Dynamic characteristics of the daily yen-dollar exchange rate

Takamitsu Kurita
Faculty of Economics, Fukuoka University

WP-2012-002

Center for Advanced Economic Study
Fukuoka University
(CAES)
8-19-1 Nanakuma, Jonan-ku, Fukuoka,
JAPAN 814-0180
Dynamic characteristics of the daily yen-dollar exchange rate

Takamitsu Kurita*
Faculty of Economics, Fukuoka University

May 14, 2012

Abstract

This paper explores various dynamic properties of daily data for the yen-dollar exchange rate. This empirical study shows that quantitative information articulated with technical trading acts as market-based indicators, thus contributing to the modelling of daily fluctuations in the exchange rate. Value-at-Risk analysis is also performed to demonstrate that allowing for data properties such as skewness is essential for representing the underlying volatility of the yen-dollar rate.

JEL classification: F31, C22
Keywords: Daily yen-dollar exchange rates, Technical trading, GARCH models, Value at Risk

1 Introduction

The objective of this study is to investigate the underlying characteristics of daily time series data for the Japanese yen - US dollar exchange rate. It is demonstrated that quantitative information associated with technical trading works as observable significant factors in the exchange rate dynamics. This study also shows that taking account of various statistical properties such as skewness is important in obtaining a well-formulated time series model for the exchange rate. The introductory section provides a brief review of related literature and also informs the research objective pursued in this paper.

It is well known that technical or chartist analysis is popular among practitioners taking part in foreign exchange markets. See Allen and Taylor (1990), Cheung and Wong (2000) and Gehrig and Menkhoff (2006) for extensive survey studies concerning technical trading. Thus, the potential importance of technical analysis has been growing and recognised in literature on foreign exchange market microstructure. For instance, De Grauwe and Dewachter (1993) develop a class of theoretical exchange rate models, in which economic agents who are categorised as chartists play important roles. Recent empirical

*Correspondence to: Faculty of Economics, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka 814-0180, Japan. E-mail: tkurita@fukuoka-u.ac.jp
research has, indeed, paid much attention to technical analysis in accounting for short-term exchange rate behaviour and forecasts; see Chang and Osler (1999), Osler (2003, 2006) and Schulmeister (2006, 2009), \textit{inter alia}. Although significant progress has been made in this line of research, the various roles of technical trading in foreign exchange markets have not been fully studied yet, and therefore need further empirical as well as theoretical investigations.

Technical trading involves a number of analytical methods, techniques and strategies. Candlestick chart analysis is, in particular, counted as one of the most popular methods among traders in financial markets. This method was, historically, developed in Japan in the 1700s to analyse short-term price movements in rice markets. See Fiess and MacDonald (1999, 2002) as well as Marshall, Young and Rose (2006) for details of candlestick charts. Four types of prices, classified as follows, play critical roles in candlestick chart analysis: the highest and lowest prices over a given unit of time (denoted \textit{high} and \textit{low}, respectively) as well as the opening and closing prices for a given unit of time (denoted \textit{open} and \textit{close}, respectively). The name of this analysis comes from a series of special graphs depicting these prices simultaneously; each graph fairly looks like a candle accompanied with its shadow and wick. Marshall \textit{et al.} (2006) provide a number of examples of candlestick charts. It is conceivable, judging from the definition of these prices recorded on a daily basis, that the difference between \textit{high} and \textit{low} suggests the degree of intraday trend and volatility in price movements, while that between \textit{close} and \textit{low} implies the direction in which prices move until the closing time of a trading day. These differences between various prices on a certain trading day may, moreover, carry some information correlated with the movement of the central price from that day to the following trading day. Such information, if judged significant from a statistical viewpoint, can be seen as evidence for the underlying inter-day trend in the price movement. As a result, the price differences themselves can be reckoned as market-based trend indicators. This possibility leads to the primary impetus for the empirical investigation pursued in this paper.

Exchange rate data have been studied in literature using various methods and techniques in time series econometrics; see Sarno and Taylor (2002), \textit{inter alia}, for an overview of empirical exchange rate studies. This paper, along the similar lines, employs a class of autoregressive conditional heteroscedasticity (ARCH) time series models, with a view to investigating the dynamics of the daily yen-dollar exchange rate. An ARCH model, pioneered by Engle (1982), paves the way for extensive research in the fields of quantitative economics and finance; a generalised ARCH or GARCH model is then introduced by Bollerslev (1986). For details of a GARCH model and its variants, see Engle (1995) as well as Francq and Zakoian (2010). GARCH-type analysis has contributed a great deal to a better understanding of the behaviour of high-frequency financial data; see Francq and Zakoian (2010) and a number of references therein. The use of GARCH-type models has, indeed, yielded fruitful outcomes in the analysis of yen-dollar exchange rate data; see Ito, Engle and Lin (1992), Tse (1998), Nagayasu (2004), Tsui and Ho (2004), \textit{inter alia}. The existing studies are certainly informative and shed useful light on various aspects of the
yen-dollar exchange rate. Empirical research which makes explicit use of candlestick-chart information such as Fiess and MacDonald (1999, 2002), however, appears to be limited in literature about the yen-dollar exchange rate. It is important, from the viewpoint of effective exchange rate policy, to have a comprehensive understanding of the influences of technical trading on exchange rate behaviour. There remains room for further empirical investigation along these lines, which drives the research objective pursued in this paper.

This study, using recent developments in time series econometrics, shows that quantitative information related to candlestick chart analysis plays an important role in accounting for the dynamics of the daily yen-dollar rate. The empirical findings revealed in this study complement Fiess and MacDonald (1999, 2002) and provides a promising direction for further research on the roles of candlestick chart analysis in the foreign exchange markets. Accumulated research findings in this regard should be useful for policy makers as well as financial analysts.

Furthermore, both thick-tailed and skewed distributions are often viewed as the distinguishing characteristics of high-frequency financial data; see Giot and Laurent (2003, 2004). This study demonstrates that a GARCH-type model based on a skewed-Student $t$ distribution, which allows for these two statistical characteristics, turns out to be useful for describing the underlying volatility of the daily yen-dollar rate data. Value-at-Risk analysis is carried out to cast light on the importance of taking account of these properties explicitly in modelling the volatility.

The rest of this paper is organised as follows. Section 2 reviews a class of GARCH-type models adopted in this paper and Section 3 estimates various models for the daily data of the yen-dollar exchange rate by incorporating technical-trading information. Value-at-Risk analysis is conducted in Section 4 using some of the estimated models. Concluding remarks are provided in Section 5. All quantitative and graphic analyses in this paper are performed using G@RCH (Laurent, 2007) and PcGive (Doornik and Hendry, 2007).

### 2 GARCH-type time series models

This section gives a brief review of GARCH-type time series models employed in this paper. See Engle (1995) as well as Francq and Zakoian (2010) for details of the models. Let $y_t$ denote a dependent variable to be analysed, while let $x_{t-1}$ denote a $k$-dimensional vector of explanatory variables. The variable $y_t$ typically represents daily returns of financial assets. A conditional mean equation in a GARCH-type model can be given by

$$y_t = \alpha' x_{t-1} + \varepsilon_t, \quad \text{for} \quad t = 1, \ldots, T,$$

where $\varepsilon_t$ is a mean-zero and serially uncorrelated innovation process with conditional variance $\sigma_t^2$. It should be noted that $\sigma_t$ is also a time-varying process, instead of a fixed constant as is usually assumed in a standard regression model; $\sigma_t$ is referred to as volatility in this paper. Let us, then, introduce a conditional variance equation involving
the GARCH($p$, $q$) structure as follows:

\[
\begin{align*}
\varepsilon_t &= \sigma_t \eta_t, \\
\sigma_t^2 &= \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j}^2,
\end{align*}
\]

(2)

where $\eta_t$ is an innovation term, which has an independent and identical distribution with zero mean and unit variance, such as a standard Gaussian or normal distribution, while the parameters in (2) are subject to $\omega > 0$ and $\beta_i, \gamma_j \geq 0$. The GARCH model above is introduced by Bollerslev (1986), following the seminal paper on ARCH by Engle (1982). Let $L$ denote lag operator, and equation (2) is then re-expressed as

\[
\sigma_t^2 = \omega + \beta (L) \sigma_t^2 + \gamma (L) \varepsilon_t^2,
\]

(3)

where $\beta (L) = \sum_{i=1}^{p} \beta_i L^i$ and $\gamma (L) = \sum_{j=1}^{q} \gamma_j L^j$. Equation (3) leads to a number of formulations of GARCH-type models. Let us review two of them below by paying attention to time series characteristics frequently found in high-frequency financial data.

First, it is known that shocks to volatility have a tendency to decline very slowly; see Bollerslev and Mikkelsen (1996), inter alia. Such persistent shocks may also be captured by a GARCH-type model with the help of long-memory or fractionally-integrated time series methodology. For this purpose, let (3) be re-expressed as

\[
[1 - \beta (L) - \gamma (L)] \varepsilon_t^2 = \omega + [1 - \beta (L)] (\varepsilon_t^2 - \sigma_t^2),
\]

which leads to the ARCH(\infty) representation of $\sigma_t^2$, provided the roots of the equation $1 - \beta (z) = 0$ lie outside the unit circle, that is, $|z| > 1$. Suppose that the lag polynomial on the left hand side can be factorised as $1 - \beta (L) - \gamma (L) = \psi (L) (1 - L)^d$, where $d$ denotes a fractional degree of integration satisfying $0 \leq d \leq 1$ and the roots of $\psi (z) = 0$ obey $|z| > 1$. We then find that the ARCH(\infty) representation of $\sigma_t^2$ is

\[
\sigma_t^2 = \omega [1 - \beta (1)]^{-1} + \left\{ 1 - \psi (L) (1 - L)^d [1 - \beta (L)]^{-1} \right\} \varepsilon_t^2.
\]

(4)

A GARCH($p$, $q$) model subject to (4) is called a fractionally integrated GARCH or FIGARCH($p, d, q$) model introduced by Baillie, Bollerslev and Mikkelsen (1996). If $\psi (L)$ and $\beta (L)$ are polynomials of degree 1, the FIGARCH model above may simply be put as $\beta (L) = \beta_1 L$ and $\psi (L) = 1 - \psi_1 L$.

Next, it should be noted that the asymmetric behaviour of volatility may be modelled in the framework of (3). Asymmetry typically means, in the context of quantitative finance, that volatility has a tendency to increase by a larger amount in response to negative shocks than to positive shocks. Such an asymmetric response is, in particular, recognised as a stylized fact for stock price volatility and is also known as a leverage effect. In order to account for the asymmetric nature of volatility in a class of GARCH-type models, an asymmetric power ARCH or APARCH model is introduced by Ding, Granger and Engle (1993). In an APARCH($p, q$) model, equation (3) is generalised to

\[
\sigma_t^6 = \omega + \beta (L) \sigma_t^6 + \gamma (L) (|\varepsilon_t| - \zeta \varepsilon_t)^6,
\]

(5)
where $\delta > 0$, while $\zeta$ subject to $|\zeta| < 1$ varies according to $j = 1, \ldots, q$, that is, 
$$
\gamma (L) (|\varepsilon_t| - \zeta \varepsilon_t) = \sum_{j=1}^q (|\varepsilon_t| - \zeta_j \varepsilon_t) \delta.
$$
The APARCH model can be seen as a general expression encompassing various GARCH-type specifications. In equation (5), $\delta$ corresponds to a Box-Cox power transformation of $\sigma_t$, which allows a general specification of volatility, whereas $\zeta_j$ represents the presence of asymmetry; for example, $\delta = 2$ and $0 \leq \zeta_1 < 1$ for $q = 1$ indicate that past negative innovations have greater influences on the conditional variance than those with positive values. Using the same line of argument as above, we also obtain a fractionally integrated APARCH or FIAPARCH model by generalising (5), as demonstrated by Tse (1998). That is, a FIAPARCH$(p, d, q)$ model is subject to
$$
\sigma_t^\delta = \omega [1 - \beta (1)]^{-1} + \left\{ 1 - \psi (L) (1 - L)^d [1 - \beta (L)]^{-1} \right\} (|\varepsilon_t| - \zeta \varepsilon_t) \delta.
$$
Again, in practice, the selection of $\beta (L) = \beta_1 L$ and $\psi (L) = 1 - \psi_1 L$ may be general enough to provide a satisfactory representation of the data under study.

### 3 Modelling the daily data of the yen-dollar rate

This section, relying on the methodology reviewed above, attempts to build econometric models of the daily yen-dollar exchange rate. It is demonstrated that quantitative information on technical trading plays a significant role in the models for the exchange rate data. Let $s_t$ denote the log of the central spot exchange rate on a daily basis in the Tokyo foreign exchange market, i.e. the exchange rate at which the largest amount of transaction was recorded in the market on a certain trading day. Note that the rate is measured as the Japanese yen value of a unit of the US dollar. The variable of interest in this study is a daily return in the central yen-dollar rate; that is, $y_t = \Delta s_t$ in the context of equation (1). Candlestick chart analysis allows us to specify the conditional mean equation in such a way that each of explanatory variables in $x_t$ is defined as a discrepancy between a pair of various yen-dollar exchange rates recorded on a lagged trading day in the Tokyo market. That is, explanatory variables are as a whole given by

$$
x_{t-1} = \begin{pmatrix}
high_{t-1} - low_{t-1} \\
high_{t-1} - central_{t-1} \\
close_{t-1} - low_{t-1} \\
close_{t-1} - open_{t-1} \\
1
\end{pmatrix},
$$

where $close_{t-1}$ is the log of the spot rate at 5 p.m., $open_{t-1}$ is the log of the spot rate at 9 a.m., $high_{t-1}$ is the log of the highest spot rate, $low_{t-1}$ is the log of the lowest rate, while $central_{t-1}$ denotes the log of the central spot rate so we find $central_{t-1} = s_{t-1}$. Note that both $y_t$ and $x_{t-1}$ are multiplied by 100, so that they are expressed in percentage terms. Let us also note that the present study assigns importance to the roles
Conditional mean

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>$-0.764(0.165)$</td>
<td>$-0.816(0.124)$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$0.918(0.156)$</td>
<td>$0.923(0.158)$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$0.862(0.262)$</td>
<td>$0.953(0.105)$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>$0.057(0.262)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_5$ (constant)</td>
<td>$-0.084(0.040)$</td>
<td>$-0.083(0.040)$</td>
<td>$-0.026(0.023)$</td>
</tr>
</tbody>
</table>

Conditional variance

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>$0.909(0.034)$</td>
<td>$0.910(0.034)$</td>
<td>$0.906(0.038)$</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>$0.074(0.028)$</td>
<td>$0.074(0.028)$</td>
<td>$0.073(0.029)$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$0.011(0.007)$</td>
<td>$0.011(0.007)$</td>
<td>$0.014(0.009)$</td>
</tr>
</tbody>
</table>

AIC

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.118</td>
<td>2.116</td>
<td>2.241</td>
</tr>
</tbody>
</table>

SIC

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.154</td>
<td>2.148</td>
<td>2.259</td>
</tr>
</tbody>
</table>

RBD(2)

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.559[0.459]</td>
<td>1.265[0.531]</td>
<td>1.075[0.584]</td>
</tr>
</tbody>
</table>

RBD(5)

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.708[0.745]</td>
<td>2.469[0.781]</td>
<td>1.891[0.864]</td>
</tr>
</tbody>
</table>

RBD(10)

<table>
<thead>
<tr>
<th></th>
<th>GARCH-A</th>
<th>GARCH-B</th>
<th>GARCH-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.725[0.959]</td>
<td>3.465[0.968]</td>
<td>2.183[0.995]</td>
</tr>
</tbody>
</table>

Table 1: GARCH models

of $high_{t-1}$ and $low_{t-1}$ in selecting differenced variables for (7). As pointed out by Fiess and MacDonald (1999, 2002), $high_{t-1}$ and $low_{t-1}$ determine a trading range on the date $t - 1$ and play the role of resistance and support levels, respectively; they can also play the same roles on the following date $t$ if the exchange rate does not change large enough to break them through. Hence, it is conceivable that various linear combinations as shown in (7) may correspond to significant inter-day trend indicators, acting as a set of observable factors contributing to the dynamics of the yen-dollar rate. With regard to the data source, all the data analysed in this study are obtained from the web site of the Bank of Japan (http://www.boj.or.jp/en/index.htm/). A sample period for estimation runs from 4 January 2007 to 28 June 2011; the number of observations effective for estimation corresponds to that of trading days in this sample period, thus it amounts to 1,096.

The empirical study starts with the estimation of three types of GARCH(1,1) models. The models are referred to as GARCH-A, GARCH-B and GARCH-C, respectively. A standard normal distribution is adopted to represent the distribution of their innovations $\eta_t$, so that the maximum likelihood estimation of unknown parameters is feasible. The conditional mean equation (1) for GARCH-A contains all the explanatory variables given in (7), thus it is seen as a general model encompassing various types of sub-models. The conditional variance equation is given by (3) with $i = j = 1$. The results of maximum likelihood estimation are presented in the first column of Table 1. Notational conventions used in the table are defined as follows: the figures in the parentheses are robust standard errors, while those in the square brackets are $p$-values; AIC and SIC denote Akaike
(1973) and Schwartz (1978) information criteria, respectively; RBD($j$) is a residual-based diagnostic test with lag length $j$, which is proposed by Tse (2002). The diagnostic tests are all insignificant at the conventional level, suggesting the model is well-specified from a statistical viewpoint. It should be noted that the explanatory variables, apart from $close_{t-1} - open_{t-1}$, are highly significant in the conditional mean equation. They are, therefore, judged to be a set of factors contributing to the dynamics of the daily yen-dollar rate. Let us interpret the roles of the significant regressors in the conditional mean equation. The variable $high_{t-1} - low_{t-1}$ has a significant negative coefficient, so it brings about a decrease in the central yen-dollar rate on the following trading day. This finding may be interpreted as a type of adjustment towards a support level defined by $low_{t-1}$, thus playing the role of a damping factor for intraday volatility. Next, both $high_{t-1} - central_{t-1}$ and $close_{t-1} - low_{t-1}$ hold significant positive coefficients, thus leading to an upsurge in the exchange rate on the following trading day. The evidence indicates that several patterns of intraday upward trend have, in fact, further upward effects on the exchange rate behaviour on the trading day that follows. One may thus conceive that the underlying intraday trend is rather persistent, turning into inter-day trend in some degree. Overall, these price differences can be viewed as market-based trend indicators. With regard to the conditional variance equation, both $\beta_1$ and $\gamma_1$ are significant, suggesting the importance of allowing for time-varying characteristics in modelling volatility.

Next, the author estimates another GARCH model, GARCH-B, from which the insignificant regressor in the preceding model, $close_{t-1} - open_{t-1}$, has been removed. The estimation results are recorded in the second column of Table 1. Again, the diagnostic tests indicate no misspecification problem and all the explanatory variables are judged highly significant. Both of the information criteria are smaller than those for GARCH-A, thus in favour of GARCH-B as compared with the preceding model. The choice of GARCH-B is also supported by a log likelihood ratio ($LR$) test for model reduction from GARCH-A to GARCH-B: the test statistic is $0.205[0.651]$, in which the figure is the square bracket is a $p$-value according to $\chi^2(1)$.

Furthermore, the third column of the table records the estimates of GARCH-C, in which no explanatory variable relating to candlestick chart analysis is included. As expected, its information criteria are much larger than those of GARCH-A and GARCH-B; the log LR test statistic for reduction from GARCH-B to GARCH-C is $142.86[0.00]$, so that the reduction is highly rejected according to $\chi^2(3)$. Thus, reducing GARCH-B to GARCH-C is anything but acceptable from a statistical standpoint, an indication that the variables associated with technical trading play significant roles in the description of the data. The GARCH-B is therefore viewed as the best model among those estimated so far, acting as a benchmark model in the subsequent study below. The empirical results summarised in Table 1, overall, seem to carry useful information for policy makers and financial analysts who need to grasp the underlying effects of technical trading on the daily yen-dollar exchange rate.

As discussed in the previous section, it is important, in exchange rate modelling, to
allow for the possibility that the characteristics of long-memory and asymmetry may be found in its volatility estimates. The models based on FIGARCH and FIAPARCH, as reviewed in (4) and (5) above, are promising vehicles for the purpose of modelling such time series features. Table 2 summarises the results of estimated FIGARCH and FIAPARCH models in the same manner as Table 1. The first column of Table 2 records the estimation results for a FIGARCH(1, d, 1) model with its innovations distributed as standard normal, while the second column presents those for a FIAPARCH(1, d, 1) model with the same distribution assumption. According to the first column of the table, the diagnostic tests suggest no significant evidence for model mis-specification, and the three regressors related to candlestick chart analysis are all significant in the mean equation. It is worth noting, among all, that the long-memory parameter \( d \) is significantly estimated, which can be seen as statistical evidence in favour of the view that a FIGARCH-type model may be more appropriate than a standard GARCH-type model. Indeed, both of the information criteria for the FIGARCH model are smaller than those of the GARCH-B model.

Next, the second column of Table 2 shows that the estimated FIAPARCH model is also accompanied with the same set of significant regressors in its mean equation and with satisfactory diagnostic test results. It should be noted that the estimates for \( \delta \) and \( \zeta_1 \) are significantly different from 0, while the estimate for \( d \) is now insignificant at a conventional level. The study has generated mixed outcomes in the estimates of these models. The FIAPARCH model appears to be preferable to the FIGARCH model, based on the information criteria. However, from the standpoint of rigorous time series investigation, it may be inappropriate to disregard the observed mixed results in the estimates. It thus seems necessary to seek an additional methodological device for addressing this issue.

In this study various models have been estimated thus far by assuming that the distribution of the underlying innovations is standard normal. The normal distribution, however, may not be capable of representing some characteristics often witnessed in daily data of asset returns. Leptokurtosis and skewness, in addition to long-memory and a leverage effect as explored so far, are counted as such data characteristics. A variant of a Student \( t \)-distribution, instead of the normal distribution, may be useful in representing the underlying distribution of \( \eta_t \). A skewed-Student \( t \) distribution, in particular, may enable us to obtain a well-formulated GARCH-type model for the data, as demonstrated by Lambert and Laurent (2001) as well as Giot and Laurent (2003, 2004). Following them, let us suppose \( \eta_t \) has a standardised skewed-Student \( t \) distribution, whose density function is given by

\[
f(\eta_t | \theta, v) = \begin{cases} 
\frac{2^\theta}{\Gamma(\frac{\theta}{2})} h(\theta \rho \eta_t + \theta \lambda | v) & \text{if } \eta_t < -\frac{\lambda}{\rho}, \\
\frac{2^\theta}{\Gamma(\frac{\theta}{2}+1/\theta)} h((\rho \eta_t + \lambda) / \theta | v) & \text{if } \eta_t \geq -\frac{\lambda}{\rho},
\end{cases}
\]

where \( h(\cdot | v) \) represents a symmetric standardised Student \( t \) density function, \( \theta \) is an asymmetry parameter, \( v \) is a degree of freedom, while \( \lambda \) and \( \rho^2 \) denote mean and variance, respectively, and they are both defined in terms of \( \theta \) and \( v \). Let us also note that
Table 2: FIGARCH, FIAPARCH and SS-FIAPARCH models

<table>
<thead>
<tr>
<th>Conditional mean</th>
<th>FIGARCH</th>
<th>FIAPARCH</th>
<th>SS-FIAPARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$ ($high_{t-1} - low_{t-1}$)</td>
<td>-0.819(0.128)</td>
<td>-0.880(0.139)</td>
<td>-0.817(0.113)</td>
</tr>
<tr>
<td>$\alpha_2$ ($high_{t-1} - central_{t-1}$)</td>
<td>0.915(0.164)</td>
<td>0.946(0.161)</td>
<td>0.890(0.137)</td>
</tr>
<tr>
<td>$\alpha_3$ ($close_{t-1} - low_{t-1}$)</td>
<td>0.937(0.102)</td>
<td>0.924(0.106)</td>
<td>0.917(0.102)</td>
</tr>
<tr>
<td>$\alpha_4$ (constant)</td>
<td>-0.072(0.040)</td>
<td>-0.062(0.039)</td>
<td>-0.072(0.038)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conditional variance</th>
<th>$\beta_1$</th>
<th>$\psi_1$</th>
<th>$\omega$</th>
<th>$d$</th>
<th>$\delta$</th>
<th>$\zeta_1$</th>
<th>$\nu$</th>
<th>$\log \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.717(0.109)</td>
<td>0.710(0.117)</td>
<td>0.782(0.096)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>0.385(0.141)</td>
<td>0.491(0.171)</td>
<td>0.399(0.164)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.012(0.010)</td>
<td>0.023(0.030)</td>
<td>0.026(0.014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.467(0.184)</td>
<td>0.343(0.284)</td>
<td>0.537(0.258)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.437(0.416)</td>
<td>1.231(0.413)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.758(0.375)</td>
<td>0.559(0.178)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>6.872(1.286)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log \theta$</td>
<td>-0.130(0.050)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| AIC | 2.114 | 2.093 | 2.020 |
| SIC | 2.151 | 2.139 | 2.075 |
| RBD(2) | 0.732[0.694] | 0.577[0.749] | 0.766[0.682] |
| RBD(5) | 1.764[0.881] | 2.001[0.849] | 2.029[0.845] |
| RBD(10) | 2.749[0.987] | 2.623[0.989] | 2.491[0.991] |

$f(\eta_t | 1/\theta, \nu)$ is seen as the mirror image of $f(\eta_t | \theta, \nu)$ with regard to the zero mean, so that the equality $f(\eta_t | 1/\theta, \nu) = f(-\eta_t | \theta, \nu)$ is observed. Hence, the sign of $\log \theta$ suggests the direction of skewness such that the density function is skewed to the left if $\log \theta < 0$, and vice versa. A FIAPARCH model based on a skewed-Student t distribution provides us with a flexible method for describing the data under study, thus enhancing the possibility that we can arrive at a satisfactory data-representation.

The third column of Table 2 reports the estimation results of a FIAPARCH($1, d, 1$) model based on the skewed-Student t distribution introduced above, which is denoted SS-FIAPARCH. The estimates for $\nu$ and $\log \theta$ are significantly different from 0, suggesting that both leptokurtosis and skewness can be viewed as the distinguishing features of the yen-dollar rate data. The parameter $\log \theta$ has a negative estimate, which indicates the underlying distribution is skewed to the left. In addition, $d$, $\delta$ and $\zeta_1$ are all estimated with statistical significance, so it is judged that the primary characteristics of FIAPARCH are also captured well by this model. As expected, the information criteria of the SS-FIAPARCH model are smaller than those of the preceding FIGARCH and FIAPARCH models. In the light of the overall statistical evidence, it is justifiable for us to select the SS-FIAPARCH model as a satisfactory data representation. Figure 1 (a) displays a
plot of daily returns in the central yen-dollar rate, while Figure 1 (b) presents a plot of conditional variance estimated by the SS-FIAPARCH model. The time-varying nature of the data seems to be represented well by the model. The next section conducts an additional quantitative analysis to give weight to the selection of the SS-FIAPARCH model.

4 Value-at-Risk analysis

In this section Value-at-Risk (VaR) analysis is performed with a view to checking the empirical validity of the estimated models. VaR at level $\varphi$ for $0 < \varphi < 1$ with respect to a sample of returns (in a certain currency value) is in general defined as the corresponding empirical quantile at this level. One can thus see that losses will be smaller, with probability $1 - \varphi$, than those indicated by the VaR.

This section, in principle, adopts the same approach as Giot and Laurent (2003, 2004) to a quantitative comparative study of the yen-dollar data. Three models are selected from those estimated in the previous section for the comparative analysis: the GARCH-B model, the FIAPARCH model and the SS-FIAPARCH model. Note that the first two models are based on the standard Gaussian distribution, as demonstrated in the previous section. These three models are subjected to 1-day ahead VaR analysis at level $\varphi$ by
Table 3: VaR analysis

computing a failure rate, which is defined as the number of times actual returns are in excess of 1-day ahead VaR in absolute values. The observed failure rate, if the model under study is properly specified, should coincide with the corresponding VaR level.

In this study, two types of failure rates are adopted: one is a failure rate for long trading positions while the other is that for short trading positions. The former corresponds to the percentage of negative returns taking smaller values than 1-day ahead VaR for long positions, while the latter to that of positive returns taking larger values than 1-day ahead VaR for short positions. Let us note that the VaR for long positions is defined using the left empirical quantile at level $\varphi$; this VaR is applied to a case in which traders in long positions suffer losses when negative returns are realised. The short side of the VaR is, in contrast, defined using the right empirical quantile at level $\varphi$, applied to a situation where traders in short positions undergo losses when positive returns are observed.

Let $FR$ denote an empirical failure rate, and the study inspects the validity of the null hypothesis $FR = \varphi$ against the alternative $FR \neq \varphi$ using a log $LR$ test by Kupiec (1995). The log $LR$ test statistic, under the null hypothesis, has an asymptotic $\chi^2(1)$
distribution, so that we are able to rely on standard asymptotic inference for the empirical investigation. If the $p$-value of the test statistic is smaller than a conventional significance level, one is then justified in concluding that the null hypothesis is rejected by the log $LR$ test statistic. Let us also note that, in the comparative study, $\varphi$ for long positions takes values ranging from 0.05 to 0.0025, while the level $\varphi$ for short positions ranges from 0.95 to 0.9975.

The results of the VaR analysis are documented in Table 3. According to the panels in the first row of the table, the GARCH-B model, overall, finds it difficult to generate satisfactory outcomes with respect to the VaR for long positions. The performance of the FIAPARCH model is not very good either in the light of the VaR for long positions, as shown in the panels in the second row. In contrast, according to the panels in the third row of the table, the SS-FIAPARCH model has succeeded in generating a battery of acceptable VaR results; one may judge that this model is a better representation of the data than the preceding GARCH-B and FIAPARCH models. Figure 2 records the plots of VaR at the level 0.025 and 0.975 estimated from the SS-FIAPARCH model. The VaR analysis above lends weight to the validity of the SS-FIAPARCH model in describing the dynamic characteristics of the yen-dollar rate volatility. Overall, the SS-FIAPARCH model is reckoned as a reliable data-representation of the exchange rate. This model can thus act as a useful empirical reference for policy makers and financial analysts who wish to have a better understanding of the yen-dollar rate behaviour on a daily basis.

Figure 2: Plots of VaR for the SS-FIAPARCH model
5 Concluding remarks

This paper has explored various dynamic properties of daily time series data for the yen-dollar exchange rate. The study shows that quantitative information on technical trading, such as the difference between the highest and lowest currency values on a lagged trading day, plays a significant part in the set of estimated GARCH-type models. These lagged differences can be viewed as market-based trend indicators, carrying significant information on the dynamics of the daily yen-dollar rate. Furthermore, the VaR analysis indicates that it is essential to allow for skewness as well as leptokurtosis in modelling the exchange rate volatility. The overall findings cast much light on the behaviour of the daily yen-dollar exchange rate and are thus informative for policy makers as well as financial analysts. They also indicate a promising direction for further research on the roles of technical trading in foreign exchange markets.

References


