Markov-switching variance models and structural changes underlying Japanese bond yields: An inquiry into non-linear dynamics

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Abstract

This study employs a Markov-switching variance method to model structural changes in Japan’s long-term government bond data and reveals three state classifications according to time-varying influences from various factors on bond yields. It examines three internal factors—Japan’s short-term interest rate, its inflation rate and stock returns—and one external factor—yields on the US long-term government bond. The results of this study highlight the non-linear nature of Japanese bond yields over approximately the past three decades.

JEL classifications: C22, C50, E43, E44.

Keywords: Markov-switching variance models, Structural changes, Non-linear dynamics, Long-term bond yields.

1 Introduction

This note seeks a well-formulated econometric representation of the dynamics of Japan’s long-term government bond yield over approximately the past thirty years. A Markov-switching variance approach is adopted for the purpose of accounting for various structural changes observed in the bond yield data. In this introductory section, an impetus is given for the empirical study of Japan’s long-term bond yield, together with a summary of the study’s significant aspects.

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Extensive empirical literature has examined long-term government bond yields from various perspectives of economics and finance. It is noteworthy, in particular, that US long-term bond rates did not rise during an era of increases in the Federal funds rate around 2004, a phenomenon that the then-chairman of the Federal Reserve Board Alan Greenspan called “conundrum” (see Greenspan, 2005). Understanding the behaviour of long-term bond yields is considered to be critical for assessing the transmission of monetary policy. It is, therefore, no surprise that Greenspan’s observation on the US long-term bond rates sparked numerous studies of US and Eurozone economies (see Backus and Wright, 2007; Bandholz, Clostermann and Seitz, 2009 and references therein). Recent studies examine the low yields that have persisted in Europe and the US for a decade or longer.

Japan is seen as a precursor of these developed economies in that it started to experience the lowering of bond rates as early as the mid-1990s after the asset-price bubble burst. See Mosk (2007, Ch.11) for further details of Japan’s economic bubble. Therefore, understanding Japan’s long-term bond rate movements is deemed to be useful for policymakers not only in Japan, but also in various other regions facing stagnant long-term interest rates. This study advances such an understanding by estimating a well-formulated econometric model for the time series data of a newly-issued 10-year government bond yield—a representative long-term interest rate in Japan’s overall bond market. Japan’s 10-year bond yield is one constituent in the leading index for the business cycle compiled by the Japanese Cabinet Office. In addition, its yield data are considered to useful in predicting the yen’s fluctuations against major currencies. Thus, modelling Japan’s long-term bond yield may assist economic forecasting. These are the foundations for the empirical investigation pursued in this study.

This study employs a Markov-switching method introduced by Hamilton (1989, 1990) to obtain an informative econometric model of Japan’s bond yield data. See Ang and Bekaert (2002), Taylor (2004), Frömmel, MacDonald and Menkhoff (2005), De Grauwe and Vansteenkiste (2007) and Schwartz (2012), *inter alia*, for the validity of this method in modelling time series data for various economic and financial variables. Following Turner, Startz and Nelson (1989), this study assigns importance to switching variance characteristics in evaluating conceivable structural changes in the data. See also Kim, Nelson and Startz (1998) and Bhar and Hamori (2003) for examples of switching variance specifications.

This study starts with a tentative regime-switching analysis of the bond yield data to provide a basis for further investigation. It then presents a formal Markov-switching econometric analysis of various factors affecting the behaviour of Japan’s long-term bond yield. This Markov-switching analysis illuminates the non-linear dynamic characteristics underlying the data, producing a more accurate understanding of Japan’s bond yield
behaviour during approximately the past thirty years.

Let us recall that short-term interest rates are generally under the control of a monetary authority. This study confirms a strong linkage between Japan’s long-term bond yield and its benchmark short-term rate, except for a deflationary period during which the short-term rate reached a zero-bound. In addition, Japan’s stock market exerts some nonlinear collateral effects on its bond market, according to the Markov-switching regression results in this study. Alongside these internal factors, this study examines the US long-term bond yield as an external factor and demonstrates that its influences on Japan’s bond yield are also regime-dependent. It is, therefore, important to take account of time-varying influences from all of these factors in order to comprehend and predict movements in Japanese bond yields. Earlier research into Japanese interest rates uses nonlinear econometrics (see Kuo and Enders, 2004; Kagraoka and Moussa, 2013, inter alia). However, few empirical researchers explicitly examine the regime-switching characteristics of Japan’s long-term bond yield. This study’s quantitative findings, therefore, illuminate the Japanese government bond market over approximately the past three decades.

This study proceeds as follows. Section 2 reviews Markov-switching variance models employed in this study. Section 3 analyses the time series data of Japan’s government bond rate in detail using the Markov-switching models. Section 4 concludes. All quantitative and graphic analyses are conducted using OxMetrics (Doornik, 2013a).

2 Review of Markov-switching variance models

Primary references for this review of Markov-switching methodology are Hamilton (1994, Ch.14) and Doornik (2013a,b). Let $s_t$ denote an unobservable random variable representing a state or regime at time $t$ and hold an integer from 0 to $N - 1$, that is $s_t \in \{0, \ldots, N - 1\}$. This variable is subject to an $N$-state Markov chain defined as

$$p_{ji} = \Pr (s_{t+1} = j | s_t = i), \quad \text{for } i, j = 0, \ldots, N - 1,$$

where $p_{ji}$ is called the transition probability that state $j$ follows state $i$. Note that a series of transition probabilities falls within the constraints of $\sum_{i=0}^{N-1} p_{ji} = 1$ and $p_{ji} \geq 0$ for $i, j = 0, \ldots, N - 1$. A representative Markov-switching variance model for an observed variable $y_t$ is then given as

$$y_t = x_t' \alpha + z_t' \beta (s_t) + \sigma (s_t) \varepsilon_t, \quad \text{for } t = 1, \ldots, T,$$

where $x_t$ is a $k$-dimensional vector of lagged and contemporaneous explanatory variables with a vector of fixed parameters $\alpha$, and $z_t$ is a $l$-dimensional vector of lagged and contemporaneous explanatory variables with a vector of state-dependent parameters $\beta (s_t)$. The
innovation term $\varepsilon_t$ is assumed to follow independent and identical normal distributions with zero mean and unit variance, that is $\varepsilon_t \sim IN[0,1]$, so that $\sigma(s_t)\varepsilon_t$ has switching variance. Both intercepts and observed autoregressive terms such as $y_{t-1}, y_{t-2}, \ldots$ may be contained in $x_t$ and $z_t$, while other explanatory variables are assumed to be weakly exogenous (see Engle, Hendry and Richard, 1983) for the parameters in (1).

Let $\theta$ represent a vector of all the unknown parameters in (1). The parameter vector $\theta$ can be estimated in a maximum likelihood framework subject to the constraints on transition probabilities. See the above references for further details of evaluating the likelihood function and estimating the unknown parameters. In addition, let $\Omega_t^j$ denote a vector consisting of the observed variables in (1) while their initial values are fixed. Using an algorithm developed by Kim (1994), one is able to calculate the probabilities $\Pr(s_t=j|\Omega_T^j, \hat{\theta})$ of a state being labelled as $j$ for $j=0, \ldots, N-1$, given information on the observed series up to the end point $T$ and on the parameter estimates $\hat{\theta}$. These probabilities, called smoothed probabilities, have time-varying features and permit inference about which regime or state the variable of interest is likely to be in over the sample period.

3 Markov-switching variance analysis of Japan’s bond yield data

This section conducts a detailed Markov-switching analysis of Japan’s government bond yield data. First, a simple tentative model is employed to depict overall bond rate behaviour from January 1986 to December 2014. The period’s starting point corresponds to when Japan was on the verge of an asset-price bubble following the 1985 Plaza Agreement on the foreign exchange market. The following simplified Markov-switching variance model, nested in (1) above, is employed for the purpose of conducting a tentative study of the data:

$$\Delta b_t = \alpha_1 \Delta b_{t-1} + \alpha_2 \Delta r_t + \alpha_3 \Delta r_{t-1} + \alpha_4 (b_{t-1} - r_{t-1}) + \mu(s_t) + \sigma(s_t) \varepsilon_t,$$

where $b_t$ is Japan’s 10-year government bond yield, $r_t$ is its uncollateralised overnight call rate representing a class of short-term interest rates, $b_{t-1} - r_{t-1}$ is the term spread between the long-term and short-term interest rates, $\Delta$ represents a first-order difference operator and $\mu(s_t)$ is a state-dependent intercept. The levels of variables are given as the linear combination $b_{t-1} - r_{t-1}$ in this equation, so any problems with non-stationarity are sorted via cointegration or equilibrium correction (see Hendry, 1995, Ch.7). The Appendix details the data. The initial model should be sufficiently general about the number of possible states, so a three-state specification (that is, $s_t \in \{0, 1, 2\}$), not a
typical two-state specification, is adopted for this Markov-switching analysis. Estimation results are summarised in Figure 1.

![Figure 1: Smoothed probabilities based on the tentative model](image)

Figure 1 (a) plots actual data for $\Delta b_t$. Figures 1 (b), (c) and (d) record smoothed probabilities for states 0, 1 and 2, respectively. The estimated classification between states 0 and 1 apparently depends upon changes in the volatility of $\Delta b_t$. As the figure shows, state 1 appears to overtake state 2 around the early 1990s, when Japan’s asset-price bubble burst. State 0 then replaces states 1 and 2 around mid-1996, several months after the Bank of Japan (BoJ) initiated its zero interest-rate monetary policy. States 1 and 2 temporarily return around year-end 1998, when the Trust Fund Bureau shock (see Shigemi, Kato, Soejima and Shimizu, 2001) gave rise to a sharp increase in long-term bond yields. This shock primarily occurred through miscommunication between participants in the Japanese bond market and the Trust Fund Bureau in the Ministry of Finance, which supervises government-debt financing. As Figures 1 (a), (b) and (d) show, state 2 recrudesces tentatively around mid-2003 due to the Value-at-Risk (VaR) shock (see Fukuda, Imakubo and Nishioka, 2012), which is said to have arisen through commercial
banks’ herd behaviour in managing financial risk using the VaR methodology. Massive sales of Japanese long-term bonds occurred in the mid-2003 when the US long-term bond yield rose and the VaR indicator surpassed permissible limits for banks’ bond balances. Figure 1 shows state 0 returning year-end 2004 and persisting to the close of the sample period.

This tentative regime-switching study set in a historical context facilitates a Markov-switching analysis of three states over the sampled period. That three-part classification updated by our formal Markov-switching study more accurately indicates structural changes in the macro economy and monetary policy than the tentative analysis summarised in Figure 1 above.

We commence a formal econometric study of the data by estimating a general three-regime Markov-switching regression for $\Delta b_t$. This study adopts Hendry’s (1995, 2000) general-to-specific modelling, guided by bond rate studies from Campbell and Shiller (1991), Evans and Lewis (1994), Enders and Granger (1998), McCallum (2005), Caporale and Williams (2002) and Bandholz et al. (2009). Bandholz et al. (2009) base their analysis of US data on Caporale and Williams (2002) and warn the possibility that the Fisher effect might not actually hold; that is, there could be an incomplete one-for-one relation between expected inflation and nominal interest rates. Hence, Bandholz et al. (2009) justify modelling US nominal bond yields and not real bond yields. In addition, McCallum (2005)’s theoretical model indicates a negative correlation between changes in nominal long-term bond yields and a lagged term for the term spread. This negative correlation endorses empirical evidence in Campbell and Shiller (1991) and Evans and Lewis (1994), and is also compatible with the concept of equilibrium correction.

This econometric study expands these earlier studies by allowing for regime changes in yield data. The following general Markov-switching model is introduced to analyse the data over the sample period:

$$\Delta b_t = z_t'\beta(s_t) + \sigma(s_t)\varepsilon_t,$$

where $s_t \in \{0, 1, 2\}$ and $z_t = (z_{1t}', z_{2t}')'$ for

$$z_{1t} = (1, \Delta b_{t-1}, \Delta r_t, \Delta \pi_{t-1}, \Delta \pi_t, \Delta q_t, \Delta q_{t-1}, \Delta b_t^{US}, \Delta b_{t-1}^{US})'$$

$$z_{2t} = (b_{t-1} - r_{t-1}, b_{t-1} - b_{t-1}^{US})'.$$

Variable $\pi_t$ is the annual inflation rate from Japan’s core consumer price index, $q_t$ is the annual growth rate of a stock price index (TOPIX) as a measure of annual stock market returns, $b_t^{US}$ is the yield on 10-year US government bonds and $b_{t-1} - b_{t-1}^{US}$ denotes the long-term yield differential between Japanese and US bonds. According to Fukuda et al. (2013), the global spillover effects on Japan’s government bond market are likely to be
significant, which allows us to include a class of regressors based on the US bond yield into the above model. Again, the variables in levels are specified as linear combinations (e.g., the term spread $b_{t-1} - r_{t-1}$ in $z_{2t}$), by anticipating the empirical effectiveness of credible equilibrium correction mechanisms in the equation. Variables $b_t$ and $q_t$ are contained within Japan’s index of leading economic indicators, and their data are from the Cabinet Office web site. Data for remaining variables originate from sources in the Appendix.

Estimating the parameters of (2) leads to a general Markov-switching representation of the data in that all parameters are treated as time-varying according to estimated regime or state classifications. This general model lays the foundations for a parsimonious expression of the data. First, this model tolerates removal of insignificant regressors. Next, we check whether estimated coefficients share constant values; that is, whether some of $z_t^i \beta (s_t)$ are regime-independent so that they can populate $x_t^i \alpha$ in (1). This reduction procedure delivers the following parsimonious three-state Markov switching regression model for $b_t$:

$$
\Delta b_t = \alpha_1 (b_{t-1} - r_{t-1}) + z_t^i \beta (s_t) + \sigma (s_t) \varepsilon_t,
$$

where $s_t \in \{0, 1, 2\}$ and $z_t = (z_{1t}, z_{2t})'$ for

$$
\begin{align*}
z_{1t} &= (1, \Delta b_{t-1}, \Delta r_t, \Delta \pi_{t-1}, \Delta q_t, \Delta b_t^{US})', \\
z_{2t} &= (b_{t-1} - b_t^{US}).
\end{align*}
$$

The term spread holds a time-invariant coefficient in this parsimonious model. Table 1 presents three residual-based diagnostic tests for the general model (2) and the parsimonious model (3); the normality test (see Doornik and Hansen, 2008), first-order and sixth-order autoregressive conditional heteroscedasticity (ARCH) tests (see Engle, 1982) and first-order and sixth-order portmanteau tests for serial correlation (see Box and Pierce, 1970). All results lack significance at 5%, indicating that neither model is judged to be mis-specified and no critical information is lost in shifting to the parsimonious model. AIC reported in Table 1 denotes an information criterion for each model based upon Akaike (1973), which favours the parsimonious model. Overall, the reduction is deemed to be acceptable.

Figure 2 (a) couples actual values of $\Delta b_t$ with a set of fitted values $\Delta \hat{b}_t$ from the parsimonious model (3). Figures 2 (b), (c) and (d) display smoothed probabilities for states 0, 1 and 2, respectively. Smoothed probabilities for state 0 are nearly identical to those in Figure 1, although the duration of state 2 here is briefer than in Figure 1. Again, differences between states classified 0 and 1, in particular, are mainly attributable to a shift in the underlying variance. See Table 2 below for the estimates $\hat{\sigma} (s_t)$ according to the derived states; state 1 exhibits relatively large volatility, while states 0 and 2 are characterised by small volatility.
<table>
<thead>
<tr>
<th>Test</th>
<th>General model</th>
<th>Parsimonious model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normality test</td>
<td>$\chi^2(2) : 5.437[0.066]$</td>
<td>$\chi^2(2) : 3.771[0.152]$</td>
</tr>
<tr>
<td>ARCH test</td>
<td>$F(3, 299) : 1.799[0.147]$</td>
<td>$F(3, 313) : 1.606[0.188]$</td>
</tr>
<tr>
<td>ARCH test</td>
<td>$F(6, 293) : 1.111[0.356]$</td>
<td>$F(6, 307) : 0.869[0.518]$</td>
</tr>
<tr>
<td>Portmanteau test</td>
<td>$\chi^2(3) : 0.280[0.964]$</td>
<td>$\chi^2(3) : 0.392[0.942]$</td>
</tr>
<tr>
<td>Portmanteau test</td>
<td>$\chi^2(6) : 1.761[0.940]$</td>
<td>$\chi^2(6) : 4.428[0.619]$</td>
</tr>
<tr>
<td>AIC</td>
<td>$-0.549858$</td>
<td>$-0.647681$</td>
</tr>
</tbody>
</table>

Note: Figures in ordinary and square brackets denote degrees of freedom and $p$-values, respectively.

Table 1: Diagnostic tests for the general and parsimonious models

As Figures 2 (a) and (c) reveal, most of the first ten years in the sample period are classified as state 1, whereas more sub-periods were state 2 in Figure 1 above. With regard to state 2 in Figure 2, recall that Japan’s economic bubble began to inflate around 1985 and burst in the early 1990s; state 2 thus may correspond to outlying bond yields outliers associated its progression. The model is judged to be successful in capturing these outliers in that it renders $\tilde{\sigma}(2)$ smaller than in the other states. As the BoJ initiated monetary easing after the bubble’s burst, the call rate dropped and the long-term bond yield decreased accordingly.

Figures 2 (a) and (b) indicate that state 0 persists from around 1996 to the end of the sample period, except for the two previously described interruptions caused by the Trust Fund Bureau and VaR shocks. This state corresponds to a deflationary period for Japan’s post-bubble economy. The bond yield stabilised throughout state 0, as monetary policy pervaded the bond market. Figures 2 (c) and (d) reveals that the effect of the VaR shock is now classified as state 1 and not as state 2 as indicated in Figure 1. Thus, it inhabits the same state as the Trust Fund Bureau shock. This finding indicates the parsimonious model depicts both shocks in a more unified manner. It also bolsters the argument that the classification of state 2 is mainly attributable to irregularity in bond yields during the speculative bubble.

Let us move on to investigate the estimated coefficients in Table 2. Coefficients for $\Delta r_t$ and $b_{t-1} - r_{t-1}$ are of interest in that they may represent influences of BoJ monetary policy on bond yields. According to Table 2, time-varying coefficients for $\Delta r_t$ are significant and positive except in state 0, whereas the coefficient for the time-invariant term spread is significant and negative, indicating that a stable internal equilibrium correction mechanism operates in the parsimonious model. Estimated coefficients for $\Delta r_t$ suggest that monetary policy, represented by the overnight call rate, substantially influences bond yield dynamics until the mid-1990s. The impact of the call rate, however, turns insignifi-
Figure 2: Smoothed probabilities based on the parsimonious model

cant afterwards as continuous monetary expansion rendered the rate nearly zero around the end of the 1990s. In contrast, the coefficient of $\Delta \pi_{t-1}$ is significant and positive coefficient in state 0 but significant and negative in state 2. One infers that the bond yield's persistent stagnation during state 0 partly reflected the economy’s underlying deflation, by admitting that state 2 is characterised by irregular bond yields during the speculative bubble. Furthermore, $\Delta q_t$ exhibits significant positive and negative effects on yields in states 0 and 2, respectively. The former may represent a flight-to-quality effect and the latter may be consistent with a standard present value relation between bond yields and stock prices. Although the interdependence of $\Delta b_t$ and $\Delta q_t$ is not taken into account in this model, the evidence suggests that the collateral effects of Japan’s stock market on its bond market are not linear over time but time-varying with changes in the underlying regimes.

Finally, note that most of state-dependent coefficients for the external variables, $\Delta b_t^{US}$ and $b_{t-1} - b_t^{US}$ in Table 2 accord with those for $\Delta r_t$ and $b_{t-1} - r_{t-1}$, although state 2 is noticeable in light of evidence for amplified influences from these external variables. In
Table 2: Estimated coefficients of the parsimonious model

<table>
<thead>
<tr>
<th></th>
<th>$s_t$</th>
<th></th>
<th>$s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{t-1} - r_{t-1}$</td>
<td>-</td>
<td>-0.039** (0.004)</td>
<td>0.104** (0.037)</td>
</tr>
<tr>
<td>$\Delta b_{t-1}$</td>
<td>0</td>
<td>-0.144* (0.066)</td>
<td>0.135** (0.012)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.082 (0.091)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.135 (0.012)</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta r_t$</td>
<td>0</td>
<td>0.290 (0.250)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.330** (0.113)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.375** (0.014)</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta \pi_{t-1}$</td>
<td>0</td>
<td>0.060* (0.027)</td>
<td>$\hat{\mu}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.006 (0.134)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.085** (0.014)</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>0</td>
<td>0.004** (0.001)</td>
<td>$\hat{\sigma}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.007 (0.004)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.036** (0.001)</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Figures in brackets denote standard errors. * and ** represent 5% and 1% levels of significance, respectively.

4 Concluding remarks

This study employed the Markov-switching variance method to model structural changes in Japan’s long-term government bond yield over approximately the past three decades. A class of internal factors consists of Japan’s short-term interest rate, its inflation rate and stock returns, whereas the US long-term government bond yield is treated as an external factor. Time-varying influences from these factors on yield dynamics have been demonstrated throughout this study. The study’s results are useful in order for us to obtain a good outline of the Japanese government bond market over approximately the past thirty years. This note also paves the way for further empirical studies of Japanese bond yields using non-linear econometric methodology.

Acknowledgement:

This work was supported by JSPS KAKENHI (Grant Number 26380349).
Appendix:
(Data definitions and sources )

Data definitions:

\( b_t \): Yield on the newly issued government bond (10 Years), Source <1>.

\( r_t \): Uncollateralised overnight call rate, Source <2>.

\( \pi_t \): 12th-order difference of the log of a consumer price index
(all items, less fresh food; base year 2010 = 100), multiplied by 100, Source <3>.

\( q_t \): 12th-order difference of the log of a Tokyo stock price index (TOPIX),
multiplied by 100, Source <1>.

\( b_t^{US} \): US government bond yield (10 Years), Source <4>.

Sources (accessed on 12 August 2015):


<2> Bank of Japan Time Series Data Search (www.stat-search.boj.or.jp/index_en.html)


References


