn_a, we get

$$\frac{dQ_q}{dn_a} = \frac{1}{|A|} n_a \frac{p_b}{p_a} \pi_a \left(- p_b \frac{\partial^2 q_b}{\partial l_b^2} \right) \frac{\partial u}{\partial q_{bj}} m^2 + q_a \left(- p_a \frac{\partial^2 q_a}{\partial l_a^2} \right) n_b p_b \frac{\partial u}{\partial q_{bj}} m^2$$

$$+ \pi_a n_a n_b \left(\frac{\partial q_a}{\partial l_a} \right)^2 \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) m \quad ,$$

and

$$\frac{dQ_b}{dn_a} = \frac{1}{|A|} n_b \frac{\partial q_b}{\partial l_b} m \left[p_a p_b \frac{\partial^2 q_a}{\partial l_a^2} l_a \frac{\partial u}{\partial q_{bj}} m + n_a \frac{p_b}{p_a} \pi_a \frac{\partial q_a}{\partial l_a} \left(- p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) \right].$$

Substituting these two equations into equation (42), we get

$$\frac{dY}{dn_a} = \frac{1}{|A|} = - \left\{ n_a \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) + n_b \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) \right\} p_b^2 \pi_a m^2 \frac{\partial u}{\partial q_{bj}}$$

$$+ n_a n_b \frac{1}{p_a} w \pi_a \frac{\partial q_a}{\partial l_a} \left\{ p_a \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) + p_b \left(- p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) \right\} m$$

where $\frac{\partial^2 q_a}{\partial l_a^2} < 0$, $\frac{\partial^2 q_b}{\partial l_b^2} < 0$, and $\frac{\partial u}{\partial q_{bj}} > 0$. A sufficient condition for the constrained utility maximization is that the determinant of the bordered Hessian is positive. Then we get

$$\left(\frac{\partial u}{\partial q_{bj}}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) + \left(\frac{\partial u}{\partial q_{aj}}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) > 0 \quad .$$

Then using this 2nd order condition for the constrained utility maximization, we get

$$\frac{dY}{dn_a} > 0 . \qquad (43)$$

This inequality means that an increase in the number of firms in the good A market raises the real GDP. It is notable that this sign condition holds if the utility function is not strictly concave but strictly quasi-concave.

This result implies that a virtue of competitive equilibrium is shown without using the concept of utility level as unobservable measure. At present a virtue of competitive equilibrium in a general equilibrium model is shown only as an efficiency gain in the sense of Pareto optimality known as the first fundamental theorem of the welfare economics. But now it is shown as an increase in the real GDP if we allow new entry or adopting a free entry policy in the general equilibrium model.

IV. Conclusion

Unambiguous effects of new entry based on this model are as follows: New entry makes individual firms in the entrant's market reduce its profits, the quantity of output, and the employment. New entry increases the market supply of the good and decreases the market price of the good. New entry raises the utility level of each household through a decrease of the price and an increase of the consumption of the new entrant's output market. In addition new entry makes an economy as a whole increases the real national income or the real GDP.

Except for the effect on real GDP all of these effects are well known in many microeconomics textbooks as results of the partial equilibrium analysis. This paper shows that these effects also observed in the general equilibrium setting where the initial endowment of the factor market is given and fixed in the economy. In the general equilibrium model new entry in one market may decrease resources available for the other firms and the other market. Therefore one of the contributions of this paper is that what is known only in the partial equilibrium model is also shown in the general equilibrium model.

It is known that a competitive equilibrium leads to the Pareto efficient resource allocation. All firms make positive profits in our model. Therefore new entry is always encouraged. This paper shows that this new entry increases the total output of the market, reduces the price of the output, and raises both of the utility level of each household and the real GDP.

There are at least two policy implications of this paper. This paper finds that new entry reduces profits of existing firms. Then existing firms have an incentive to restrict new entry. One of policy implications of this paper is that we need to keep a competitive environment in order not to impede free entry. The other policy implication is that trade and investment liberalization in a global economy leads to raise the real total income of a global economy since it promotes new entry in global markets.

What has not been shown in the traditional partial equilibrium model is that new entry raises the real GDP. The real GDP is measurable and is a much simpler and easier concept for general public to understand than the concept of utility level. One of virtues of market economy is said that the competitive equilibrium generally attains the Pareto optimal resource allocation. This is known as the first fundamental theorem of welfare economics. Now, this paper proposes the additional aspect of virtues of market economy. Economists can tell the general public that the virtue of market economy is to raise the real GDP by allowing free entry. No referring to the concept of Pareto optimality or the utility function is necessary.

There are some cautions in this model. Firstly a marginal cost curve of each firm is upward sloping starting zero output level and then each firm makes positive profits at any positive price level. This allows an infinite number of firms to enter into the market in the long run and leads to each firm's output level to an infinitesimal degree. Realistically a marginal cost curve is U-shaped and it means that there exists entry cost, fixed cost, and sunk cost or that the production function shows increasing returns to scale in the early stage of production level. Accordingly it is plausible that there exist the minimum average cost and an output level which each firm earns no profits at the price level equal to the minimum average cost. Once the output level reaches this long-run level, all firms will earn zero profits and this model will no longer apply in this case. However, this case can also avoid a possibility that all firms produce an infinitesimal output level in the long run. Secondly this paper considers a very simple two goods economy. It is surely important to study a model of many goods and many factors in order to derive much more realistic and fruitful conclusions. If we cannot derive any meaningful results even from this simple model, it will be harder to get more fruitful conclusions from general equilibrium models with more goods and factors. In this sense this study will be the first step for developing this direction of research work. However, this task will be left for further research.

Appendix:

A System of Equations

The simultaneous equations system in this paper is rewritten as

$$A\mathbf{x} = \mathbf{b} \tag{A1}$$

where

$$\mathbf{x}' = (dq_a, dQ_a, d\pi_a, dl_a, d\Pi_a, dq_b, dQ_b, d\pi_b, d\Pi_b, dq_{aj}, dq_{bj}, d\pi_j, dw, dp_a, du) \ ,$$

$$\mathbf{b}' = (0, q_a dn_a, 0, 0, \pi_a dn_a, 0, 0, 0, 0, 0, 0, 0, 0, -q_a dn_a, -l_a dn_a, -\pi_a dn_a),$$

$$A = \begin{bmatrix}
1 & 0 & 0 & a_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & 0 & 1 & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{314} & a_{315} & 0 \\
0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & a_{415} & 0 \\
0 & 0 & a_{53} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{69} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{76} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{86} & 0 & 1 & a_{89} & 0 & 0 & 0 & 0 & a_{814} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{99} & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{108} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1111} & a_{1112} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1211} & a_{1212} & -1 & a_{1214} & a_{1215} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1311} & a_{1312} & 0 & 0 & a_{1315} & 0 \\
a_{141} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{1411} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{154} & 0 & 0 & 0 & 0 & a_{159} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{162} & 0 & 0 & 0 & 0 & a_{168} & 0 & 0 & 0 & 0 & a_{1613} & 0 & 0 & 0
\end{bmatrix}$$

where $a_{14} = -\frac{\partial q_a}{\partial l_a}$, $a_{21} = -n_a$, $a_{31} = -p_a$, $a_{34} = w$, $a_{314} = l_a$, $a_{315} = -q_a$,

$a_{44} = -p_a \frac{\partial^2 q_a}{\partial l_a^2}$, $a_{53} = a_{21} = -n_a$, $a_{69} = -\frac{\partial a_b}{\partial l_b}$, $n_{76} = -n_b$, $n_{86} = -p_b$,

$a_{89} = a_{34} = w$, $a_{814} = l_b$, $a_{99} = p_b \frac{\partial^2 q_b}{\partial l_b^2}$, $a_{108} = a_{76} = -n_b$, $a_{1111} = -\frac{\partial u}{\partial q_{aj}}$,

$$a_{1112} = -\frac{\partial u}{\partial q_{bj}} \quad , \quad a_{1211} = p_a \quad , \quad a_{1212} = p_b \quad , \quad a_{1214} = -\bar{l} \quad , \quad a_{1215} = q_{aj} \quad ,$$

$$a_{1311} = p_b \frac{\partial^2 u}{\partial q_{aj}^2} - p_a \frac{\partial^2 u}{\partial q_{bj} \partial q_{aj}} \quad , \quad a_{1312} = p_b \frac{\partial^2 u}{\partial q_{bj} \partial q_{aj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \quad , \quad a_{1315} = a_{1112} = -\frac{\partial u}{\partial q_{bj}} \quad ,$$

$$a_{141} = -a_{21} = n_a \quad , \quad a_{1411} = -m \quad , \quad a_{154} = -a_{21} = n_a \quad , \quad a_{159} = -a_{76} = n_b \quad ,$$

$a_{163} = -a_{21} = n_b$, $n_{168} = -a_{76} = n_b$, and $a_{1613} = a_{1411} = -m$.

A new entry in the good A market is considered as an increase in the number of firms producing the good A, n_a. Therefore effects of new entry are examined by solving this system of equations for endogenous variables.

Results of Comparative Static Analysis

By assuming the law of diminishing marginal product in the production function and diminishing marginal utility in the utility function for simplicity, we have $\frac{\partial^2 q_a}{\partial l_a^2} < 0$, $\frac{\partial^2 q_b}{\partial l_b^2} < 0$, $\frac{\partial^2 u}{\partial q_{aj}^2} < 0$, $\frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} > 0$, and $\frac{\partial^2 u}{\partial q_{bj}^2} < 0$.

By applying Cramer's rule to solving this simultaneous equations system (A1), we get the following results.

$$\frac{dq_a}{dn_a}|A| = -n_b w l_a \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) \left(\frac{\partial q_a}{\partial l_a} \right)^2 m$$

$$- n_b p_b q_a \left(- p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} \right) \left(\frac{\partial q_a}{\partial l_a} \right)^2 m + \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) p_b^2 l_a \frac{\partial u}{\partial q_{bj}} \frac{\partial q_a}{\partial l_a} m^2 < 0.$$

$$\frac{dQ_a}{dn_a}|A| = -n_b p_a p_b q_a \left(\frac{\partial^2 q_a}{\partial l_a^2} \right) \frac{\partial u}{\partial q_{bj}} m^2 - n_a \frac{p_b^2}{p_a} \pi_a \left(\frac{\partial^2 q_b}{\partial l_b^2} \right) \frac{\partial u}{\partial q_{bj}} m^2$$

$$+ \pi_a n_a n_b \left(\frac{\partial q_a}{\partial l_a} \right)^2 \left(p_b \frac{\partial^2 u}{\partial q_{aj} \partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2} \right) m > 0.$$

$$\frac{d\pi_a}{dn_a}\left|A\right| = n_a\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{p_b^2}{p_a}\pi_a q_a\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)m$$

$$+n_b\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)p_a p_b q_a^2\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)m$$

$$-\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)p_a p_b^2 l_a^2\frac{\partial u}{\partial q_{bj}}m^2$$

$$+n_b\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)p_a p_b l_a q_a\frac{\partial q_b}{\partial l_b}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)m<0.$$

$$\frac{dl_a}{dn_a}\left|A\right| = p_b^2\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)l_a\frac{\partial u}{\partial q_{bj}}m^2 - n_b w l_a\frac{\partial q_a}{\partial l_a}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)m$$

$$-n_b p_b q_a\frac{\partial q_a}{\partial l_a}\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)m<0.$$

$$\frac{d\Pi_a}{dn_a}\left|A\right| = -m[n_a n_b q_a w^2\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)+n_b p_a p_b\pi_a\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}}m$$

$$+n_a p_b\pi_a\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)mq_{aj}+n_b p_a\pi_a\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)mq_{aj}$$

$$+n_a\pi_a p_b^2\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{\partial u}{\partial q_{bj}}m-n_a n_b p_b\pi_a\left(\frac{\partial q_a}{\partial l_a}\right)^2\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$-n_a^2 p_a q_a\frac{p_b}{p_a}\pi_a\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a^2 p_b^2 q_a^2\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)+n_a n_b w^2 l_a\frac{\partial q_a}{\partial l_a}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a p_b\pi_a\frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)m\bar{l}$$

$$-n_a^2 p_a p_b l_a^2\left(\frac{\partial q_a}{\partial l_a}\right)^2\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a^2 p_b^2 l_a^2\left(\frac{\partial q_a}{\partial l_a}\right)^2\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)-n_a n_b p_a w q_a\frac{\partial q_a}{\partial l_a}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a n_b p_b w q_a\frac{\partial q_a}{\partial l_a}\left(-p_b\frac{\partial^2 u}{\partial q_{aj}^2}+p_a\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)-n_a n_b p_b\pi_a\frac{\partial q_a}{\partial l_a}\frac{\partial q_b}{\partial l_b}\left(p_b\frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}-p_a\frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a n_b p_a^2 q_a^2 \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) - n_a n_b p_a p_b q_a^2 \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$+n_a n_b p_b \pi_a l_b \frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$+n_a n_b p_b p_a q_a \left(\frac{\partial q_a}{\partial l_a}\right)^2 \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$-n_a n_b p_a w l_a \left(\frac{\partial q_a}{\partial l_a}\right)^2 \left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$+n_a^2 p_b^2 q_a l_a \frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$+n_a^2 p_a p_b q_a l_a \left(\frac{\partial q_a}{\partial l_a}\right)\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$+n_a^2 p_b^2 q_a l_a \frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) + n_a p_a p_b^2 l_a^2 \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{\partial u}{\partial q_{bj}} m].$$

$$\frac{dq_b}{dn_a}|A| = p_b \frac{\partial q_b}{\partial l_b} m\left[p_a l_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m + n_a \frac{1}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right].$$

$$\frac{dQ_b}{dn_a}|A| = n_b w m\left[p_a l_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m + n_a \frac{1}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right].$$

$$\frac{d\pi_b}{dn_a}|A|$$

$$= -m p_b^2 \left(\frac{\partial^2 q_b}{\partial l_b^2}\right) l_b \left[p_a l_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m + n_a \frac{1}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right].$$

$$\frac{d\Pi_b}{dn_a}|A|$$

$$= -n_b m p_b^2 \left(\frac{\partial^2 q_b}{\partial l_b^2}\right) l_b \left[p_a l_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m + n_a \frac{1}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right].$$

$$\frac{dq_{aj}}{dn_a}|A| = -n_a\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{p_b^2}{p_a}\pi_a\frac{\partial u}{\partial q_{bj}}m - \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)q_a n_b p_a p_b \frac{\partial u}{\partial q_{bj}}m$$

$$+ n_a n_b \frac{p_b}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\frac{\partial q_b}{\partial l_b}\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) > 0.$$

$$\frac{dq_{bj}}{dn_a}|A| = n_b w\left[n_a \frac{1}{p_a}\pi_a \frac{\partial q_a}{\partial l_a}\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) + p_a l_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m\right].$$

$$\frac{d\pi_j}{dn_a}|A| = -[-n_a^2 p_b w l_a \left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$-n_a^2 q_a \frac{p_b^2}{p_a}\pi_a \left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$+ n_a p_b^2 \pi_a \left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\frac{\partial u}{\partial q_{bj}} m + n_b p_a p_b \pi_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\frac{\partial u}{\partial q_{bj}} m$$

$$- n_a p_b w l_a m q_{aj}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$- n_a^2 \frac{p_b}{p_a^2} w^2 l_a^2 \left\{p_a\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) + p_b\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right\}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)$$

$$- n_a n_b p_a w l_a q_a \left(\frac{\partial^2 q_a}{\partial l_a^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)$$

$$- n_a n_b p_a p_b q_a^2 \left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\left(\frac{\partial^2 q_a}{\partial l_a^2}\right)$$

$$- n_a p_b \pi_a \frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) m\bar{l}$$

$$- n_a n_b w l_a \left(\frac{\partial q_a}{\partial l_a}\right)^2 \left\{p_a\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) + p_b\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right\}$$

$$+ n_a^2 p_b^2 q_a l_a \frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)$$

$$- n_a n_b w \frac{\partial q_a}{\partial l_a} q_a \left\{p_a\left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right) + p_b\left(-p_b \frac{\partial^2 u}{\partial q_{aj}^2} + p_a \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right\}$$

$$+ n_a p_a p_b (n_a l_a + n_b l_b) q_a \left(p_b \frac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \frac{\partial^2 u}{\partial q_{bj}^2}\right)\frac{\partial q_a}{\partial l_a}\left(\frac{\partial^2 q_b}{\partial l_b^2}\right)$$

$$- n_a n_b p_a p_b l_a^2 \left(\tfrac{\partial q_a}{\partial l_a}\right)\left(p_b \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \tfrac{\partial^2 u}{\partial q_{bj}^2}\right)\tfrac{\partial q_a}{\partial l_a}\left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right)$$

$$+ n_a n_b l_b \tfrac{p_b^2}{p_a} \pi_a \left(- p_b \tfrac{\partial^2 u}{\partial q_{aj}^2} + p_a \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\tfrac{\partial q_a}{\partial l_a}\left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right)$$

$$+ p_a p_b^2 (n_a l_a + n_b l_b) l_a \left(\tfrac{\partial^2 q_a}{\partial l_a^2}\right)\left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right)\tfrac{\partial u}{\partial q_{bj}} m].$$

$$\tfrac{dw}{dn_a}\left|A\right|$$

$$= p_b^2 \left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right) m \left[p_a l_a \left(\tfrac{\partial^2 q_a}{\partial l_a^2}\right)\tfrac{\partial u}{\partial q_{bj}} m + n_a \tfrac{1}{p_a} \pi_a \tfrac{\partial q_a}{\partial l_a}\left(- p_b \tfrac{\partial^2 u}{\partial q_{aj}^2} + p_a \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right)\right].$$

$$\tfrac{dp_a}{dn_a}\left|A\right| = n_a \left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right)\tfrac{p_b^2}{p_a} \pi_a \left(- p_b \tfrac{\partial^2 u}{\partial q_{aj}^2} + p_a \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) m$$

$$+ \left(\tfrac{\partial^2 q_a}{\partial l_a^2}\right) n_b p_a p_b q_a \left(- p_b \tfrac{\partial^2 u}{\partial q_{aj}^2} + p_a \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) m$$

$$+ \left(\tfrac{\partial^2 q_a}{\partial l_a^2}\right) l_a n_b p_a p_b \tfrac{\partial q_b}{\partial l_b}\left(p_b \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \tfrac{\partial^2 u}{\partial q_{bj}^2}\right) m < 0.$$

$$\tfrac{du}{dn_a}|A|$$

$$= \left\{\left(\tfrac{\partial u}{\partial q_{bj}}\right)\left(- p_b \tfrac{\partial^2 u}{\partial q_{aj}^2} + p_a \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}}\right) + \left(\tfrac{\partial u}{\partial q_{aj}}\right)\left(p_b \tfrac{\partial^2 u}{\partial q_{aj}\partial q_{bj}} - p_a \tfrac{\partial^2 u}{\partial q_{bj}^2}\right)\right\} \pi_a^2 \left(\tfrac{\partial q_a}{\partial l_a}\right)^2 n_b$$

$$- \left(\tfrac{\partial^2 q_a}{\partial l_a^2}\right) p_a \pi_a n_b \tfrac{\partial u}{\partial q_{bj}} \tfrac{\partial u}{\partial q_{bj}} m - n_a \left(\tfrac{\partial^2 q_b}{\partial l_b^2}\right)\tfrac{p_b^2}{p_a} \pi_a \tfrac{\partial u}{\partial q_{aj}} \tfrac{\partial u}{\partial q_{bj}} m > 0.$$

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