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Dynamic econometric modelling and the impact of a structural break: Evidence from Japanese time series data

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Abstract

This paper aims to achieve a small-scale empirical system representing macroeconomic dynamics in Japan. The study also examines the impact of a structural break on the system's equilibrium correction mechanism. A cointegrated vector autoregressive analysis reveals an interpretable mechanism for equilibrium correction in the data. The cointegrated system is reduced to a trivariate equilibrium correction system, which is judged to be a reliable representation of dynamic interactions between internal and external demands. It is also inferred that a structural break observed in the data has a limited impact on the equilibrium correction mechanism itself due to possible co-breaking effects. A forecasting analysis using the estimated system is performed to give weight to the validity of this argument.

Keywords: equilibrium correction mechanism, cointegrated vector autoregressive analysis, structural breaks, co-breaking, export-led economic growth.
JEL Classification Codes: C32, C50.

1 Introduction

The purpose of this paper is to obtain a small-scale econometric system representing dynamic interactions between internal and external aggregate demands in the Japanese economy. The study also assigns importance to the inspection of whether or not the equilibrium correction mechanism of the estimated system is stable over sample periods covering recent years of economic turmoil. In order to achieve these objectives, this study conducts a detailed multivariate analysis of quarterly macroeconomic data from Japan. This introductory section explains the primary impetus for the empirical study conducted

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in this paper and describes the significant aspects of this research, coupled with a brief review of related literature.

It is well known that the Japanese economy suffered from decade-long stagnation after the burst of an asset price bubble in the early 1990s. The economy, however, experienced a significant recovery between the period of 2002-2008, during which the shadow of long-lasting economic slowdown finally receded. The underlying driving force of Japan's economic recovery in this period can be attributed to a steady increase in aggregate export and investment. Indeed, Japan's export increased a great deal during this period possibly due to an upsurge in external demands from emerging economies as well as the US and Europe. A depreciation of the Japanese yen against major currencies around this period, in addition, appears to have contributed to a rise in Japan's export. In accordance with a steady growth of export, both domestic capital investment and industrial production also turned up and increased steadily. It is, therefore, conceivable that there was a stable interdependent circle between external and internal demands in the Japanese economy in this recovery period. However, this circle turned into a vicious one when international trade declined a great deal in the early phase of global economic depression, triggered by a financial crisis in the US in September 2008; see Levchenko, Lewis and Tesar (2010), *inter alia*, for further details of the trade collapse around this period. Japan's export then plummeted, accompanied by a large-scale decrease in domestic fixed investment and industrial production. It seems that export and fixed investment, combined with production activities, have played key roles in macroeconomic dynamics in Japan.

Turning to literature on macroeconomics, one recognises that researchers often attach importance to various kinds of roles played by export in open economies. The idea of export-led economic growth, in particular, has been under extensive empirical study in literature; see Kunst and Marin (1989), Marin (1992), Giles and Williams (2000a, 2000b), Hatemi-J (2002), Reppas and Christopoulos (2005) and Awokuse (2006), *inter alia*. Export-oriented macroeconomic views, which also underlie the review of the recent Japanese economy presented above, lead to the possibility that export and fixed investment may act as a twin-engine to fuel macroeconomic growth. Thus, given the underlying interdependent nature of export and fixed investment, one may find it appropriate to pursue an econometric model representing dynamic interactions between them. Modelling these two variables jointly, however, does not seem to be popular empirical methodology in the macroeconomic literature; thus one may argue that its potential importance fails to be appreciated. It is true that the existing empirical research in the literature sheds light on various important aspects of aggregate export, but there still remains room for further explorations in the light of export-investment nexus dynamics. The econometric study in this paper is, thus, expected to contribute to a deeper understanding of empirical macroeconomics as well as the Japanese economy.

This paper conducts a cointegrated vector autoregressive (CVAR) analysis, which is reviewed below, using Japan's quarterly time series data for fixed investment, export,

industrial production and a real effective exchange rate. Given the nature of production activities, it is likely that the level of industrial production reflects the combination of the underlying external and internal orders. Thus, a rise in production to order can turn to an increase in the amount of exports and also stimulate fixed capital investment. In addition, the value of the Japanese yen in real terms can be closely related to the price competitiveness of export goods. This may have significant influences not merely on exports but also on fixed investment associated with the production of export goods. Hence, including the indices of industrial production and a real effective exchange rate in the model is considered as a promising approach to an appropriate representation of the data. The joint CVAR analysis of the four variables, indeed, results in the finding that there exists a stable long-run economic linkage characterising the underlying interactions among these variables. The CVAR system paves the way for an equilibrium correction system representing short-run as well as long-run dynamics of the variables.

The empirical investigation in this paper casts much light on critical roles played by internal and external demand in the Japanese economy, giving support to the usefulness of Keynesian macroeconomic views. See Aoki and Yoshikawa (2002) as well as Yoshikawa (2003), *inter alia*, for recent research developments in the role of aggregate demand in macroeconomics. To the best of the author's knowledge, the present paper can be viewed as the first empirical study that has led to a data-congruent representation of interactions between Japan's export, fixed investment and industrial production. The results of this paper also complement Kurita (2011), in which external factors are not explicitly taken into account in its analysis. This paper will be useful as an empirical reference for the understanding of the dynamics of internal and external demands in the Japanese economy.

Next, let us make a brief review of econometric methodology employed in this paper, paying particular attention to the analysis of parameter stability. Economic and financial time series data are subject to stochastic trend, so that they should often be perceived as non-stationary processes integrated of order 1 (denoted as $I(1)$ henceforth). A modelling approach based on cointegration is introduced by Granger (1981), an approach enabling us to investigate long-run stationary combinations consisting of non-stationary integrated variables. Johansen (1988, 1996) then develops the likelihood-based analysis of a CVAR system, which has played a key role in the recent advances of methodology in time series analysis. For empirical research based on CVAR models, see Juselius (2006). It is also known that the CVAR analysis is compatible with general-to-specific econometric modelling; see Hendry and Mizon (1993), Hendry (1995), Juselius (2006) and Kurita (2007, 2011), *inter alia*. Thus, the CVAR analysis can provide us with a promising route towards the attainment of a parsimonious data-congruent representation.

Furthermore, the constancy of cointegrating parameters in a multivariate system needs to be checked so as to ensure the validity of standard asymptotic inferences. It is therefore important, in empirical research, to ascertain that a set of estimated cointegrating parameters are stable over some sample periods of economic interest. Hansen and Johansen

(1999), employing the technique of recursively estimating a CVAR system, introduce and explore various tests for parameter stability over time. Some of their recursive test statistics, which are based on Nyblom (1989), are formulated as quasi Lagrange multiplier (LM) tests and can be used as diagnostic methods for the purpose of checking the constancy of cointegrating parameters. This paper, using this recursive-estimation technique as well as some other methods, investigates the stability of an empirical mechanism for long-run equilibrium correction.

The rest of this paper is organised as follows. Section 2 reviews a likelihood-based CVAR analysis of non-stationary time series data. Section 3 carries out a CVAR analysis of Japanese data prior to the trade decline in 2008 and also seeks a parsimonious representation of the data. Section 4 then extends the sample period for estimation and examines the stability of an identified long-run economic linkage. Concluding remarks are presented in Section 5. The empirical and graphical analyses of this paper are performed using *CATS* in *RATS* (Dennis, Hansen, Johansen and Juselius, 2005) and *PcGive* (Doornik and Hendry, 2007).

2 An overview of econometric framework

This section makes a brief review with regard to the likelihood-based analysis of non-stationary time series data. The primary references for the review in this section are Johansen (1988, 1996) and Juselius (2006). First, a joint unrestricted VAR(k) model for a p-dimensional economic time series $X_{-k+1}, \cdots, X_T$ is given by

$$\Delta X_t = (\Pi, \Pi_l) \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t, \quad \text{for} \quad t = 1, ..., T, \tag{1}$$

where $\varepsilon_1, \cdots, \varepsilon_T$ denote errors distributed as independent and identical normal $N(0, \Omega)$ conditional on the initial values $X_{-k+1}, ..., X_0$, while the parameters Π, Γ_i, $\Omega \in \mathbf{R}^{p \times p}$ and Π_l, $\mu \in \mathbf{R}^p$ all vary freely, with Ω being a positive definite matrix. Let us then introduce three regularity conditions for the likelihood-based $I(1)$ cointegration analysis using equation (1):

1. The roots of a characteristic equation $\left|(1-z) I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_j (1-z) z^i\right| = 0$ satisfy either $|z| > 1$ or $z = 1$.

2. $(\Pi, \Pi_l) = \alpha(\beta', \gamma)$, where $\alpha, \beta \in \mathbf{R}^{p \times r}$ are of full column rank $r < p$, while $\gamma \in \mathbf{R}^r$.

3. $rank\,(\alpha_\perp' \Gamma \beta_\perp) = p - r$, where $\alpha_\perp, \beta_\perp \in \mathbf{R}^{p \times p-r}$ are orthogonal complements such that $\alpha' \alpha_\perp = 0$ and $\beta' \beta_\perp = 0$ with $(\alpha, \alpha_\perp)$ and $(\beta, \beta_\perp)$ being of full rank p, while $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$.

The first condition ensures that the process is neither explosive nor seasonally cointegrated. The second is perceived as a reduced rank condition, implying that at least $p-r$ common stochastic trends exist and cointegration arises for $r \geq 1$. Let us define $\beta^{*\prime} = (\beta', \gamma)$ and $X_{t-1}^{*} = \left(X_{t-1}', t\right)'$ for future reference. The parameters β^{*} and α are called cointegrating and adjustment vectors respectively, while r is referred to as cointegrating rank. The third condition is a full rank condition, preventing the process from being $I(2)$ or of higher order. If these conditions are all satisfied, the Granger-Johansen representation of equation (1) can be derived as follows:

$$X_t = C \sum_{i=1}^{t} \varepsilon_i + \xi_t + \tau_c + \tau_l t + A, \tag{2}$$

where $C = \beta_{\perp}(\alpha_{\perp}'\Gamma\beta_{\perp})^{-1}\alpha_{\perp}'$ is a composite parameter for cumulated innovations, ξ_t is seen as a mean-zero stationary process by choosing its initial value appropriately, and A depends on the initial values such that $\beta' A = 0$. According to Johansen, Mosconi and Nielsen (2000), the parameters for deterministic terms, τ_c and τ_l, satisfy the following relationships:

$$\beta'\tau_c = \overline{\alpha}'\left(\Gamma C - I_p\right)\mu + \overline{\alpha}'\left(\Gamma C \Gamma - \Gamma\right)\overline{\beta}\gamma - \gamma, \tag{3}$$

$$\tau_l = C\mu + \left(C\Gamma - I_p\right)\overline{\beta}\gamma, \tag{4}$$

where $\overline{\alpha} = \alpha(\alpha'\alpha)^{-1}$ and $\overline{\beta} = \beta(\beta'\beta)^{-1}$ so $\alpha'\overline{\alpha} = \beta'\overline{\beta} = I_r$. It is found that $\beta'\tau_l = -\gamma$ holds due to $\beta' C = 0$. Note that $\beta' X_t + \gamma$ or $\beta^{*\prime} X_{t-1}^{*}$ can be seen as a stationary process with its mean given as $\beta'\tau_c$, which is defined in equation (3). See also Hansen (2005) for the explicit expression of the Granger-Johansen representation (2). Under the three conditions above, the following CVAR model is well defined as a sub-model of (1) with the representation (2) being fulfilled:

$$\Delta X_t = \alpha\beta^{*\prime} X_{t-1}^{*} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t, \tag{5}$$

Equation (5) paves the way for subsequent cointegration analysis and model reduction.

The index r, which is called cointegrating rank, is usually unknown to researchers; thus it is often necessary to determine r based on a formal statistical analysis. The method of reduced rank regression (see Anderson, 1951) allows us to find that maximising the log-likelihood function for (5) boils down to solving a generalised eigenvalue problem. A log-likelihood ratio ($\log LR$) test statistic is then composed of the null hypothesis of r cointegration rank, $\mathsf{H}(r)$, against the alternative hypothesis, $\mathsf{H}(p)$. The statistic's limiting quantiles are provided by Johansen (1996, Ch.15); see also Nielsen (1997) and Doornik (1998) for gamma approximation methods for calculating the quantiles. Hence it is possible to conduct a likelihood-based asymptotic inference for the choice of r. Determining r in (5) then enables us to test various restrictions on α, β and γ in order to seek the

adjustment structure and cointegrating relationships subject to economic interpretations. It should be noted that cointegrating relationships, $\beta^{*\prime} X_{t-1}^{*}$, which represent a set of stationary linear combinations of the variables, can be seen as a mechanism for equilibrium correction in equation (5).

Next, let the process be broken down into $X_t = (Y_t', Z_t')'$ for $Y_t \in \mathbf{R}^m$, $Z_t \in \mathbf{R}^n$, $m \geq r$, with a view to deriving a partial or conditional data-representation from the cointegrated VAR system. The parameters and error terms of (5) are, in the corresponding manner, expressed as $\alpha = \left(\alpha_y', \alpha_z'\right)'$, $\Gamma_i' = \left(\Gamma_{y,i}', \Gamma_{z,i}'\right)'$ and $\varepsilon_t = \left(\varepsilon_{y,t}', \varepsilon_{z,t}'\right)'$. The variances of $\varepsilon_{y,t}$ and $\varepsilon_{z,t}$ are given as Ω_{yy} and Ω_{zz} respectively, while its covariance is expressed as Ω_{yz} or Ω_{zy}. Suppose that $\alpha_z = 0$ holds, so that (5) is split into a conditional model for Y_t given Z_t and a marginal model for Z_t:

$$\Delta Y_t = \omega \Delta Z_t + \alpha_y \beta^{*\prime} X_{t-1}^{*} + \sum_{i=1}^{k-1} \widetilde{\Gamma}_{y,i} \Delta X_{t-i} + \widetilde{\varepsilon}_{y,t}, \tag{6}$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \varepsilon_{z,t}, \tag{7}$$

where $\omega = \Omega_{yz}\Omega_{zz}^{-1}$, $\widetilde{\Gamma}_{y,i} = \Gamma_{y,i} - \omega \Gamma_{z,i}$ and $\widetilde{\varepsilon}_{y,t} = \varepsilon_{y,t} - \omega \varepsilon_{z,t}$. The conditional normal innovation $\widetilde{\varepsilon}_{y,t}$ has variance $\Omega_{yy} - \Omega_{yz}\Omega_{zz}^{-1}\Omega_{zy}$ and is thus independent of $\varepsilon_{z,t}$ by construction. Note that the marginal model, (7), is free of $\beta^{*\prime} X_{t-1}^{*}$. If the condition of $\alpha_z = 0$ is satisfied, Z_t is then said to be weakly exogenous for the parameters of the conditional model, (6). These parameters can be estimated from the conditional model (6) alone without any loss of information. It is not necessary, as a result, to estimate the marginal model (7), provided that the parameters of interest correspond to those in (6). This property can facilitate model reduction in econometric research and thus will be utilised in this empirical study. See Johansen (1992) and Urbain (1992) for further details of weak exogeneity in cointegrated VAR systems.

3 Dynamic econometric modelling of Japanese data

This section, consisting of three sub-sections, aims to seek a stable empirical model for Japan's macroeconomic interactions over a sample period preceding the global economic depression in 2008. A detailed analysis of quarterly time series data is performed using econometric methodology reviewed in the previous section. A set of macroeconomic variables to be studied is given as $X_t = (in_t, ex_t, ip_t, re_t)'$, where in_t is the log of real fixed investment, ex_t is the log of real export of goods and services, ip_t is the log of an index of industrial production, and re_t is the log of a real effective Japanese yen exchange rate. See the Appendix for further details of the data and their graphical overview. The sample period runs from the first quarter in 1994 to the second quarter in 2008, denoted by 1994.1-2008.2 hereafter. The end point in the sample period reflects the US financial

crisis starting in September 2008. A full-sample analysis using the data up to 2012.1 is conducted in Section 4.

3.1 Selecting the cointegrating rank

This sub-section investigates the underlying cointegrating rank by estimating a well-formulated unrestricted VAR model. The lag order of the VAR model is chosen for 2, judging from F test statistics for the model reduction. The sample period effective for estimation is thus 1994.3-2008.2. The unrestricted VAR model may be seen as a *statistical* representation of the data, with the result that the estimated coefficients are not necessarily subject to meaningful economic interpretations. In order to seek and justify such interpretations, it is essential to identify the underlying cointegrating relations and subsequently obtain a parsimonious model by removing irrelevant regressors. The unrestricted VAR model, thus, needs to represent the data adequately to validate such cointegration analysis and model reduction.

Single equation tests	in_t	ex_t	ip_t	re_t
Autocorr.[F_{ar}(4,42)]	0.70 [0.60]	2.30 [0.07]	0.78 [0.54]	1.64 [0.18]
ARCH [F_{arch}(4,38)]	0.42 [0.79]	0.45 [0.77]	0.85 [0.51]	0.46 [0.76]
Hetero.[F_{het}(18,27)]	0.27 [1.00]	0.53 [0.92]	0.33 [0.99]	0.82 [0.66]
Normality [$\chi^2_{nd}(2)$]	0.99 [0.61]	4.19 [0.12]	5.34 [0.07]	3.77 [0.15]

System tests			
Autocorr.[F_{ar}(16,119)]	0.67 [0.82]	Hetero.[F_{het}(180,179)]	0.61 [1.00]
- [F_{ar}(64,107)]	0.88 [0.71]	Normality [$\chi^2_{nd}(8)$]	10.2 [0.25]

Note. The figures in the square brackets are p-values.

Table 1: Residual diagnostic tests for the unrestricted VAR model

Table 1 summarises residual diagnostic tests for the unrestricted VAR model. Most of the test statistics are presented in the form $\mathrm{F}_j(k, T-l)$, which signifies an approximate F test against the alternative hypothesis j: kth-order serial correlation (F_{ar}: see Godfrey, 1978, Nielsen, 2006), kth-order ARCH (F_{arch}: see Engle, 1982), heteroscedasticity (F_{het}: see White, 1980). A chi-square test for normality (χ^2_{nd}: see Doornik and Hansen, 2008) is also reported. All the mis-specification test statistics reported in the table are insignificant at the 5% level, thus permitting us to conclude that the model is well fitted as a whole.

The model is now put in a position to be utilised for formal cointegration analysis. Table 2 reports a set of two types of $\log LR$ test statistics for cointegrating rank, together with some relevant numerical outputs. The first panel of the table displays standard

$\log LR$ test statistics for the rank, according to Johansen (1996, Ch.6), which are denoted by $-2\log Q\left[\mathsf{H}\left(r\right)|\mathsf{H}\left(p\right)\right]$. The panel also provides Bartlett-corrected $\log LR$ test statistics based on Johansen (2002), *i.e.* $\log LR$ test statistics adjusted for the small sample size, which are represented by $-2\log Q^{BC}\left[\mathsf{H}\left(r\right)|\mathsf{H}\left(p\right)\right]$. Judging from the results in this panel, one finds that the null hypothesis of $r=0$ is highly rejected by both types of test statistics at the 5% significance level, while that of $r\leq 1$ is not rejected by both test statistics at the same level, although the non-rejection by the standard test is rather marginal. By attaching more importance to the Bartlett-corrected tests than the standard ones in this small-sample analysis, one can be justified in choosing $r=1$ as the appropriate cointegrating rank.

In order to consolidate the argument for the choice of $r=1$, the second panel in Table 2 reports modulus (denoted by *mod*) of the six largest eigenvalues of the CVAR model's companion matrix, under the restriction of $r=1$. These are the reciprocal values of the roots of a characteristic equation for the estimated CVAR model. All the eigenvalues, except for the imposed three unit roots ($p-r=3$), are distinct from 1, which suggests that the selection of $r=1$ is appropriate with no $I(2)$ roots being involved in the model. In addition, there is no eigenvalue over 1, indicating that the model does not include any explosive root. These results are all in favour of the adequacy of the $I(1)$ CVAR model with $r=1$.

The overall findings enable us to conclude that the cointegrating rank is set at 1 or $r=1$, which lays the foundations for a further cointegrated VAR analysis of the set of $I(1)$ data.

	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
$-2\log Q\left[H\left(r\right)\|H\left(p\right)\right]$	97.94[0.00]**	42.45[0.05]	18.55[0.31]	3.34[0.83]
$-2\log Q^{BC}\left[H\left(r\right)\|H\left(p\right)\right]$	84.28[0.00]**	38.35[0.13]	17.07[0.42]	3.18[0.85]

mod $(r=1)$	1	1	1	0.68	0.68	0.37

Note. The figures in the square brackets are p-values.

Table 2: Determination of the cointegrating rank

3.2 Revealing the long-run economic relationship and its feedback mechanism

The choice of the cointegrating rank ($r=1$) allows us to examine various restrictions on the adjustment and cointegrating vectors, relying on standard χ^2-based asymptotic inferences. In order to draw some useful implications from the estimated model, it is essential to identify an interpretable cointegrating relationship and inspect how its feed-

	in_t	ex_t	ip_t	re_t
$\log LR$	26.70[0.00]**	5.34[0.02]*	0.04[0.84]	0.82[0.37]

Note. The figures in the square brackets are p-values according to $\chi^2(1)$.

Table 3: Testing for weak exogeneity

back mechanism works in the model. As the starting point of investigation, the model is subject to tests for weak exogeneity. Testing for weak exogeneity is synonymous with testing a zero-restriction on the adjustment vector α, as demonstrated in Section 2. If either or both of the variables ip_t and re_t are judged to be weakly exogenous with respect to the parameters of interest such as β^*, it justifies estimating a partial model centring upon ex_t and in_t given weakly exogenous variables. This reduction, augmented with further model reduction, will lead us to a parsimonious empirical representation of Japan's export-investment mechanism. Table 3 reports a series of $\log LR$ test statistics for the null hypothesis of weak exogeneity. The hypothesis is rejected for ex_t and in_t at the 5% significance level, whereas it is not rejected for ip_t and re_t at the same level. One is, therefore, able to infer that both ip_t and re_t are weakly exogenous with respect to the parameters of the bivariate partial model for ex_t and in_t. This result paves the way for a parsimonious empirical model for Japan's macroeconomic dynamics.

	in_t	ex_t	ip_t	re_t	t	$\chi^2(3)$
$\widehat{\alpha}'$	-0.862 (0.134)	0.409 (0.133)	0 (−)	0 (−)	−	1.05[0.79]
$\widehat{\beta^*}'$	1 (−)	-0.137 (0.017)	-0.984 (0.059)	0.036 (0.027)	0 (−)	

Note. The figures in the parentheses are standard errors.

Table 4: Restrictions on α and β^*

Identifying a set of weakly exogenous variables then leads us to inspect joint restrictions on the adjustment and cointegrating vectors in the CVAR model. Since the interpretation of deterministic trend is not always straightforward from the standpoint of economic theory, it is worthwhile to test for the exclusion of the trend from the estimated cointegrating combination. Table 4 displays a set of parameter estimates, in which a zero restriction is imposed on the cointegrating parameter γ for deterministic trend, with zero restrictions being simultaneously imposed on the adjustment parameters for ip_t and re_t. Let us also note that the cointegrating vector is normalised with respect to in_t. The table provides the corresponding $\log LR$ test statistic and p-value, according to which the null hypothesis of the joint restrictions is not rejected at the 5% level; this evidence indicates that

	in_t	ex_t	ip_t	re_t	t	$\chi^2(5)$
$\widehat{\alpha}'$	-0.793 (0.128)	0.396 (0.124)	0 (−)	0 (−)	−	2.28[0.81]
$\widehat{\beta^*}'$	1 (−)	-0.152 (0.009)	-1 (−)	0 (−)	0 (−)	

Note. The figures in the parentheses are standard errors.

Table 5: Identification of the equilibrium correction mechanism

the deterministic trend can be eliminated from the cointegrating combination. According to the Granger-Johansen representation (2), $\gamma = 0$ implies that the parameters for the deterministic terms, τ_c and τ_l, can be simplified as

$$\beta'\tau_c = \overline{\alpha}'\left(\Gamma C - I_p\right)\mu \quad \text{and} \quad \tau_l = C\mu, \tag{8}$$

which will be utilised in the study of a structural break in Section 4.

Furthermore, according to Table 4, the estimate of the cointegrating parameter for ip_t turns out to be close to -1, whereas that for re_t is not significantly different from 0. These results suggest a direction for another set of restrictions to be imposed on the cointegrating vector; that is, the value of -1 is imposed on the estimate for ip_t and a zero restriction is also placed on the estimate for re_t, in addition to those restrictions already employed in Table 4. Under these joint restrictions the CVAR model is re-estimated and its results are summarised in Table 5. Again, the null hypothesis of the overall restrictions is not rejected at the 5% level.

The set of restricted estimates in Table 5 allows us to find an empirical equilibrium correction mechanism (ECM) acting on both in_t and ex_t as follows:

$$\widehat{\beta^*}X^*_{t-1} = \widehat{\beta}X_{t-1} = in_{t-1} - ip_{t-1} - 0.152ex_{t-1}, \tag{9}$$

which can be seen as a stationary deviation from a long-run equilibrium combination. Note that equation (9), normalised for in_{t-1}, is a single identified cointegrating relationship embedded in the estimated model. It is, therefore, possible to solve equation (9) with respect to in_{t-1} to infer that fixed investment tends to increase with both industrial production and export; see Johansen (2005) for details of the interpretation of cointegrating relationships. It seems likely that the growth of industrial production and export raises expected future profitability in the manufacturing industry. The outlook for positive business conditions leads to a rise in the level of desired capital stock for the future, thus resulting in an increase in fixed capital investment. A deviation from the equilibrium mean, which corresponds to equation (9), turns to a stationary error process, acting as an effective mechanism for equilibrium correction. The revealed ECM, (9), indicates that there is a close linkage among fixed investment, industrial production and export. The

ECM can also be seen as evidence suggesting that the real effective exchange rate does not play a significant role in the long-run structure of the system.

3.3 Estimating a dynamic trivariate system

The cointegration study conducted above has led us to a position where a parsimonious representation of Japan's macroeconomic dynamics can be pursued. The model reduction commences with transforming the set of variables to $I(0)$ series by taking the first-order differences of the four variables and using the disequilibrium error defined in (9). Next, the author estimates a trivariate $I(0)$ equilibrium correction system for Δin_t, Δex_t and Δip_t conditional on one of the weakly exogenous variables, Δre_t, by regarding the first three variables as the variables of research interest; this classification can be justified by the fact that the long-run equilibrium relationship (9) consists of these three variables. In the regression analysis of the trivariate simultaneous equation system, a set of insignificant contemporaneous and lagged regressors are dropped from the system step by step; it turns out that Δre_t and its past values play no significant roles in the trivariate model. As a consequence of the model reduction, a parsimonious equilibrium correction representation is achieved as follows:

$$\begin{aligned}
\Delta in_t &= -\underset{(0.089)}{0.718}\,\widehat{\beta}X_{t-1} - \underset{(0.087)}{0.121}\,\Delta in_{t-1} + \underset{(0.43)}{3.487} + \widehat{\varepsilon}_{1,t}, \\
\Delta ex_t &= \underset{(0.249)}{0.513}\,\Delta ip_t + \underset{(0.118)}{0.361}\,\widehat{\beta}X_{t-1} + \underset{(0.211)}{0.911}\,\Delta ip_{t-1} - \underset{(0.572)}{1.74} + \widehat{\varepsilon}_{2,t}, \\
\Delta ip_t &= -\underset{(0.068)}{0.148}\,\Delta in_{t-1} + \underset{(0.083)}{0.435}\,\Delta ex_{t-1} + \underset{(0.118)}{0.241}\,\Delta ip_{t-1} - \underset{(0.002)}{0.004} + \widehat{\varepsilon}_{3,t},
\end{aligned} \tag{10}$$

$$System\ tests: F_{ar}(36,115) = 1.20[0.23],\ \chi^2_{nd}(6) = 4.17[0.65],\ F_{het}(72,196) = 1.00[0.49],$$

where the small figure under each coefficient is the corresponding standard error, while the standard errors of the equations for Δin_t, Δex_t and Δip_t are 0.018, 0.018 and 0.013, respectively. The diagnostic test statistics are all insignificant, suggesting that the parsimonious system (10) is seen as a satisfactory representation of the data. Furthermore, Figure 1 (a), (b) and (c) plot the model's fitted values against the actual values, Figure 1 (d), (e) and (f) display residual quantile-quantile (QQ) plots against the standard normal, while Figure 1 (g), (h) and (i) record residual correlograms. All the graphs indicate no evidence against congruency, which is in line with the set of insignificant test statistics above. All the diagnostic analyses support the validity of the view that the overall system (10) is a data-congruent representation.

Let us inspect some of the estimated coefficients of the parsimonious system (10). As expected, the equilibrium correction term holds a significant coefficient in each of the equations for Δin_t and Δex_t, suggesting the existence of stable adjustment towards the long-run equilibrium mean. Furthermore, it should be noted that the coefficients

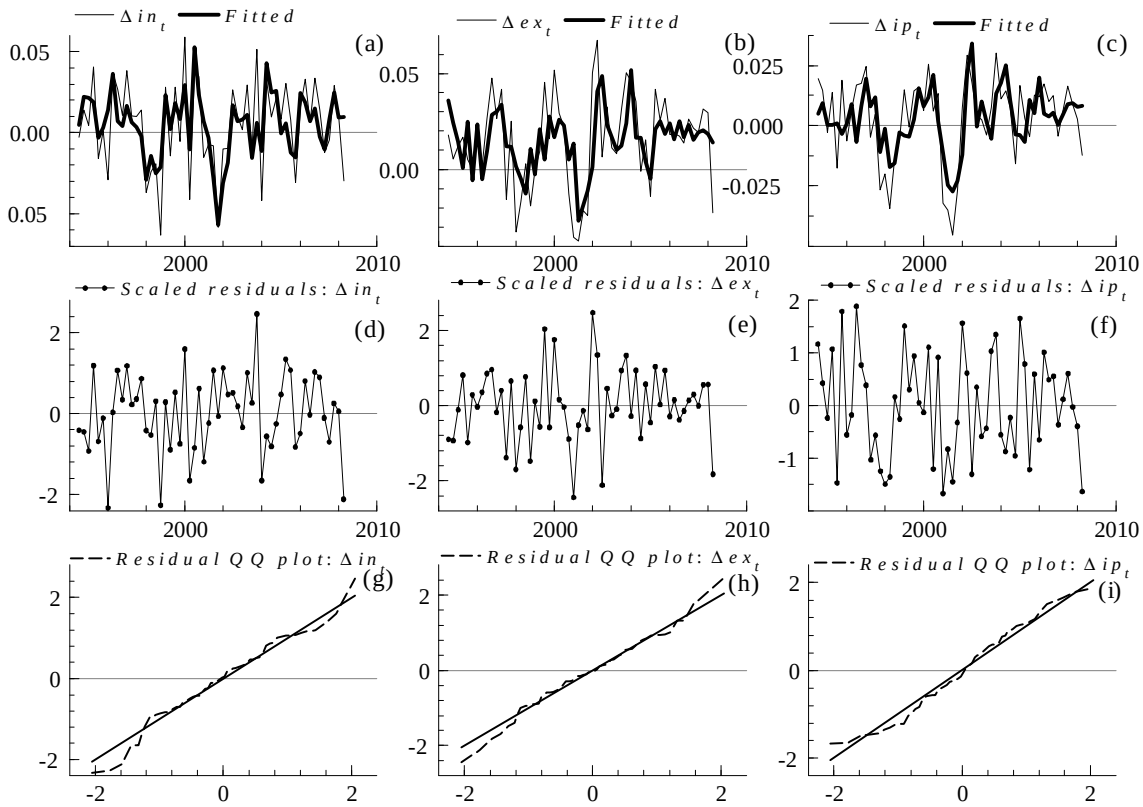

Figure 1: Fitted values, scaled residuals and residual QQ plots

for Δip_{t-1} and Δip_t are positive and highly significant in the equation for Δex_t; this finding indicates that lagged and contemporaneous increases in industrial production are closely linked to the growth of export. The underlying structure of this linkage would be that Japan's industrial production is heavily influenced by the amount of orders received from overseas. Domestic products in response to such overseas demand are successively exported, thereby generating the observed effects of Δip_{t-1} and Δip_t on Δex_t in the second equation in the system (10). It is thus inferred that the underlying true causality should run from overseas demand to domestic production. This inference seems to be supported by the finding that Δex_{t-1} has a highly significant positive coefficient in the Δip_t equation; that is, the lagged growth of export leads to an increase in domestic industrial production.

As demonstrated above, the equilibrium correction system (10) is judged to be free from any significant mis-specification problems and can also be subject to some economic interpretations. The system can thus be seen as a reliable data-representation prior to the worldwide depression starting in late 2008.

4 The stability of the long-run economic linkage

This section investigates the stability of the ECM, which is specified as equation (9), over the pre and post crisis periods. First, the study attempts to ensure that the mechanism is stable over the sub-sample period ending in 2008.2, adopted in the analysis in Section 3 above. All the parameters of the CVAR model under the restrictions in Table 5 are recursively estimated, in order to generate a time series plot of a quasi LM supremum or sup test statistic for the stability of $\widehat{\beta}$. For details of the recursive analysis of the CVAR model, see Hansen and Johansen (1999) as well as Juselius (2006). The recursive graph over 1999.2 -2008.2 is presented in Figure 2 (a). According to the figure, the sup test statistic stays well below the 5% critical value, indicating that no evidence is found against the null hypothesis of constancy of the cointegrating parameters. Moreover, the ECM is plotted over the whole sub-sample period in Figure 2 (b). As expected, the term seems to be a stationary process with neither noticeable break nor shift, exhibiting a stable mean-reverting behaviour. These results are all in line with the diagnostic tests conducted for equation (10) in Section 3.3. The overall evidence, thus, allows us to conclude that the ECM is judged stable over the sub-sample period, providing a good starting point for a further examination of this issue.

We are now in a position to extend the sample period up to 2012.1 so that the stability of the ECM can be inspected over the post-crisis period. The estimation of the CVAR model is recursively conducted to the point 2012.1. A recursive plot of the supremum test statistic is displayed in Figure 3 (a) in the same manner as Figure 2. It seems, judging from the figure, that the trade decline in the early phase of the global economic depression has an impact on the mechanism of equilibrium correction. The supremum test statistic surges beyond the 5% critical value in 2009.1, implying some instability in the mechanism around this period. The test statistic, however, goes down quickly after 2009.1 and then lies below the level of the 5% critical value. In addition, a plot of the ECM is displayed in Figure 3 (b) over the same period as that adopted in Figure 3 (a), so that one can compare its movement in the recovery period of the Japanese economy, as discussed in the Introduction, with the movement of the ECM in the post-crisis period. According to the figure, the ECM spikes and reaches its peak in 2009.1, so the effect of the external shock is evident again. However, the ECM turns back to the pre-crisis level afterwards and moves smoothly like a stationary process as observed in the pre-crisis period. Overall, these observations suggest that the mechanism for equilibrium correction may be restored swiftly after the external shock hits the economy.

Considering the interdependent nature of export, fixed investment and industrial production, one finds it likely that all of these three variables are affected in almost the same degree and direction by the large-scale external shock. According to Hendry (1997), Hendry and Mizon (1998) and Clements and Hendry (1999, Ch.9), the concept of co-breaking indicates a situation where structural breaks are cancelled out when taking

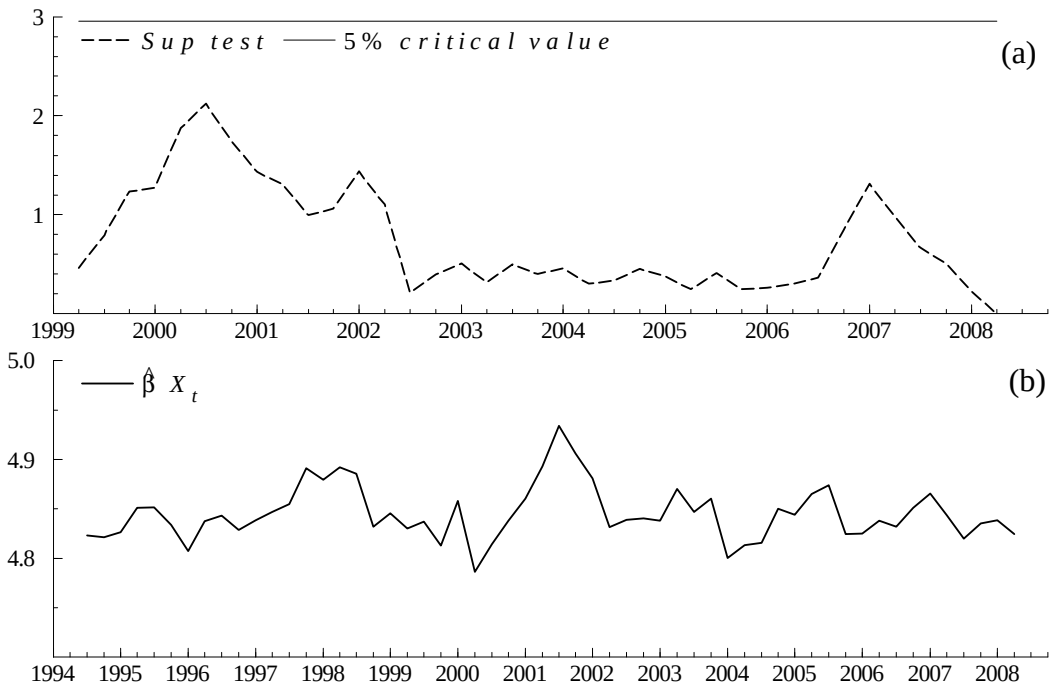

Figure 2: Stability of the ECM before the external shock

some linear combinations of variables in a system. See Hendry and Massmann (2007) for details of co-breaking and its latest research developments. The overall evidence in Figure 3 suggests that a type of empirical co-breaking may have an effect on the ECM after the external shock. In other words, the influence of the crisis on each macroeconomic variable is significant and persistent, but its impact on the long-run economic linkage itself may be short-lived as a consequence of co-breaking effects.

With a view to revealing the structure of co-breaking possibly taking place in the Japanese data, let us recall the set of parameters (8) for the representation (2) with $\gamma = 0$ and also introduce a shift parameter μ^*, the specification of which is made possible by following the thought experiments of Johansen (2005). Suppose that μ is shifted to $\mu + \mu^*$ as a result of the external shock hitting the economy; namely, the impact may be seen as a step change in the constant term. Recalling $C = \beta_\perp \left(\alpha_\perp' \Gamma \beta_\perp\right)^{-1} \alpha_\perp'$, one may define $\mu^* = \Gamma \beta_\perp \delta$ for some $\delta \in \mathbf{R}^{p-r}$ as shown in Proposition 1 in Johansen (2005), so that the expansion of τ_c in the direction of β can bring about

$$\begin{aligned}
\beta' \tau_c &= \overline{\alpha}' \left(\Gamma C - I_p\right)\left(\mu + \mu^*\right) \\
&= \overline{\alpha}' \left(\Gamma C - I_p\right)\mu + \overline{\alpha}' \left\{\Gamma \beta_\perp \left(\alpha_\perp' \Gamma \beta_\perp\right)^{-1} \alpha_\perp' \Gamma \beta_\perp \delta - \Gamma \beta_\perp \delta\right\} \\
&= \overline{\alpha}' \left(\Gamma C - I_p\right)\mu.
\end{aligned}$$

That is, although the impact of the external shock itself may be rather persistent, μ^* as

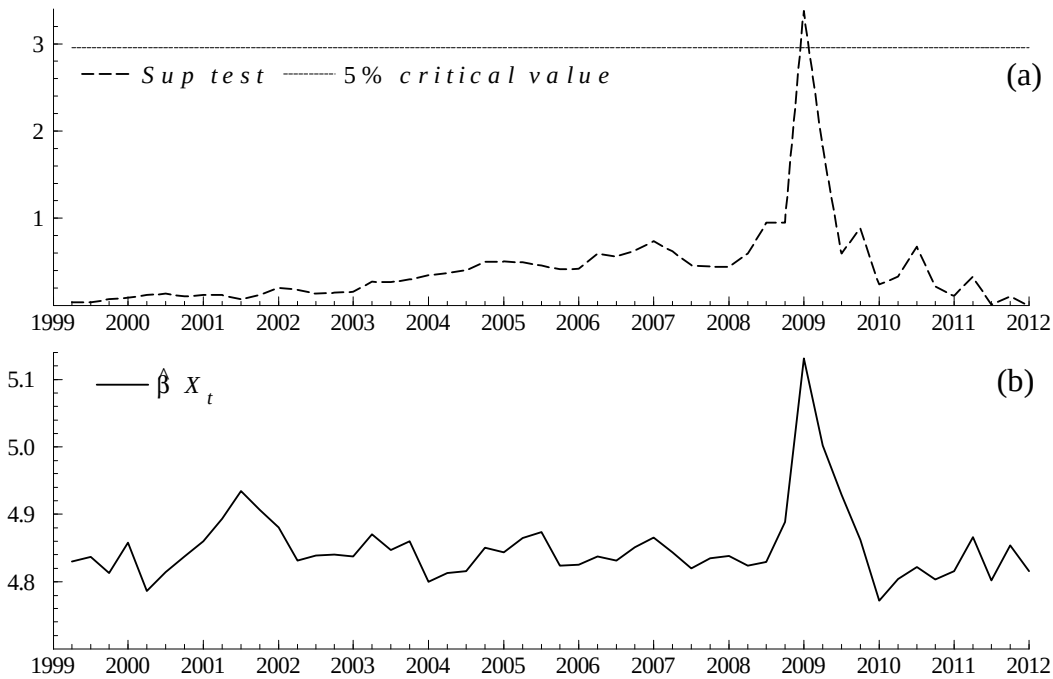

Figure 3: Stability of the ECM after the external shock

defined above is eliminated from the linear combination $\beta'\tau_c$, which corresponds to the equilibrium mean of the ECM; the cointegrating relationship is recognised as a co-breaking relationship as well in this case. This may be considered to be a type of cointegration co-breaking as demonstrated by Hendry and Massmann (2007). This reasoning explains the possibility that the mechanism for equilibrium correction is recovered quickly after the external shock, although the influence of the shock itself remains in each variable in the system. It may also be inferred that the shift parameter μ^* should have some impacts on the series shown in Figure 3, giving rise to some irregular observations, despite the stability of the cointegrating parameters themselves.

In addition, let us inspect the trend parameter τ_l in the Granger-Johansen representation. Owing to $\gamma = 0$ and $\mu^* = \Gamma\beta_\perp\delta$, the parameter τ_l after the shock is expressed as

$$\tau_l = C\left(\mu + \mu^*\right) = C\mu + \beta_\perp\left(\alpha_\perp'\Gamma\beta_\perp\right)^{-1}\alpha_\perp'\Gamma\beta_\perp\delta = C\mu + \beta_\perp\delta,$$

which confirms that the trend shift is embodied in the space of $\beta_\perp$, thus one can see that its impact is also removed when taking the cointegrating combination of the variables in the system.

This line of reasoning allows us to infer that, except for a large spike at the point of the trade decline, the ECM may remain largely intact and act as an empirical anchor for the trivariate equilibrium correction system even after the crisis point. Hypothetically

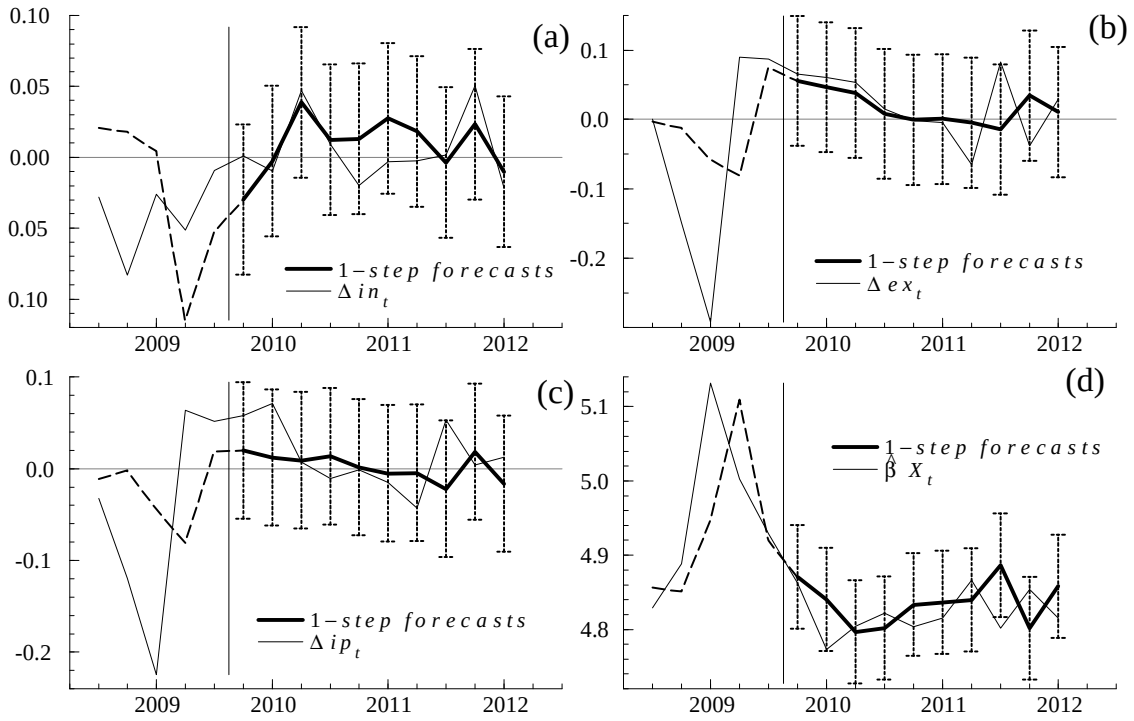

Figure 4: Sequences of 1-step forecasts

speaking, if the equilibrium mean were shifted after this point, which is contrary to the argument above, the model would yield a sequence of systematic mis-forecasts as a result of adjusting towards the imposed equilibrium rather than correcting deviations from the new equilibrium (see Clements and Hendry, 1999, Ch.3). In order to examine this issue empirically, the equilibrium correction system (10) is re-estimated from the data up to 2009.3 with $\widehat{\beta}$ being fixed at the values given in (9), so that 1-step ahead forecasts can be calculated from it from 2009.4 onwards. The sequences of forecasts for Δin_t, Δex_t, Δip_t and $\widehat{\beta}X_t$ are displayed in Figure 4, together with their approximate 95% confidence intervals. According to the figure, no systematic mis-forecast seems to be observed over the out-of sample horizon. This evidence is in support of the view that the ECM restores its inherent function after the crisis point, owing to plausible co-breaking effects. Thus, it is inferred that the trivariate equilibrium correction system can be used as a forecasting device over the post-crisis period.

5 Summary and conclusion

Analysing Japan's quarterly macroeconomic data, this paper attains a small-scale econometric system for dynamic interactions between internal and external aggregate demands.

It is also examined whether or not the ECM of the empirical system is judged stable in the recent years characterised by global economic depression. For this objective, the former part of the empirical analysis (Section 3) focuses on a sub-sample period prior to late 2008, when a structural break triggered by the US financial crisis hit the Japanese economy. The latter part of the analysis (Section 4) extends the sample period up to the end of 2011 and examines the influence of this break on the identified ECM. The econometric methodology used in this paper is briefly reviewed in Section 2 before launching the econometric investigations.

In Section 3, a detailed CVAR analysis is carried out using Japan's quarterly time series data for fixed investment, export, industrial production and a real effective exchange rate. The analysis successfully reveals a long-run economic relationship embedded in the data, a relationship characterising the underlying structural interactions among the macroeconomic variables of interest. It is also discussed that the identified long-run economic linkage can be subject to some meaningful economic interpretations. The CVAR system is then reduced to a trivariate dynamic system incorporating the long-run linkage as its ECM. The trivariate equilibrium correction system passes a series of diagnostic tests and also demonstrates various short-run dynamic properties of the data. Overall, the system can be viewed as a reliable representation of the data prior to the global depression starting in late 2008.

Moving on to Section 4, the study ensures, as a preliminary examination, that the ECM is judged to be stable over the sub-sample period analysed in the preceding section. Next, the sample period is extended to the end of 2011 so that the stability of the ECM in the post-crisis period can be investigated. The empirical investigation indicates that, although the impact of the external shock is significant, the mechanism for equilibrium correction may be restored quickly after that shock hits the economy. It is therefore inferred that a type of co-breaking may take place inside the ECM, thereby eliminating an equilibrium-mean break. With a view to consolidating this inference, the Granger-Johansen representation of the system is utilised to comprehend possible co-breaking effects occurring in the data. The subsequent forecasting exercise also provides some evidence in support of the validity of this inference.

The overall empirical results presented in this paper shed light on dynamic interactions among fixed investment, export and industrial production. Thus, this paper will be viewed as a useful empirical reference for the purpose of grasping the dynamics of internal and external demands in the Japanese economy. The study also contributes to paving the way for further empirical studies of the economy employing recent developments in time series econometrics.

Appendix:

(Data overview)

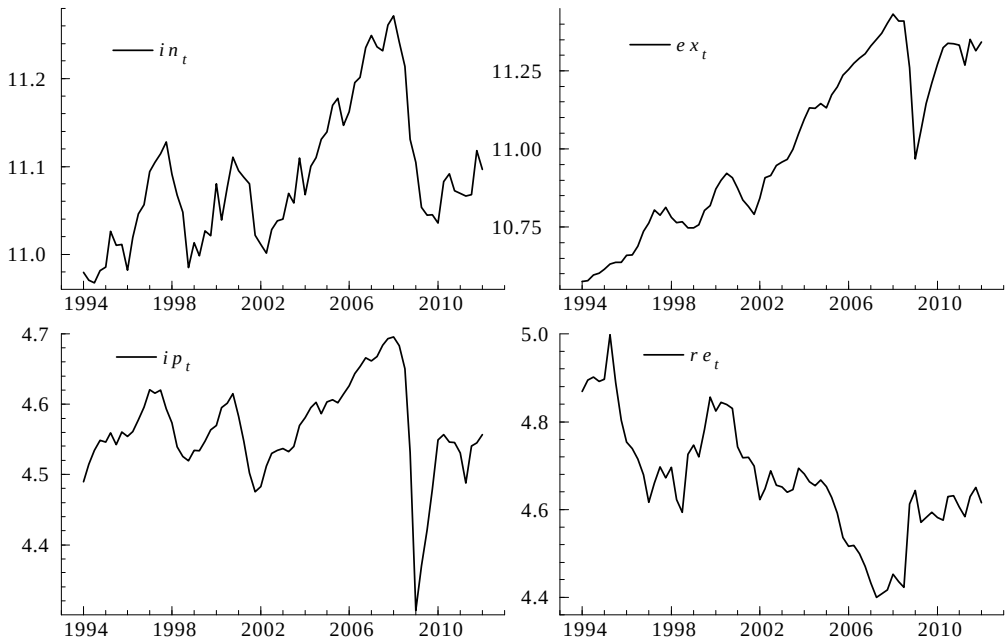

Figure 5: An overview of the data

(Data definitions, sources and notes)

Data definitions:
in_t: the log of real fixed investment (billions of chained 2005 yen prices), <1>
ex_t: the log of real export of goods and services (billions of chained 2005 yen prices), <1>
ip_t: the log of an index of industrial production (mining and manufacturing; year 2005 = 100), <2>
re_t: the log of a real effective Japanese yen exchange rate (year 2010 = 100), <3>

Sources:
<1> National Accounts of Japan (www.esri.cao.go.jp/en/sna/menu.html)
<2> Indices of Industrial Production (www.meti.go.jp/english/statistics/tyo/iip/index.html)
<3> Bank of Japan Time Series Data Search (www.stat-search.boj.or.jp/index_en.html)

Notes:
All the data except for those of re_t are seasonally adjusted. The data for ip_t and re_t are given as the three-month averages of the corresponding monthly series. The data for ip_t from 1994 to 2002 are taken from Connected Indices of Industrial Production in the web site for <2> above.

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