CAES Working Paper Series

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WP-2014-007

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Modeling the nexus of US monetary policy rule and inflation over the past quarter century

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Abstract
This paper performs a time series analysis centering on US monetary policy rule and inflation over the past quarter century. A dynamic system is estimated to reveal the underlying long-run linkage, interpretable as an interest-based monetary policy rule as well as a Phillips-curve representation. It is shown that an effective exchange rate plays a critical role in the long-run analysis. The system is subjected to various econometric investigations, which indicate that inflation is controllable by employing not only the Federal fund rate but a term spread as monetary policy instruments. Finally, the study seeks a dynamic nexus representation.

JEL classifications C32; C51; E31; E52

Keywords Monetary policy rules; Inflation controllability; Term spread; Cointegrated vector autoregressive analysis; Exogeneity; General-to-specific econometric modeling

1. Introduction

This paper makes econometric investigations into the dynamics of US monetary policy rule and inflation over the past quarter century. A dynamic nexus model is estimated as a result of detailed studies of US time series data. This introduction briefly reviews the preceding research on monetary policy rules and the term structure of interest rates. The significant aspects of the present paper are then explained.

Concerning monetary policy rules, it is essential to refer to Taylor (1993) as a seminal paper, which presents an expression of the underlying US monetary policy rule known as a "Taylor rule"; this paper lays the foundations for the vast amount of the subsequent literature on policy rules in monetary economics. See Clarida *et al.* (1998, 2000), Orphanides (2001, 2003), Ball and Tchaidze (2002), to name but a few, for various empirical studies associated with monetary policy rules. From the standpoint of modern time series econometrics, Österholm (2005) argues the possibility that empirical Taylor rules estimated from time series data may be subject to spurious regression (see Phillips, 1986, *inter alia*), a phenomenon which gives rise to doubts about the reliability of estimated monetary policy rules. By addressing the issue of spurious regression, Christensen and Nielsen (2009) have made a significant contribution to time series analysis of monetary policy rules. Christensen and Nielsen (2009), using cointegrated vector autoregressive (CVAR) methodology pioneered by Johansen (1988, 1996),

analyze the US monthly time series data over the period of Alan Greenspan's tenure as the chairman of the US Federal Reserve System, *i.e.* September 1987 to January 2006. They reveal a stable cointegrating relationship among the Federal funds rate, the rate of unemployment and the long-term bond rate; this relationship is interpretable as a Taylor-type monetary policy rule, based on the concepts of the natural rate of unemployment and the term structure of interest rates. An econometric analysis performed by Christensen and Nielsen (2009) has led to remarkable empirical outcomes, which provide impetus to our empirical investigations of the US quarterly time series data. At the same time, various issues can arise regarding the validity of simple policy rules for real-world central banks operating in open economies. See Svensson (2000) and Taylor (2001), *inter alia*, for possible roles of exchange rates in policy rule functions. The present paper, therefore, allows for information on exchange rates in the econometric modeling of the US time series data.

A term spread is, in general, defined as the difference between the long-term and the short-term interest rates and reflects the slope of the underlying yield curve. The term structure of interest rates, frequently represented by the term spread, has been subjected to comprehensive empirical explorations. The spread's predictability of future economic activities, in particular, has attracted much attention in empirical macroeconomic research; see Mishkin (1990), Estrella and Mishkin (1997, 1998), *inter alia*. In view of monetary policy implications of the term spread, we find it important to refer to Estrella and Mishkin (1997), Estrella (2005) and Adrian, Estrella and Shin (2010). According to Estrella and Mishkin (1997), monetary policy acts as one of the important determinants of the term spread behavior, while the spread is also linked to the future rate of inflation. Estrella (2005) presents a theoretical model in which the monetary policy rule and the Phillips curve play important roles in determining the predictability of the term spread. Furthermore, Adrian, Estrella and Shin (2010) argue that the term spread is seen as an indicator of profitability of financial intermediaries in the supply of credit. In order to understand their argument, it is important to note that financial intermediaries such as commercial banks are subject to balance sheet constraints. The shrinkage of the term spread, for example, indicates that the cost of short-term liabilities rises in comparison with the rate of return in long-term loans. This means a decrease in net interest margin obtained from typical short-term borrowing and long-term lending activities. The term spread's contraction, thus, renders marginal long-term loans less profitable than before and results in the reduction of credit supplied into the overall economy. See also English (2002) for this type of argument and some empirical evidence.

In this paper, we conduct an econometric analysis of the US quarterly time series data that are associated with the underlying monetary policy rule and inflation dynamics. The data set includes a US-dollar nominal effective exchange rate, in addition to a class of standard macroeconomic variables such as real gross domestic product (GDP). See Appendix for details of the data analyzed in this paper. A CVAR analysis is applied to the set of presumably non-stationary time series data containing the effective exchange rate as an external factor. As a result, we are able to conduct rigorous time series investigations to consider the possibility that the external factor can play significant roles in accounting for the dynamics of monetary policy and internal inflation. Moreover, this paper pays close attention to the dual aspects of the term structure of interest rates, as indicated by Estrella and Mishkin (1997): one is that the term spread behavior can be associated with the conduct of monetary policy, while the other is that the term spread can contain information on future economic activities, thereby accounting for the dynamics of inflation. As regards the sample period of the data that is effective for estimation in the present paper, it ranges from the fourth quarter in 1987 to

the first quarter in 2013. It should be noted that this sample period covers the duration of Alan Greenspan's chairmanship and also covers most of the period of his successor, Ben Bernanke.

The CVAR analysis in this paper leads to a cointegrating relationship among the Federal funds rate, the long-term bond rate, the rate of inflation, real GDP and the effective US dollar exchange rate. It is noteworthy that the effective exchange rate plays a critical role in the estimated long-run relationship. Overall, this relationship is interpretable as a Taylor-type monetary policy rule including the term structure, which appears to be consistent with the finding of Christensen and Nielsen (2009). Moreover, we argue that the estimated cointegrating relationship can also be interpreted as a representation of the long-run Phillips curve incorporating both the term spread and the effective exchange rate. Thus, the dual roles of the term spread are illuminated in the estimated long-run representation, and it turns out that a set of estimated feedback mechanisms is consistent with these dual aspects of the term spread. These empirical findings are in favor of the argument of Estrella and Mishkin (1997). The estimated CVAR system is reduced to a conditional vector equilibrium correction model (VECM) as a result of the model reduction. The conditional VECM is perceived as a nexus of the dynamics of the monetary policy rule and inflation in the US. This nexus model allows us to have a data-based comprehension of the dynamic characteristics of these two variables, so it can be viewed as a significant contribution made in this paper.

Whether or not inflation can be controlled by the US monetary authority is also a subject of interest in this study. Johansen and Juselius (2001) developed a method for checking the controllability of a target variable by using an instrument variable in the context of a CVAR model. According to the results of their empirical illustrations, the Federal Reserve System seems to be able to lower domestic inflation by decreasing the Federal funds rate, which is against a standard view about monetary policy influences. Christensen and Nielsen (2009), however, find significant evidence indicating the controllability of inflation in a manner consistent with the standard view. The present paper also confirms this standard type of controllability of inflation in a CVAR setting different from that of Christensen and Nielsen (2009). Moreover, this paper demonstrates the controllability of inflation by employing the term spread as an explicit policy instrument. This additional evidence on the role of the term spread can be viewed as supporting the following view of monetary transmission: the term spread is directly affected by monetary policy and the subsequent change in the term spread, not in the level of long-term bond rate, influences the lending activities of financial intermediaries; such an influence on credit creation in the economy has a significant spillover effect on the underlying inflationary pressure.

As indicated by Adrian, Estrella and Shin (2010), it is probably not necessary for the central bank to control the level of the long-term bond rate in order to make monetary policy effective; what is essential, instead, is that the monetary authority is able to exercise a significant influence on the slope of the yield curve. Geenspan (2005) indicated that long-term bond rates didn't increase in response to a rise in the Federal funds rate, and described this phenomenon as a "conundrum". According to Adrian, Estrella and Shin (2010), Greenspan's conundrum does not prevent monetary tightening from taking place, but rather indicates effective tight monetary policy as a result of influencing credit creation by financial intermediaries under balance sheet constraints. Both Laurent (1988) and Bernanke and Blinder (1992) also suggest that the term spread can be perceived as an indicator of the US monetary policy stance. This paper's analysis of inflation controllability casts light on the role of the term spread in the underlying monetary transmission mechanism in the US economy, and supports the views of the

preceding studies such as Adrian, Estrella and Shin (2010). We recognize this analysis as a contributing factor to a central argument developed in this paper.

Furthermore, we conduct a generalized impulse response analysis developed by Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998), by allowing a shock in the residuals of an equation of interest in the empirical system. It should be noted, in particular, that a positive shock in the exchange rate equation has a significant negative effect on the rate of inflation. This is consistent with the standard view that an overall appreciation of the US dollar gives rise to a deflationary pressure in the domestic economy. The generalized impulse response analysis has cast light on various dynamic properties of the US data, and the results of this analysis may also be viewed as a set of notable contributions made in this paper.

The rest of this paper is organized as follows. Section 2 briefly reviews econometric methodology employed in this paper. Section 3 makes econometric investigations into the US quarterly macroeconomic data and explores various time series characteristics of monetary policy and inflation. The overall summary and conclusion are presented in Section 4. This paper used *CATS in RATS* (Dennis, Hansen, Johansen and Juselius, 2005), *Microfit* (Pesaran and Pesaran, 2009) and *PcGive* (Doornik and Hendry, 2007) to perform the econometric analyses of the data.

2. Review of econometric methods

A likelihood-based analysis of a CVAR model is employed as the primary econometric technique in our empirical exploration. This technique is briefly reviewed in this section, coupled with a concise explanation of a procedure for reducing the CVAR model to a parsimonious data-representation. It is well known that economic time series data are liable to exhibit non-stationary trending behavior; for this reason, they should often be viewed as the realizations of stochastic processes integrated of order 1, denoted by $I(1)$ henceforth. In regard to modeling $I(1)$ time series data, a likelihood-based method for analyzing CVAR systems was developed by Johansen (1988, 1991); see Johansen (1996, 2006) and references therein. This method is widely recognized as essential for researchers in time series econometrics, applied macroeconomics and various other fields; see Juselius (2006), Kurita (2010), Choo and Kurita (2011) for various empirical applications using the CVAR methodology.

We introduce here a general VAR(k) system for a p-dimensional time series $X_{-k+1},\cdots,X_T$, which is given as

$$\Delta X_t = (\Pi, \Pi_l)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \Phi D_t + \varepsilon_t, \text{ for } t = 1,\cdots,T, \qquad (1)$$

where ε_t represents an error vector that has an independent and identical normal $N(0,\Sigma)$ distribution given the initial values $X_{-k+1},\cdots,X_0$, and D_t denotes a q-dimensional vector consisting of various dummy variables. Note that the parameters in equation (1) all vary freely, which are given as $\Pi, \Gamma_i, \Sigma \in \boldsymbol{R}^{p\times p}$, $\Phi \in \boldsymbol{R}^{p\times s}$ and $\Pi_l, \mu \in \boldsymbol{R}^p$, with the matrix Σ being positive definite.

Next, we present three regularity conditions, which need to be satisfied in order to perform multivariate cointegration analysis of $I(1)$ data by utilizing equation (1). In order to give the first condition, we introduce the equation $|\mathrm{A(z)}| = 0$, in which $\mathrm{A(z)}$ derives from a lag polynomial for equation (1) and is explicitly expressed as

$$A(z) = (1-z)I_p - \Pi\, z + \sum_{i=1}^{k-1} \Gamma_i\,(1-z)z^i.$$

The first condition is that the characteristic roots of $|\mathrm{A(z)}| = 0$ fulfill either $|\mathrm{z}| > 1$ or $\mathrm{z} = 1$. It is ensured, under this condition, that explosive roots and seasonal cointegration are ruled out. Let us move on to the second condition. It is presented as

$$rank(\Pi, \Pi_l) \leq r \quad \text{or} \quad (\Pi, \Pi_l) = \alpha(\beta', \delta),$$

where $\alpha, \beta \in \mathbf{R}^{p\times r}$ are of full column rank for $r < p$ and $\delta \in \mathbf{R}^r$, α represents adjustment vectors, β^* denotes cointegrating vectors and r is referred to as cointegrating rank. Let us also introduce notation $\beta^{*\prime} = (\beta', \delta)$ and $X_{t-1}^* = \left(X_{t-1}', t\right)'$ for the sake of future reference. This second condition signifies the existence of at least $p - r$ common stochastic trends, leading to the phenomenon of cointegration when $r \geq 1$. Lastly, the third condition is

$$rank\left(\alpha_\perp' \Gamma \beta_\perp\right) = p - r,$$

where $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$ and $\alpha_\perp, \beta_\perp \in \mathbf{R}^{p\times(p-r)}$ are orthogonal complements given in such a way as $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$ with $(\alpha, \alpha_\perp)$ and $(\beta, \beta_\perp)$ being of full rank, respectively. As a result of this condition, we can preclude cases where higher orders than $I(1)$ hold.

According to Theorem 4.2 in Johansen (1996), if these three conditions are satisfied, the following Granger-Johansen representation is derived from equation (1):

$$X_t = C \sum_{i=1}^{t} \varepsilon_i + \varphi_t + \tau_c + \tau_l t + B_0, \tag{2}$$

where $C = \beta_\perp\left(\alpha_\perp' \Gamma \beta_\perp\right)^{-1} \alpha_\perp'$ is referred to as an impact matrix, representing how the underlying cumulated innovations affect the overall process, φ_t is viewed as a mean-zero stationary process, τ_c and $\tau_l t$ represent a constant and a linear trend term respectively, and B_0 depends on the staring values of the process, satisfying $\beta' B_0 = 0$. This representation lays the foundation for $I(1)$ cointegration analysis using equation (1). Let us also note that an interpretation of the impact matrix C is pursued by Johansen and Juselius (2001) in light of control theory for non-stationary economic series. Under the three regularity conditions above, we also find that the following $I(1)$ CVAR model is derived as a sub-model of equation (1):

$$\Delta X_t = \alpha\beta^{*\prime} X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i\, \Delta X_{t-i} + \mu + \Phi D_t + \varepsilon_t, \tag{3}$$

where $\beta^{*\prime} X_{t-1}^*$ represents a group of stationary combinations of non-stationary variables. They are called cointegrating relationships, which can correspond to the underlying *long-run* economic relationships, hence working as the mechanism of equilibrium correction in equation (3). In empirical analysis, the cointegrating rank r is usually an unknown parameter and its value is often investigated through the data analysis in the maximum likelihood framework. A log-likelihood ratio ($\log LR$) test statistic is formulated by the null hypothesis of r rank or $\mathrm{H}(r)$ against the alternative hypothesis $\mathrm{H}(p)$, which is denoted as $\log LR[\mathrm{H}(r)|\mathrm{H}(p)]$. The maximization of the log-likelihood function involves solving a generalized eigenvalue problem. The limiting quantiles of the $\log LR$ test statistic are provided by Johansen (1996, Ch.15); with

regard to the method of gamma approximations to simulate the quantiles, see Nielsen (1997) as well as Doornik (1998). Relying upon these simulated quantiles for $\log LR$ statistics, one is able to determine the cointegrating rank for equation (1) estimated from the data. The subsequent step in the analysis is to check valid restrictions on the maximum likelihood estimates of α and β^* in equation (3). Economic interpretations of the CVAR model are pursued by checking whether or not a set of restrictions based on economic theories can be imposed on the estimates of these parameters.

Let us move on to a concise review of a conditional or partial CVAR system and a procedure for reducing the system. Equation (3) is broken down into $X_t = \left(Y_t^{'}, Z_t^{'}\right)'$ for $Y_t \in R^m$ and $Z_t \in R^{p-m}$, and $r \leq m < p$, with the consequence that the parameters and innovations are represented as

$$\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix},\ \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix},\ \mu = \begin{pmatrix} \mu_y \\ \mu_z \end{pmatrix},\ \Phi = \begin{pmatrix} \Phi_y \\ \Phi_z \end{pmatrix},\ \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix},\ \Sigma = \begin{pmatrix} \Sigma_{yy} & \Sigma_{yz} \\ \Sigma_{zy} & \Sigma_{zz} \end{pmatrix},$$

and $\omega = \Sigma_{yz}\Sigma_{zz}^{-1}$ is also defined. Suppose that $\alpha_z = 0$ holds, so that equation (1) is presented as a set of a partial CVAR system for Y_t conditional on Z_t and a marginal system for Z_t in the following manner:

$$\Delta Y_t = \omega \Delta Z_t + \alpha_y \beta^{*'} X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_{y,i}^* \Delta X_{t-i} + \mu_y^* + \Phi_y^* D_t + \varepsilon_{y,t}^*, \tag{4}$$

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta X_{t-i} + \mu_z + \Phi_z D_t + \varepsilon_{z,t}, \tag{5}$$

where both the parameters and the innovation vector are defined as

$$\Gamma_{y,i}^* = \Gamma_{y,i} - \omega\Gamma_{z,i},\ \mu_y^* = \mu_y - \omega\mu_z, \Phi_y^* = \Phi_y - \omega\Phi_z \text{ and } \varepsilon_{y,t}^* = \varepsilon_{y,t} - \omega\varepsilon_{z,t},$$

and the variance of $\varepsilon_{y,t}^*$ is given as $\Sigma_{yy} - \Sigma_{yz}\Sigma_{zz}^{-1}\Sigma_{yz}$. If the condition $\alpha_z = 0$ is observed, Z_t is said to be weakly exogenous for the parameters of interest in equation (4) such as β^*. In other words, the condition $\alpha_z = 0$ allows us to concentrate on the estimation of equation (4) without estimating equation (5), provided that the parameters of interest are all nested in a class of parameters for equation (4). See Engle, Hendry and Richard (1983), Johansen (1992) and Urbain (1992) for further details of weak exogeneity. If the weak exogeneity condition $\alpha_z = 0$ holds empirically, one is able to proceed from the joint model to the partial model containing the cointegrating relationships $\beta^{*'}X_{t-1}^*$ as a set of long-run regressors. Furthermore, the partial model can be reduced to a parsimonious VECM by eliminating insignificant lagged regressors. This model reduction procedure is based on a general-to-specific modeling approach; see Hendry (1995) and Campos, Ericsson and Hendry (2005), *inter alia*, for further details of this approach. A well-formulated parsimonious VECM is viewed as an econometric representation of the underlying data generating mechanism; a set of first-order regressors in the VECM is interpreted in light of *short-run* dynamics, while the cointegrating relationship is viewed as working as the long-run driving force in the model.

3. Econometric analysis of the US time series data

This section performs a detailed econometric analysis of the US quarterly time series

data to reach a parsimonious representation of the US monetary policy rule and inflation dynamics. Let us introduce a set of economic variables under study as follows:

$$X_t = (i_t^s \quad i_t^l \quad \pi_t \quad y_t \quad e_t)',$$

where i_t^s is the log of the Federal fund rate, i_t^l is the log of the 10-year government bond yield, π_t is the rate of quarter inflation, y_t is the log of real GDP and e_t is the log of a nominal effective US dollar exchange rate; see Appendix for further details of the data for these five variables. Note that i_t^s and i_t^l are also referred to as short-term and long-term interest rates, respectively. The empirical study including lagged dynamics covers a period running from the second quarter in 1987 to the first quarter in 2013, which is represented as 1987.2 - 2013.1, hereafter. In regard to the data studied here, they are seen as *ex post* revised data, in contrast to real-time data analyzed by Orphanides (2001). GDP related series, in particular, are subject to constant revisions, so that the analysis of such series could lead to different empirical outcomes depending on which revised versions are studied. In spite of such limitations, the analysis of *ex post* data is nonetheless valuable in that it may enable us to obtain pieces of empirical evidence well interpretable from economic theory. Such theory-consistent evidence will then give impetus to further empirical and theoretical research, which can contribute to improving our understanding of economic issues in question.

This section adopts the general-to-specific modeling approach to the data and is composed of five sub-sections. Section 3.1 estimates a well-formulated VAR system as the starting point of this study and proceeds to determine the cointegrating rank for the system. Section 3.2 estimates a CVAR system and checks various restrictions on estimated parameters, such as those related to weak exogeneity, with an intent to shed light on the underlying time series properties of each variable in the system. Section 3.3, based on the test results obtained in the preceding sub-sections, attempts to reveal interpretable long-run relationships coupled with the corresponding adjustment mechanisms. The long-run impact matrix for the CVAR system is derived in Section 3.4, with an intent to examine the empirical controllability of inflation and the term spread by the US monetary authority. Section 3.5 performs impulse response analysis and 3.6 explores strong exogeneity. Lastly, Section 3.7 arrives at a parsimonious VECM for the short-term interest rate and inflation, and examines time series properties of the data in light of equilibrium correction and short-run dynamics.

3.1 Estimating a VAR system and examining its cointegrating rank

A tentative VAR model including constant and linear trend is fitted to the data by setting the model's lag length at 3 or $k = 3$. We find that the lagged dynamic terms at length 3 appear to be insignificant, while those at length 2 play clear significant roles in the model. Hence, we conclude that $k = 2$ is appropriate as the lag length of the VAR model. The determination of $k = 2$ results in 102 observations (over 1987.4 - 20013.1) available for estimation.

In addition, it appears that the US data contain several outliers in 1989.1, 1990.1, 2007.1, 2008.4 and 2011.4. The outlier in 1989.1 is spotted in the residual series of the equation for e_t, while the outliers for 1990.1, 2007.1 and 2011.4 are found in the residuals of the equation for π_t; it should be noted that the outliers in 2007.1 and 2011.4 are of almost the same volume with the opposite direction. The outlier in 1989.1, is caused by a rapid appreciation of the US dollar, which could be a backlash from the

dollar's depreciation since the Plaza accord in September 1985. The set of outliers found in the residuals of the inflation equation might possibly be attributed to relatively large changes in the underlying food prices. With regard to the time point 2008.4, all the variables in the model except for i_t^l suffer from large outliers caused by the US financial crisis starting in September 2008, which led to a subsequent global-scale economic depression. Hence, the following set of dummy variables, taking either 1 or -1 at some specific points and 0 otherwise, is included in the VAR model in the same way as equation (1): $D_{1,t} = 1(1989.1)$, $D_{2,t} = 1(1990.1)$, $D_{3,t} = 1(2008.4)$, $D_{4,t} = [1(2007.1),\ -1(2011.4)]$ and 0 otherwise.

Table 1 presents a set of mis-specification test results for the VAR(2) system containing all of these dummy variables. Most of the test results are given in the form F_j, representing an approximate F test against the j alternative hypothesis: *k*th-order autocorrelation (F_{ARk}: see Godfrey, 1978, Nielsen, 2006), *k*th-order autoregressive conditional heteroskedasticity (ARCH) (F_{ARCHk}: see Engle, 1982), heteroskedasticity (F_{HET}: see White, 1980). In addition, a chi-square test for normality (χ^2_{ND}: see Doornik and Hansen, 2008) is also recorded here. According to this table, most of the diagnostic tests indicate no problems at the 5% significance level, and there is no significant test statistic at the 1% level. Judging from the results in Table 1, we may be justified in concluding that the overall VAR(2) system is seen as a satisfactory representation of the data. This VAR system is utilized for the subsequent cointegration analyses.

TABLE 1 MIS-SPECIFICATION TESTS FOR THE VAR(2) SYSTEM

(a) INDIVIDUAL TESTS

	i_t^s	i_t^l	π_t	y_t	e_t
Autocorr.[F_{AR5}(5,81)]	1.99[0.09]	2.39[0.04]*	1.02[0.41]	3.17[0.01]*	0.37[0.87]
ARCH [F_{ARCH4}(4,78)]	0.72[0.58]	0.59[0.67]	1.45[0.22]	0.47[0.76]	0.13[0.97]
Hetero.[F_{HET}(22,63)]	1.29[0.21]	0.75[0.77]	0.51[0.96]	0.86[0.64]	0.86[0.64]
Normality [χ^2_{ND}(2)]	5.72[0.06]	0.53[0.77]	5.40[0.07]	0.05[0.97]	1.04[0.60]

(b) SYSTEM TESTS

Autocorr.[F_{AR1}(25,287)]	1.49[0.07]	Autocorr.[F_{AR5}(125,285)]	1.31[0.03]*
Hetero.[F_{HET}(330,656)]	0.90[0.87]	Normality [χ^2_{ND}(10)]	10.96[0.36]

Notes: Figures in square brackets are *p*-values.
* denote significance at the 5% level.

TABLE 2 TESTS FOR COINTEGRATING RANK IN THE VAR(2) SYSTEM

	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$
log LR[H(r)|H(p)]	113.06[0.00]**	60.01[0.10]	29.01[0.57]	9.96[0.92]	2.19[0.94]
log LR^*[H(r)|H(p)]	101.13[0.00]**	54.08[0.25]	26.19[0.73]	8.58[0.96]	1.92[0.96]
mod ($r = 1$)	1	1	1	1	0.71	0.71	0.36

Notes: Figures in square brackets are *p*-values.
** denotes significance at the 1% level.

Next, a series of log *LR* tests is performed to check the cointegrating rank of the VAR(2) system, and all the test statistics are recorded in the first panel of Table 2. The first row of this table documents standard log *LR* test statistics (see Johansen, 1996),

while the second row records log LR test statistics corrected for the sample size (see Johansen, 2002), which are labeled as $\log LR^{*}[\mathrm{H}(r)|\mathrm{H}(p)]$. Since both of the test statistics reject the null hypothesis of $r = 0$ at the 5% level of significance, we can proceed to the examination of the second null hypothesis $r \leq 1$. It is then found that, according to both of the test statistics, the second null hypothesis is not rejected at the 5% level. For the purpose of ascertaining the choice of $r = 1$, the second panel of Table 2 provides the modulus (denoted by *mod*) of the seven largest roots calculated from a companion matrix for the VAR(2) model under the restriction of $r = 1$. These roots correspond to the reciprocal values of the roots of $|\mathrm{A(z)}| = 0$. Apart from the imposed four unit roots, the roots are much smaller than 1, which is seen as evidence indicating that neither explosive nor $I(2)$ root is involved in this model. The choice of $r = 1$ appears to be reasonable.

FIGURE 1 COINTEGRATING RELATIONSHIP AND RECURSIVE STATISTICS

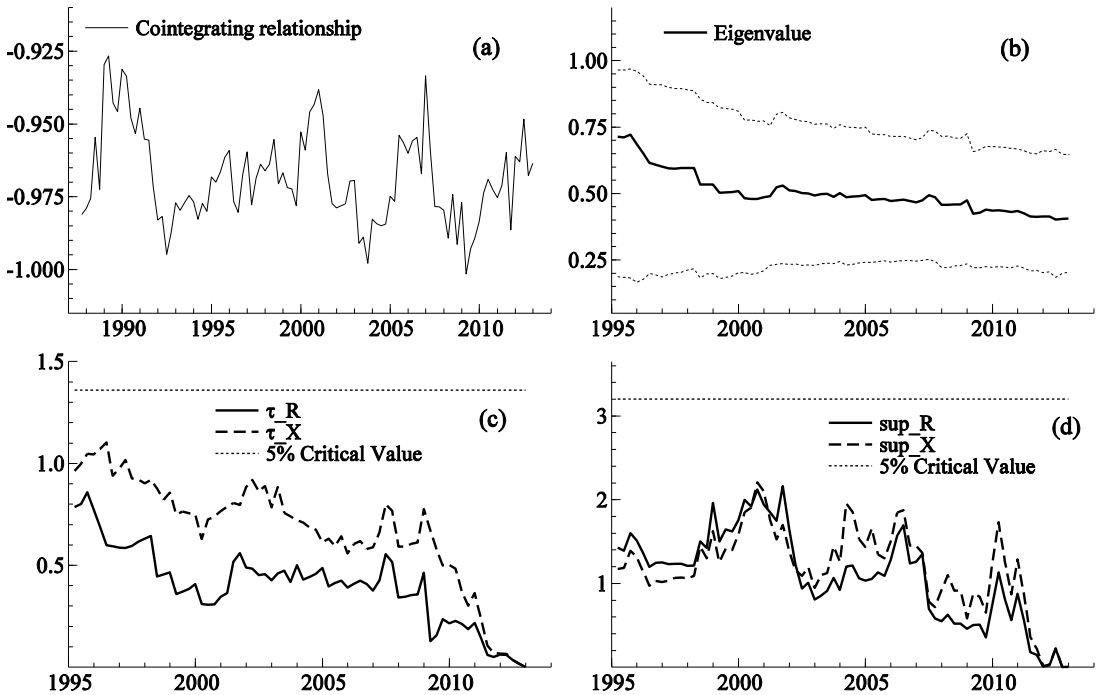


Figure 1(a) records the estimated cointegrating combination when $r = 1$ is selected. It doesn't exhibit any trending behavior and so appears to be stationary. Recursive estimation is performed under the restriction of $r = 1$, and various recursive statistics are calculated based on Hansen and Johansen (1999). They are also presented in Figure 1. A plot of the estimated eigenvalue, which is a critical element of the maximized likelihood, and its 95% confidence band are displayed in Figure 1(b), while Figure 1(c) shows a couple of fluctuation tests for the eigenvalue, denoted τ_R and τ_X, respectively. The former test is obtained by estimating recursively all of the parameters, including those for short-run dynamics, whereas the latter by concentrating out the effects of all short-run dynamics before recursive estimation. Figure 1(d), in addition, records a couple of supremum tests for the constancy of the cointegrating vector, represented by sup_R and sup_X, respectively. The notation symbols, R and X, indicate the same estimation procedures as those for Figure 1(c). All of these tests stay below the corresponding 5% critical values. See Hansen and Johansen (1999) for further

details of these recursive statistics. The results in Figure 1 are all in support of parameter stability in the CVAR system under the restriction of $r = 1$.

We have, therefore, arrived at the conclusion that $r = 1$ is chosen for this VAR model. This choice also agrees to that of Christensen and Nielsen (2009). The determination of the cointegrating rank ($r = 1$) leads us to the analysis of a CVAR model formulated as equation (3), by means of which we will now pursue a further investigation of the underlying time series structure of the data.

3.2 Checking the validity of various hypotheses of interest

Since the cointegrating rank $r = 1$ is determined, it is possible to examine the validity of various restrictions on the adjustment and cointegration vectors in the CVAR model, with an intent to extract useful information from the data. A class of hypotheses under study here is referred to as stationarity, long-run exclusion and weak exogeneity (see Juselius, 2006, Chs. 10, 11 and 19); the corresponding test results are denoted in Table 3 as *ST*, *LE* and *WE*, respectively. The test for stationarity (*ST*) investigates whether or not any of the variables in X_t can be individually seen as a stationary series when the deterministic trend is present in the cointegrating relationship; next, the test for long-run exclusion (*LE*) examines whether or not any of the variables in X_t can be excluded from the cointegrating combination; lastly, the test for weak exogeneity (*WE*) examines whether or not any of the variables are weakly exogenous with respect to the parameters of interest such as β^*. Note that the final test is concerned with a zero restriction on α, instead of β^*, as reviewed in the last section.

TABLE 3 TESTS FOR *ST*, *LE* and *WE* IN THE CVAR(2) SYSTEM

	i_t^s	i_t^l	π_t	y_t	e_t
ST	21.80[0.00]**	25.49[0.00]**	27.33[0.00]**	44.87[0.00]**	42.01[0.00]**
LE	14.06[0.00]**	0.85[0.36]	11.19[0.00]**	15.47[0.00]**	12.32[0.00]**
WE	13.18[0.00]**	0.16[0.69]	17.77[0.00]**	0.87[0.35]	0.03[0.85]

Notes: Figures in square brackets are *p*-values.
** denote significance at the 1% level.

According to Table 3, the null hypothesis for *ST* is rejected at the standard 5% level with respect to all the variables, and no variable is judged to be individually stationary and the joint analysis aiming at an interepretable cointegrating relationship is crucial. With regard to *LE*, it is found that no variable, apart from i_t^l, can be long-run excluded from the cointegrating space at the 5% level; thus, most of the variables in the system are indispensable in the long-run analysis. It is noteworthy that e_t cannot be long-run excluded, which is viewed as evidence in support of the idea that a measure of exchange rate behavior should be included in the CVAR model. Although a zero restriction can be imposed on the coefficient for i_t^l under the condition that no other restriction is introduced, the inclusion of i_t^l in the long-run analysis is still justified in light of its economic implications based on theories such as the term structure of interest rates and the Fisher equation. Turning to *WE*, in contrast, we notice that the null hypothesis of weak exogeneity is not rejected at the same level with respect to i_t^l, y_t and e_t. Consequently, these variables are judged to be weakly exogenous for the parameters of interest. As reviewed in Section 3, from the standpoint of statistical inference with no loss of information, we can focus on modeling i_t^s and π_t conditional on these three

exogenous variables. The finding that both the short-term interest rate and inflation are non-weakly exogenous variables coincides with that in Christensen and Nielsen (2009). Conditional modeling is an important step towards a parsimonious empirical representation of the data. We make the most of the property of exogeneity in reducing the CVAR system in Section 3.7.

3.3 Interpreting the estimated long-run economic relationship

Economic interpretations of the estimated cointegrating relationship are pursued in this sub-section. In doing so, we place joint zero restrictions on the adjustment coefficients for i_t^l, y_t and e_t; the validity of these joint restrictions is expected from the results of weak exogenity tests in Table 3 above. The cointegrating vector is then normalized with respect to i_t^s, which is a monetary policy instrument and thus viewed as one of the variables of primary interest in this study. A set of restricted estimates, coupled with the corresponding standard errors reported in parentheses, is summarized as follows:

$$\hat{\alpha}\,\hat{\beta}^{*'}X_{t-1}^{*} = \begin{bmatrix} \underset{(0.018)}{-0.090} \\ \underset{(-)}{0} \\ \underset{(0.011)}{-0.056} \\ \underset{(-)}{0} \\ \underset{(-)}{0} \end{bmatrix} \begin{bmatrix} \underset{(-)}{1} & \underset{(0.375)}{-0.941} & \underset{(1.039)}{4.045} & \underset{(0.078)}{-0.340} & \underset{(0.027)}{0.105} & \underset{(0.0007)}{0.0024} \end{bmatrix} \begin{pmatrix} i_{t-1}^{s} \\ i_{t-1}^{l} \\ \pi_{t-1} \\ y_{t-1} \\ e_{t-1} \\ t \end{pmatrix},$$

where the sign ^ denotes a set of parameter estimates in question. A log *LR* test for these joint restrictions is $0.89[0.83]$ with its *p*-value according to $\chi^2(3)$ reported in the square bracket; the null hypothesis is therefore not rejected at the standard 5% level, as expected from the preceding test results recorded in Table 3. All of the coefficient estimates reported above are significant, judging from a set of relatively small standard errors for these estimates. Note that the coefficient for i_{t-1}^l in the cointegrating vector above is highly significant, indicating that i_{t-1}^l cannot be long-run excluded anymore under the set of weak exogeneity restrictions. Moreover, the absolute value of this coefficient is fairly close to unity, suggesting the possibility that an additional restriction yielding a one-for-one relationship between the two interest rates, *i.e.* $i_{t-1}^s - i_{t-1}^l$, can be applied to the cointegrating vector. Hence, this restriction is imposed on the corresponding parameter, resulting in the following set of updated estimates:

$$\hat{\alpha}\,\hat{\beta}^{*'}X_{t-1}^{*} = \begin{bmatrix} \underset{(0.018)}{-0.090} \\ \underset{(-)}{0} \\ \underset{(0.012)}{-0.057} \\ \underset{(-)}{0} \\ \underset{(-)}{0} \end{bmatrix} \begin{bmatrix} \underset{(-)}{1} & \underset{(-)}{-1} & \underset{(1.024)}{4.030} & \underset{(0.072)}{-0.335} & \underset{(0.025)}{0.103} & \underset{(0.0005)}{0.0023} \end{bmatrix} \begin{pmatrix} i_{t-1}^{s} \\ i_{t-1}^{l} \\ \pi_{t-1} \\ y_{t-1} \\ e_{t-1} \\ t \end{pmatrix},$$

for which the log *LR* test statistic is $0.91[0.92]$ with its *p*-value based upon $\chi^2(4)$. Again, the null hypothesis is not rejected at the conventional 5% level, so that the formulation of the term spread in the cointegrating combination is judged as a valid

representation. Furthermore, the coefficient for π_{t-1} in $\widehat{\beta}^*$ seems to suggest that annualized inflation, *i.e.* $4\pi_{t-1}$, is relevant in the long-run relationship, which leads to a real long-term interest rate if combined with i^l_{t-1}. This additional restriction is placed on the cointegrating vector, and the following structure is revealed:

$$\hat{\alpha}\,\widehat{\beta}^{*'}X^*_{t-1} = \begin{bmatrix} \underset{(0.018)}{-0.091} \\ \underset{(-)}{0} \\ \underset{(0.012)}{-0.057} \\ \underset{(-)}{0} \\ \underset{(-)}{0} \end{bmatrix} \begin{bmatrix} \underset{(-)}{1} & \underset{(-)}{-1} & \underset{(-)}{4} & \underset{(0.061)}{-0.334} & \underset{(0.022)}{0.103} & \underset{(0.0004)}{0.0023} \end{bmatrix} \begin{pmatrix} i^s_{t-1} \\ i^l_{t-1} \\ \pi_{t-1} \\ y_{t-1} \\ e_{t-1} \\ t \end{pmatrix}, \quad (6)$$

for which the log *LR* test statistic is $0.91[0.97]$ with the *p*-value according to $\chi^2(5)$; once again, the null hypothesis is anything but rejected at the standard level. Judging from the estimation results reported in equation (6), all the other estimates are significant and no further additional restriction can lead to more interesting outcomes as a long-run economic representation than equation (6). The linear combination of the variables based on equation (6) is rebelled as ecm_{t-1}, which is expressed as

$$ecm_{t-1} = i^s_{t-1} - \left(i^l_{t-1} - 4\,\pi_{t-1}\right) - 0.334(y_{t-1} - 0.007t) + 0.103\,e_{t-1}. \quad (7)$$

Equation (7) represents a stationary disequilibrium error, to which two endogenous variables (i^s_t and π_t) are reacting according to the structure of $\hat{\alpha}$. Since the significant coefficients in $\hat{\alpha}$ are both negative, an equilibrium correction mechanism is properly working in this empirical system.

We are now in a position to interpret the cointegrating combination (7) as a long-run economic representation. Solving (7) for i^s_{t-1} and re-expressing ecm_{t-1} as a stationary error term v_{t-1}, we find

$$i^s_{t-1} = \left(i^l_{t-1} - 4\,\pi_{t-1}\right) + 0.334(y_{t-1} - 0.007t) - 0.103\,e_{t-1} + v_{t-1}, \quad (8)$$

which can be viewed as a long-run representation of the underlying US monetary policy rule. The left hand side of equation (8) is the Federal fund rate, which corresponds to the monetary policy instrument; the right hand side terms are considered to be the long-run determinants of the overall behavior of the policy instrument variable.

Let us check the first term on the right hand side of equation (8), the real long-term interest rate $(i^l_{t-1} - 4\,\pi_{t-1})$, which is viewed as measuring the deviation of the nominal long-term interest rate i^l_{t-1} from the realized annual inflation rate $4\,\pi_{t-1}$. If the nominal long-term interest rate goes up and is greater than the actual inflation rate, this may indicate an upsurge in the underlying expected inflation rate as compared with the realized rate of inflation. We assume here that information on inflationary expectations is contained in the nominal long-term interest rate; see Goodfriend (1993), Ireland (1996) and Mehra (2001), *inter alia*, for the validity of this view with regard to the US economy. In response to the rise in the real long-term interest rate, the monetary authority raises the short-term interest rate to fight against the anticipated inflation which may possibly occur in the future horizons. Therefore, it appears reasonable that the first term holds a positive coefficient, moving in tandem with the policy instrument. The observation that the real long-term interest matters in the conduct of monetary

policy appears to be consistent with the model of Romer (2000), in which a standard LM curve is replaced by a monetary policy rule centering on the real interest rate.

Next, the second term on the right hand side of equation (8) represents a measure of the output gap ($y_{t-1} - 0.007t$), which indicates whether the macro economy is in boom or recession on the basis of the estimated linear trend. If this output gap measure is positive, this implies that the overall economy has heated up, leading the monetary authority to raise the short-term interest rate as a deflationary policy. The reverse policy is implemented when the output gap measure is negative. Thus, we find it reasonable that the second term representing the output gap possesses a positive coefficient in the empirical policy rule function (8) above.

Furthermore, the third term on the right hand side of equation (8) corresponds to the effective exchange rate e_{t-1}, an increase of which means the overall appreciation of the US dollar against various other currencies, implying the creation of a deflationary pressure in the US economy. The monetary authority will lower the short-term interest rate in response to the appreciation of the home currency, while the opposite policy response will be taken in the face of the currency depreciation. Hence, the finding that e_{t-1} holds a negative coefficient in the long-run relationship makes sense, indicating the presence of monetary policy responses to the movements of the foreign exchange rate.

In view of the observation that the adjustment coefficient for π_t is highly significant in equation (6), in addition to i_t^s, we may reinterpret ecm_{t-1} as a long-run relationship centering on π_t. That is, solving equation (8) for π_t and disregarding rounding errors lead to

$$\pi_{t-1} = 0.25\left(i_{t-1}^l - i_{t-1}^s\right) + 0.084(y_{t-1} - 0.007t) - 0.026e_{t-1} + u_{t-1},$$

for $u_{t-1} = 0.25v_{t-1}$. This expression appears to be interpreted as a Philips-curve type relationship augmented with the term spread $(i_{t-1}^l - i_{t-1}^s)$ and the effective exchange rate e_{t-1}. Thus, the estimated long-run linkage appears to be subject to the dual economic interpretations according to the significant adjustment coefficients of the system. This dual structure is consistent with the argument of Estrella and Mishkin (1997), indicating critical roles played by the term spread in determining the dynamics of the US monetary policy and inflation. The combination ecm_{t-1} acts as a key dynamic factor in a nexus model pursued in Section 3.7.

3.4 Checking the controllability of inflation

As reviewed in Section 2, it is possible to check the controllability of inflation π_t by means of the Federal fund rate i_t^s, based upon the control theory for a CVAR system developed by Johansen and Juselius (2001). Let us refer to the impact matrix C in equation (2) for this purpose. According to Johansen and Juselius (2001), the condition for the controllability of π_t by means of i_t^s is given as the rejection of the following null hypothesis H_0:

$$H_0: \ b'Ca = 0, \tag{9}$$

for

$$a = (1,0,0,00)' \text{ and } b = (0,0,1,00)'.$$

These selection vectors pick out π_t and i_t^s from X_t, respectively. That is, the element corresponding to $b'Ca$ is an impact matrix coefficient that measures the effect of the cumulated innovations for i_t^s upon π_t. The null hypothesis (9) needs to be rejected statistically, in order to infer that inflation can be turned into a stationary process by a series of interventions using the short-term interest rate; the controllability of inflation is defined in this sense. Inference for the hypothesis (9) is based on Paruolo (1997). See Johansen and Juselius (2001) for further details of the controllability of a variable of interest in the CVAR framework.

We examine whether or not the condition of controllability empirically holds in our CVAR model. In doing so, we maintain the restrictions imposed on the cointegrating vector while those on the adjustment vector are lifted. A set of parameter estimates for the impact matrix C, combined with the corresponding standard errors in parentheses, is given below:

$$\hat{C} = \begin{bmatrix} \underset{(0.329)}{0.908} & \underset{(0.296)}{1.208} & \underset{(0.512)}{-1.608} & \underset{(0.181)}{0.157} & \underset{(0.033)}{-0.065} \\ \underset{(0.200)}{-0.139} & \underset{(0.180)}{1.195} & \underset{(0.311)}{0.173} & \underset{(0.110)}{0.020} & \underset{(0.020)}{-0.024} \\ \underset{(\mathbf{0.055})}{\mathbf{-0.235}} & \underset{(0.049)}{0.124} & \underset{(0.085)}{0.356} & \underset{(0.030)}{0.057} & \underset{(0.005)}{-0.026} \\ \underset{(0.426)}{-0.093} & \underset{(0.383)}{0.375} & \underset{(0.663)}{-0.730} & \underset{(0.234)}{1.549} & \underset{(0.043)}{-0.031} \\ \underset{(1.816)}{-1.368} & \underset{(1.634)}{-3.934} & \underset{(2.828)}{1.216} & \underset{(0.998)}{1.412} & \underset{(0.182)}{1.360} \end{bmatrix} for\ X_t = \begin{pmatrix} i_t^s \\ i_t^l \\ \pi_t \\ y_t \\ e_t \end{pmatrix}, \qquad (10)$$

where the coefficient of interest $b'\hat{C}a$ is given in bold, -0.235, for which the *t*-type test statistic for the null hypothesis (9) is $4.27[0.00]^{**}$ and the *p*-value reported in the square bracket is according to the standard normal distribution. Hence, the null hypothesis is rejected at the 5% significance level, indicating that π_t is controllable by making use of i_t^s as an instrument. The coefficient is negative, which enables us to argue that a rise in the short-term interest rate brings about monetary contraction, thereby having a downward pressure on inflation. Thus, this finding implies that the US monetary authority is capable of stabilizing inflation's path around its target level, in a manner consonant with the conventional view of monetary policy influences on inflation. This finding is also consistent with that found by Christensen and Nielsen (2009). In addition, focusing on the conditional modeling of π_t and i_t^s may be justified by the evidence for controllability here, as well as by the preceding evidence for weak exogeneity of the remaining three variables.

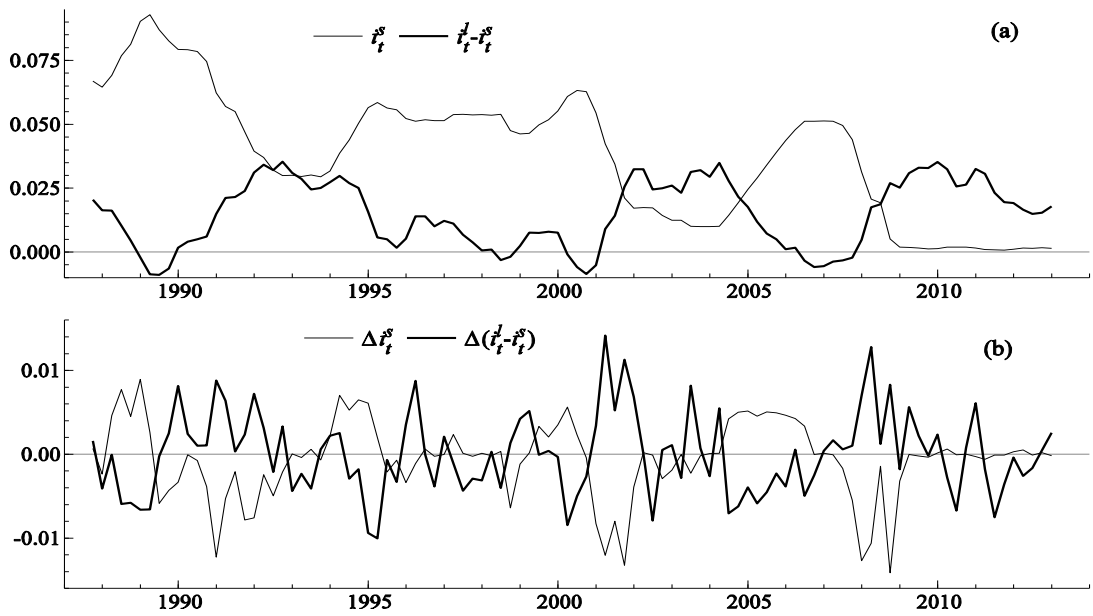
Let us turn attention to the controllability of the long-term bond rate i_t^l by means of the Federal fund rate i_t^s; we need to redefine the selection vector b as

$$b = (0,1,0,00)',$$

with the vector a unchaned. According to equation (10), the test statistic in this case is $0.70[0.48]$ and the null hypothesis cannot be rejected at the 5% level. Thus, it is possible to conclude that i_t^l is not empirically controllable by i_t^s. This finding may not be surprising in view of the well-known Greenspan's conundrum (see Greenspan, 2005) concerning the behavior of the US long-term bond yields; see Warnock and Warnock (2009) and Byrne, Fazio and Fiess (2012) for various inferences about the underlying causes of this phenomenon, such as the growing influences of international capital flows on the US bond market.

Adrian, Estrella and Shin (2010) argue that, for the purpose of implementing effective monetary policy, the monetary authority does not necessarily have to control the level of the long-term bond rate. They indicate, instead, that exercising a significant influence on the term spread leads to effective monetary policy, as a result of affecting the credit creation process of financial intermediaries facing balance sheet constraints. Let us examine here the relationship between the Federal fund rate i_t^s and the term spread $i_t^l - i_t^s$ by looking at Figure 2. Figure 2(a) displays an overview of i_t^s and $i_t^l - i_t^s$, while Figure 2(b) shows that of the corresponding first-order differences, Δi_t^s and $\Delta(i_t^l - i_t^s)$. In view of the countercyclical features of the data displayed in Figure 2, we find it possible to conclude that the behavior of the term spread is dominated by the Federal fund rate. Thus, in the rest of this sub-section, we will concentrate our attention on the term spread, instead of the Federal fund rate.

FIGURE 2 FEDERAL FUND RATE AND TERM SPREAD



In order to highlight the role of the term spread in the US economy, we introduce a new set of variables $\tilde{X}_t$ defined as

$$\tilde{X}_t = K'X_t = (i_t^l - i_t^s \quad \pi_t \quad y_t \quad e_t)'.$$

where

$$K' = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Thus, $\tilde{X}_t$ can be seen as a vector process derived from X_t by focusing on an explicit role of the term spread in the system. For the purpose of conducting a comparative analysis, a VAR model for $\tilde{X}_t$ is newly estimated to ensure that its cointegrating rank is unity. We also make certain of the acceptability of those restrictions on its cointegrating vector which yield a long-run relationship consistent with equation (7). Representing

the impact matrix for $\tilde{X}_t$ as $\tilde{C}$, we find that the condition for the controllability of π_t by means of $i_t^l - i_t^s$ corresponds to the rejection of a null hypothesis H_0 given as

$$H_0: \ b'\tilde{C}a = 0, \tag{11}$$

for

$$a = (1,0,0,0)' \text{ and } b = (0,1,00)'.$$

A set of parameter estimates and standard errors for $\tilde{C}$ is

$$\hat{\tilde{C}} = \begin{bmatrix} \underset{(0.181)}{0.701} & \underset{(0.362)}{1.652} & \underset{(0.123)}{-0.366} & \underset{(0.023)}{0.024} \\ \underset{(\mathbf{0.045})}{\mathbf{0.198}} & \underset{(0.089)}{0.344} & \underset{(0.030)}{0.038} & \underset{(0.006)}{-0.029} \\ \underset{(0.335)}{0.183} & \underset{(0.669)}{-0.693} & \underset{(0.228)}{1.615} & \underset{(0.042)}{-0.028} \\ \underset{(1.360)}{-0.251} & \underset{(2.712)}{0.345} & \underset{(0.924)}{0.332} & \underset{(0.171)}{1.243} \end{bmatrix} for\ \tilde{X}_t = \begin{pmatrix} i_t^l - i_t^s \\ \pi_t \\ y_t \\ e_t \end{pmatrix}, \tag{12}$$

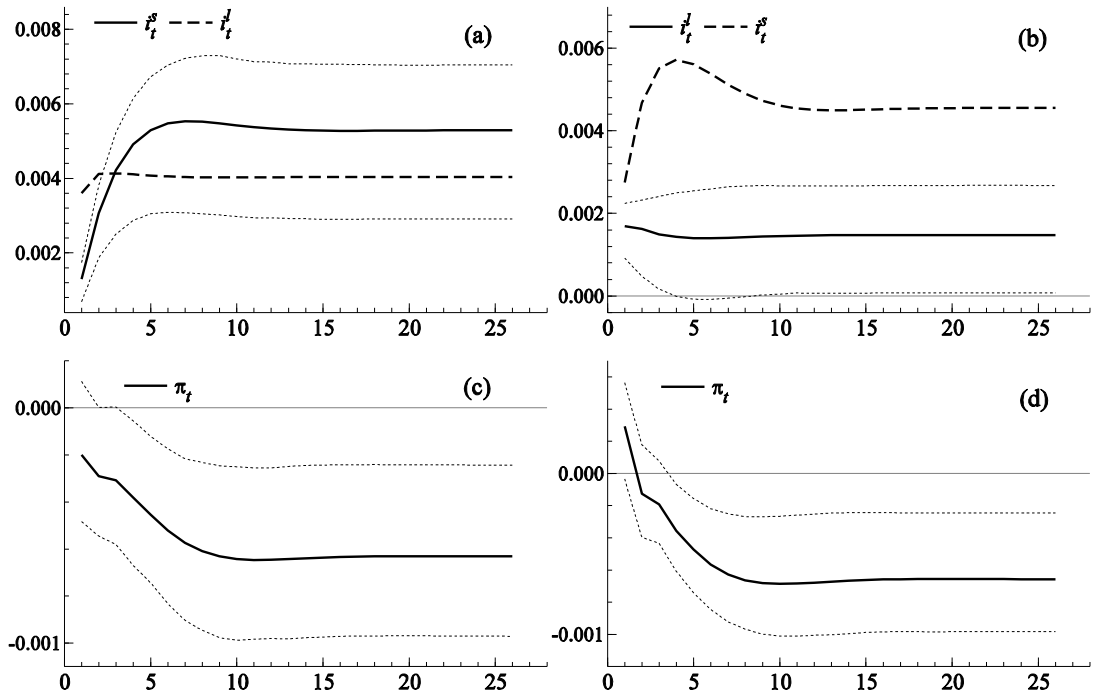
where the coefficient of interest $b'\hat{\tilde{C}}a$ is given in bold. Since the test statistic is 4.4[0.00]**, the null hypothesis (11) is rejected at the 5% significance level. We are, therefore, justified in concluding that π_t is controllable by employing $i_t^l - i_t^s$ as a policy instrument. The coefficient of interest in equation (12) is positive, as expected, which leads us to infer that a steep rise in the slope of the yield curve caused by a cut in the Federal fund rate moves inflation upwards. Let us recall here the preceding finding that the long-term bond rate is uncontrollable by the Federal fund rate. Thus, the transmission of the US monetary policy over the sample period may largely hinge on the mechanism suggested by Adrian, Estrella and Shin (2010), rather than the conventional interest rate channel, in which monetary policy's influences are usually seen as running from short-term interest rates to long-term ones. Both Laurent (1988) and Bernanke and Blinder (1992) also suggest that the term spread can be perceived as an indicator of the US monetary policy stance. In addition, McCallum (2005) presents an economic model in which the term spread influences the dynamics of the short-term interest rate. As in line with these preceding research papers, the econometric study in this sub-section casts light on the importance of the term spread in comprehending the underlying monetary transmission mechanism in the US economy.

3.5 Checking impulse responses

Encouraged by the results given in Section 3.4, this sub-section performs a generalized impulse response analysis introduced by Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998). The generalized impulse response analysis enables us to understand how an external shock dynamically influences a set of variables to be studied in a VAR framework. The shock is generated in a residual term of interest with all other shocks being integrated out. As a result, it is possible to interpret the generated shock as a variable-specific one, although its structural interpretation is not necessarily justified. The generalized impulse response analysis can help us to gain an insight into the interdependent nature of variables from a viewpoint different from a standard long-run cointegration analysis. Furthermore, it should be noted that the generalized impulse response analysis is not affected by the ordering of variables, a property missing in a

standard orthogonalized impulse response analysis in structural VAR modeling. Hence, the generalized impulse response analysis is viewed as a flexible method that allows us to obtain useful information from the data; see Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998) for further details. For various caveats concerning orthogonalized impulse response analysis in structural VAR models, see Ericsson, Hendry and Mizon (1998).

FIGURE 3 GENERALIZED IMPULSE RESPONSE FUNCTIONS



This sub-section introduces an external shock to each of the residual terms of the equations for some selected variables and then checks how the shock influences other variables of interest in the empirical system. Figure 3, overall, displays a set of point estimates of various impulse response functions (thick lines), coupled with the corresponding bootstrapped 95% confidence intervals (thin dotted lines). The number of the bootstrap replications is 2,000. Let us note that both Figures 3(a) and (b) have additional features for reference purposes. Figure 3(a) plots the responses of the Federal fund rate i_t^s to a one-standard error shock in the equation for the long-term interest rate i_t^l. The plotted response function indicates that the short-term interest rate rises significantly in response to the shock. An upsurge in the long-term bond yield, under ordinary circumstances, may indicate the outlook that the rate of inflation is rising, so it can induce a positive response of the Federal fund rate, as witnessed in Figure 3(a). This finding is consistent with the long-run monetary policy function given by equation (7) and also suggests the existence of a close connection between the long and short term interest rates. Figure 3(a) contains, as a reference, a time series profile of responses of i_t^l itself, which is denoted by a thick dotted line and its confidence band is omitted for ease of exposition. The total response of i_t^s turns out to be much larger than that of i_t^l, so it is judged that the slope of the underlying yield curve has become flattened as a result of this external shock. Hence, the monetary contraction in this case is accompanied by a decrease in the term spread, which is in line with the reasoning

developed in the previous sub-section.

Next, Figure 3(b) allows a one-standard error shock in the equation for the Federal fund rate and checks its influences on the long-term bond rate, coupled with those on the Federal fund rate itself, which is displayed here for reference purposes. In contrast to Figure 3(a), the responses of i_t^l to the shock are judged to be marginally significant, and its total volume is much smaller than that for i_t^s, which is reported here as a thick dotted line. This observation indicates the possibility that the Federal fund rate may have a limited impact on the bond rate, an inference which is consistent with Greenspan's conundrum and the evidence for the uncontrollability of i_t^l in the last sub-section. In spite of this observation, we can still argue the plausibility of effective monetary policy based on the idea of controlling the slope of the yield curve, as discussed above by referring to Adrian, Estrella and Shin (2010). We also check the responses of inflation π_t to the shock in the equation for the Federal fund rate. The results are displayed in Figure 3(c). The rate of inflation exhibits significant negative responses to the shock, in line with the last sub-section's observation that inflation is controllable by utilizing the Federal fund rate as an instrument.

Lastly, an external impulse is realized in the equation for the effective exchange rate e_t, and we examine its subsequent influences on the rate of inflation. The impulse is given as a one standard error shock; see Figure 3(d) for the estimates of responses. A series of negative responses of inflation is observed in this figure, although its initial responses are not found to be statistically significant. Thus, we can argue that an appreciation of the US dollar gives rise to the overall deflationary influences in the economy. This observation may allow us to infer that the exchange rate plays a significant role in the policy rule function (7) as a result of having influences on the dynamics of the inflation rate. This point is worth further empirical investigations and may indicate a possible research direction to be pursued in the future.

3.6 Testing strong exogeneity

Given the well-formulated CVAR system, we move on to mapping the data to *I(0)* series by making use of both first-order differencing and the restricted cointegrating combination (ecm_{t-1}). A full system in a VECM form is estimated here, which is employed to examine the property of strong exogeneity, before proceeding to a conditional data-representation pursued in Section 3.7.

If the combined condition of weak exogeneity and Granger non-causality is fulfilled with respect to a variable under investigation, the variable is said to be strongly exogenous for the parameters of interest such as β^*. Valid multi-step forecasts of a set of endogenous variables can be made, given the forecasts of strongly exogenous variables. See Engle, Hendry and Richard (1983) for further details of strong exogenity. The null hypothesis in a log *LR* test statistic in this case is formulated as the exclusion of the set of lagged endogenous variables from an equation for a weakly exogenous variable in question. In this study i_t^s and π_t correspond to endogenous variables (that is, variables modeled in a partial system), while i_t^l, y_t and e_t are viewed as weakly exogenous variables based on the test results in Section 3.2. Thus, the study in this sub-section checks the null hypothesis of excluding Δi_{t-1}^s , $\Delta\pi_{t-1}$ and ecm_{t-1} from the equations for Δi_t^l, Δy_t and Δe_t. The calculated log *LR* test statistics for the equations of Δi_t^l, Δy_t and Δe_t are 1.67[0.64], 5.26[0.15] and 2.79[0.43], respectively, where the figures in brackets represent *p*-values according to $\chi^2(3)$. The null hypothesis is not rejected at the standard 5% level with respect to all of these three

variables, leading us to conclude that these variables are all seen as strongly exogenous for the set of parameters of interest. Combined with the empirical evidence reported in Sections 3.2 and 3.4, these test results encourage us to model a nexus of Δi_t^s and $\Delta\pi_t$ conditional on the information of Δi_t^l, Δy_t and Δe_t.

3.7 Reducing the full system to a parsimonious nexus model

Finally, this section arrives at a data-congruent nexus model for Δi_t^s and $\Delta\pi_t$ and pursues its economic interpretations. First, a bivariate VECM for Δi_t^s and $\Delta\pi_t$ is estimated conditional on the three strongly exogenous variables, Δi_t^l, Δy_t and Δe_t. Next, by referring to log LR test statistics for the model reduction, insignificant contemporaneous and lagged regressors are dropped from the bivariate system. After conducting a series of the model reduction tests, we have reached a parsimonious VECM for Δi_t^s and $\Delta\pi_t$ as follows:

$$\widehat{\Delta i_t^s} = \underset{(0.067)}{0.352}\Delta i_t^l + \underset{(0.005)}{0.019}(\Delta e_t + \Delta e_{t-1}) - \underset{(0.017)}{0.081}ecm_{t-1} + \underset{(0.055)}{0.643}\Delta i_{t-1}^s + \underset{(0.070)}{0.160}\Delta i_{t-1}^l - \underset{(0.016)}{0.074} - \underset{(0.003)}{0.015}\, D_{3,t},$$

$$\widehat{\Delta\pi_t} = -\underset{(0.029)}{0.049}\Delta y_t + \underset{(0.005)}{0.011}\Delta e_t - \underset{(0.011)}{0.060}ecm_{2,t-1} - \underset{(0.067)}{0.229}\Delta\pi_{t-1} + \underset{(0.027)}{0.063}\Delta y_{t-1} - \underset{(0.010)}{0.055} + \underset{(0.002)}{0.008}\, D_{2,t} - \underset{(0.002)}{0.008}\, D_{3,t} + \underset{(0.001)}{0.006}\, D_{4,t},$$

where the sign ^ denotes a set of fitted values and the standard errors of coefficients are reported in parentheses. Diagnostic test statistics for this bivariate system are

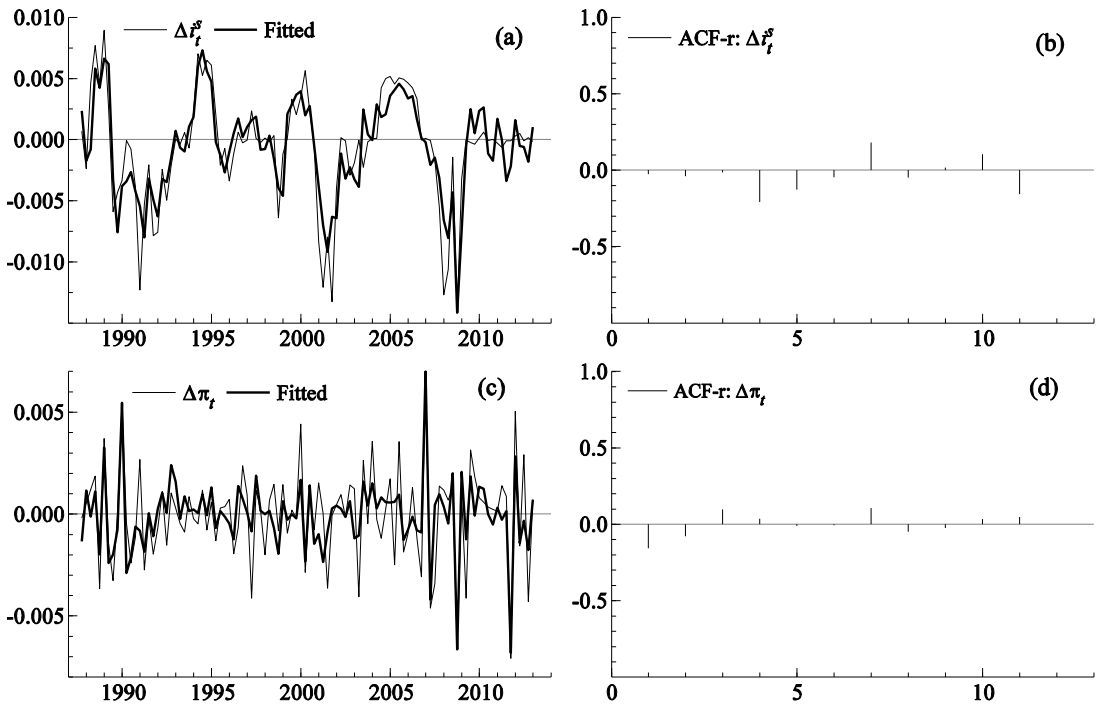
$$\text{Autocorr.}\,[F_{AR5}(20,166)] = 1.38\ [0.14],$$
$$\text{Hetero.}\,[F_{\text{HET}}(69,204)] = 0.89\ [0.71],$$
$$\text{Normality}[\chi^2_{ND}(4)] = 5.13\ [0.27].$$

Thus, the hypotheses indicating the absence of model mis-specification problems are not rejected at the 5% level. The parsimonious VECM's fitted values, together with the corresponding actual data, are recorded in Figures 4(a) and (c), while autocorrelation functions for the equations' residuals are displayed in Figures 4(b) and (d). According to Figure 4 (a), the tracking of the data for Δi_t^s is remarkably good, which indicates the success of modeling the dynamics of the instrument of the US monetary policy. In light of the overall statistical evidence reported here, it is possible to conclude that the parsimonious VECM is conceived of as an empirical nexus model congruent with the data.

Let us move on to economic interpretations of the estimated coefficients of the nexus above. First, as expected, the adjustment structure of each equation in the nexus agrees with the results recorded in equation (6). That is, the coefficient of ecm_{t-1} in each equation is negative and highly significant, indicating the presence of a mechanism of equilibrium correction in the dynamics of Δi_t^s and $\Delta\pi_t$. The adjustment structure in the first equation is consistent with the normalization of ecm_{t-1} centering on i_{t-1}^s; thus, the structure appears to be in favor of the interpretation that ecm_{t-1} represents a disequilibrium error from the underlying long-run monetary policy rule. We also conclude, in view of the second equation, that there is a significant influence of the deviation from the monetary policy rule, or from an implicit Phillips-curve-type relationship, upon the dynamics of inflation; hence, the presence of ecm_{t-1}

contributes to stabilizing the overall inflation behavior. This conclusion is also backed up by the empirical evidence indicating inflation controllability, as reported in Section 3.4.

FIGURE 4 FITTED VALUES AND RESIDUAL AUTOCORRELATIONS



Next, we proceed to checking the short-run dynamics of each equation in the nexus above. Looking at the short-run components of the Δi_t^s equation, we notice that two contemporaneous terms (Δi_t^l and Δe_t), coupled with their lagged terms (Δi_{t-1}^l and Δe_{t-1}), are all significant and thus play important roles in accounting for the behavior of the short-term interest rate. The initial estimate of a coefficient for Δe_t turns out to be fairly close to that for Δe_{t-1}, and therefore a restriction generating the common coefficient is introduced in the estimation of this nexus model. Since a rise in the nominal long-term bond rate can imply an upsurge in expected inflation, it appears reasonable that both Δi_t^l and Δi_{t-1}^l hold positive coefficients here, reflecting monetary policy responses to the presence of inflation pressure. With regard to the effective exchange rate, it may be viewed as a summary of leading economic indicators, *i.e.* positive signals found in various leading economic indicators can be reflected in the overall appreciation of the US dollar and *vice versa*. This may be the reason why the common coefficient for Δe_t and Δe_{t-1} proves to be positive, not negative as indicated by the conventional hypothetical relationship between an interest rate and an exchange rate. Turning attention to the short-run dynamics of the equation for $\Delta \pi_t$, we find that Δy_{t-1} has a significant positive influence on inflation acceleration, together with a marginal negative effect of Δy_t. The sum of their coefficients is positive, hence indicating the existence of inflationary pressure caused by the growth of the US macro economy. The positive coefficient for Δe_t in this equation is subject to the same interpretation as above, so it may be seen as a reflection of the overall behavior of various leading economic indicators.

Judging from the evidence reported in this sub-section, it is possible to conclude that

the nexus model is a meaningful economic representation of the dynamics of the data. Thus, we are able to rely upon this model in order to gain empirical insight into the dynamic interactions between monetary policy and inflation in the US economy.

4. Summary and conclusion

This paper has applied formal multivariate econometric methods to the analysis of the US macroeconomic time series data over the period of 1987-2013. A long-run economic linkage underlying the data is estimated as a restricted cointegrating relationship, which is subject to dual economic interpretations: one is a monetary policy rule determining the behavior of the Federal fund rate, while the other is a Phillips-curve type relationship influencing the dynamics of inflation. It is worth noting that the effective exchange rate plays a critical role in the long-run cointegrating linkage in a theory-consistent manner. The estimated CVAR system is utilized to investigate various economic characteristics of the data. Among all of the empirical findings in this investigation, it is noteworthy that the inflation process is judged to be controllable by employing the Federal fund rate as well as the term spread as policy instruments. The analysis of inflation controllability sheds light on the role of the term spread in the underlying monetary transmission mechanism; the revealed evidence also conforms to the views of important empirical studies such as Adrian, Estrella and Shin (2010). Furthermore, the generalized impulse response analysis generates a set of estimated response functions consistent with the preceding study of inflation controllability. The importance of the effective exchange rate in the determination of inflation dynamics is illuminated as well in this analysis. Finally, a parsimonious VECM is obtained from the CVAR system by following the general-to-specific modeling approach. The final parsimonious model is perceived as a data-congruent econometric nexus of the dynamics of the monetary policy and inflation in the US economy. Overall, the empirical evidence documented in this paper allows us to gain insight into the underlying nature of the US macroeconomic time series data. Further empirical research along the line of this paper will lead us to have a deeper comprehension of the US economy in the future.

Acknowledgement:

The second author gratefully acknowledges a financial support from JSPS KAKENHI (Grant Number: 26380349).

Appendix: Detailed information on the data

A.1. Source

All of the data analyzed in this paper were obtained from International Financial Statistics CD-ROM (September, 2013) published by International Monetary Fund. International Financial Statistics is abbreviated to IFS below.

A.2. Data definitions

$i_t^s =$ the natural logarithm of the Federal funds rate (IFS code: 11160B..ZF...),

$i_t^l =$ the natural logarithm of 10 year government bond yield (IFS code: 11161...ZF...),

$\pi_t =$ the first-order difference of the natural logarithm of GDP deflator (IFS code: 11199BIRZF...), with the base year 2005,

$y_t =$ the difference between the natural logarithm of seasonally adjusted nominal gross domestic product (IFS code: 11199BACZF...) and the natural logarithm of GDP deflator (IFS code: 11199BIRZF...), with the base year 2005,

$e_t =$ the natural logarithm of a US dollar nominal effective exchange rate (IFS code: 111..NELZF...).

A.3 Note

The interest rate series are converted such that $i_t^s = ln(1 + I_t^s/100)$ and $i_t^l = ln(1 + I_t^l/100)$ respectively, in which I_t^s and I_t^l denote the corresponding original series (in percent) available in IFS.

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