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Formulating testable hypotheses on inflation expectations with application to New Zealand's macroeconomic data

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Abstract

This note presents an operational testing procedure for investigating roles played by inflation expectations in the determination of actual inflation dynamics. A class of hypothetical equilibrium correction models is introduced as a primary building-block of the procedure. The models are then subject to various statistical tests focusing on the specification of adjustment and cointegrating parameters. The suggested procedure sheds light on a data generation process which underlies the interaction between the rates of expected and actual inflation. An empirical illustration is also presented using New Zealand's macroeconomic time series data.

JEL classification: C12; C32; D84; E31.
Keywords: Inflation expectations; Inflation dynamics; Inflation-coupling models; Equilibrium correction models; Cointegration.

1 Introduction

This note focuses on the analysis of inflation expectations to illuminate their roles in the determination of actual inflation dynamics. As a framework for the pursuit of price stability, a number of central banks have adopted the policy of anchoring nominal variables such as rates of money growth and inflation. Inflation targeting in the presence of forward guidance, in particular, has played a central role in modern monetary policy; see Bernanke and Gertler (2001), Mishkin (2001), and Corden (2002, Ch.3) for further details. In addition to this standard nominal-anchor approach, Bernanke (2007) argues the importance of anchoring long-run *inflation expectations* in terms of sustaining price stability. According to Mishkin (2007), inflation expectations have been stabilized in the US in recent years, thus resulting in the phenomenon of a flattened Phillips curve. Bernanke (2007), however, also pointed out the possibility that inflation expectations may not have been completely anchored, by referring to Gűrkaynak, Sack and Swanson (2005), *inter alia*. The overall assessment of the behavior of inflation expectations in the context of empirical macroeconomics has not yet been fixed, and there will be continuous arguments about this important variable from the standpoints of academics and practitioners.

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This study aims to contribute to a better understanding of the dynamics of survey-based time series data for inflation expectations. An operational testing procedure is presented for this purpose. As suggested by Bernanke (2004, 2007) and Orphanides and Williams (2005), the public's inflation expectations can be formed through a learning process, in which required adjustments may take time due to a lack of complete knowledge of current economic situations, the effectiveness of monetary policy and so forth. This note aims to capture such a learning process by utilizing equilibrium correction models (ECMs; see Hendry, 1995, Ch.7, *inter alia*) on inflation expectations, an approach similar to Grant and Thomas (1999). First, a class of multivariate ECMs is introduced as a main building-block of the suggested procedure. The models are then utilized for various statistical tests specifying adjustment and cointegrating parameters. Finally, an empirical illustration of the procedure is provided. The suggested procedure, using the set of ECMs as a medium of investigation, illuminates the underlying data generation process (DGP) for the interaction between the rates of actual and expected inflation. Note that Grant and Thomas (1999) focused on a small system consisting of two variables for actual inflation and inflation expectations. This study, in contrast, employs a large multivariate system including various other macroeconomic variables such as aggregate money and interest rates, with a view to allowing for their influences on the rates of actual and expected inflation. This study can thus be viewed as a useful complement to Grant and Thomas (1999).

As an empirical illustration of the procedure, New Zealand's time series data are analyzed using a set of empirical ECMs. New Zealand is well known as one of the front runners in inflation targeting policy (see Bernanke, Laubach, Mishkin and Posen, 1999, Ch.5, *inter alia*). The Reserve Bank of New Zealand has made public, on its web site, a number of survey-based time series data for expectations of future economic situations; the web site, for example, provides time series data for one year ahead expected inflation. We use this data set to demonstrate the validity of the proposed methodology. New Zealand's data were also analyzed in one of our previous working papers, Choo and Kurita (2012), and this working paper acted as precursory research, which gave rise to the present study focusing on the illumination of the underlying dynamics of New Zealand's data.

The rest of this note is organized into four sections. Section 2 explains a strategy for illuminating roles played by inflation expectations by introducing ECMs. Section 3 then presents a testing procedure applicable to the ECM, with view to specifying adjustment and cointegrating parameters in a manner consistent with Section 2. An empirical illustration of the procedure is provided in Section 4. Finally, Section 5 concludes the paper. Econometric analyses in this note are performed by utilizing *CATS* in *RATS* (Dennis, Hansen, Johansen and Juselius, 2005) and *PcGive* (Doornik and Hendry, 2013).

2 Econometric modeling strategy

This section introduces a class of economic variables to be analyzed in the ECM framework. A quick review is also made regarding a method using a cointegrated vector autoregressive (CVAR) system pioneered by Johansen (1988, 1996). Let us recall that this note aims to present a procedure for investigating the roles played by inflation expectations in empirical inflation dynamics. From this point of view, it is important to build our modelling strategy on economic linkages empirically conceivable between the rates of expected and actual inflation. This paves the way for the introduction of ECMs incorporating interpretable long-run economic relationships; see Davidson, Hendry, Srba and Yeo (1978), Hendry and von Ungern-Sternberg (1981) and Hendry (1995, Ch.7) for further details of ECMs. These hypothetical relationships should be plausible enough to be relevant to empirical cointegrating combinations derived from data investigation. Assigning importance to the view that actual inflation is under the influence of inflation expectations (see Bernanke, 2004 and 2007, *inter alia*), we assume that the formulation of baseline inflation dynamics for $t = 1, \ldots, T$ is given, with the initial value for π_t fixed, as an ECM in the following way:

$$\Delta\pi_t = \lambda\left(\pi_{t-1} - \pi^e_{t-1}\right) + u_t \quad \text{for} \quad -1 < \lambda < 0, \tag{1}$$

where π_t is actual inflation, π^e_t is one-period ahead expected inflation available from survey data, u_t is an error term representing a set of unspecified stationary explanatory variables for $\Delta\pi_t$. Eq.(1) implies that inflation reacts to lagged disequilibrium $\pi_{t-1} - \pi^e_{t-1}$ in such a way that the disequilibrium error is corrected at every time point, so that actual inflation tends to move in line with inflation expectations. In other words, the baseline stochastic behavior of π_t is anchored by that of π^e_t. Concerning the dynamics of π^e_t with its initial value given, we assume the following ECM:

$$\Delta\pi^e_t = \phi\left(\pi^e_{t-1} - \eta' W_{t-1}\right) + v_t \quad \text{for} \quad -1 < \phi < 0, \tag{2}$$

where W_{t-1} is a vector of macroeconomic variables associated with monetary policy such as interest rates and monetary aggregates, η is a vector of parameters conformable to W_{t-1}, and v_t is an error term consisting of unspecified stationary explanatory variables for $\Delta\pi^e_t$. Note that (2) is formulated based on the idea that inflation expectations are moored by those economic fundamentals which are considered to be subject to monetary policy. The set of ECMs, (1) and (2), indicates that inflation expectations play the role of a long-run coupler connecting economic fundamentals W_t and the rate of actual inflation π_t. The ECMs can also be interpreted as a representation of the forward-looking nature of the economy that underlies the transmission of monetary-policy influences. Moreover, the ECMs may be justified as a reflection of economic agents' learning process; see Orphanides and Williams (2005), and Bernanke (2004, 2007), *inter alia*, for further details of the process of learning.

The above ECMs seem to be general enough to be analyzed using real-life data, but a caveat should be mentioned when employing them for the purpose of empirical research: there is no guarantee that W_t can be treated as a set of pure explanatory or exogenous variables. In other words, W_t should, at the outset, be regarded as the members of endogenous variables determined in an economic system, and a subsequent analysis should seek evidence for the justification of treating some of them as exogenous variables. In this modeling scheme, it is possible to introduce the following economic system (with lag length k and a set of initial values fixed) in which both Eqs. (1) and (2) are nested and W_t is endogenized:

$$\begin{pmatrix} \Delta\pi_t \\ \Delta\pi^e_t \\ \Delta W_t \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \phi \\ 0 & \xi \end{pmatrix} \begin{pmatrix} \pi_{t-1} - \pi^e_{t-1} \\ \pi^e_{t-1} - \eta' W_{t-1} \end{pmatrix} + \textstyle\sum_{i=1}^{k-1} \Gamma_i \begin{pmatrix} \Delta\pi_{t-i} \\ \Delta\pi^e_{t-i} \\ \Delta W_{t-i} \end{pmatrix} + \varepsilon_t, \tag{3}$$

where ξ and Γ_i for $i = 1, \ldots, k-1$ are vector and matrix parameters conformable to subsequent sets of variables respectively, and ε_t is a vector error sequence following independent and identical distribution (i.i.d.). For Eq.(3), note that u_t and v_t are expressed as ε_t in combination with the past dynamics of all the variables in the system. Eq.(3) is referred to as an inflation-coupling model (ICM) in the rest of this study.

The above argument can be expressed using a CVAR system for stochastic processes integrated of order 1. Let us define a p-dimensional vector series X_t as

$$X_t = (\pi_t, \pi_t^e, W_t)',$$

so that W_t is seen as a $(p-2)$-dimensional vector series. Referring to Johansen (1988, 1996), Kurita (2010) and Choo and Kurita (2011), we introduce a CVAR(k) model incorporating deterministic terms such as linear trend, given a set of initial values, as follows:

$$\Delta X_t = \alpha\,(\beta', \rho)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1}\Gamma_i \Delta X_{t-i} + \mu + \Phi D_t + \varepsilon_t, \tag{4}$$

where ε_t is a p-dimensional random sequence following i.i.d. normal $N(0, \Omega)$ with Ω being a positive definite symmetric matrix, and D_t is an s-dimensional vector of deterministic dummy variables. The adjustment and cointegrating parameters, $\alpha, \beta \in \mathbf{R}^{p\times r}$, are assumed to be full column rank r (cointegrating rank) for $r < p$, while the remaining parameters are defined as $\Gamma_i \in \mathbf{R}^{p\times p}$ for $i = 1, \ldots, k-1$, $\Phi \in \mathbf{R}^{p\times s}$, $\rho \in \mathbf{R}^r$ and $\mu \in \mathbf{R}^p$. See, for instance, Juselius (2006), Kurita (2010), and Choo and Kurita (2011) for empirical studies using CVAR models. In order for Eq.(4) to be consistent with the ICM, the cointegrating rank needs to be 2 ($r = 2$), although we cannot assume the rank *a priori* before launching an empirical investigation.

Under a set of regularity assumptions, it is possible to derive a moving average representation of (4), which is called the Granger-Johansen representation of the CVAR system (see Theorem 4.2 in Johansen, 1996). This representation enables us to make a likelihood-based inference for r. That is, we can rely on a log-likelihood ratio test statistic ($\log LR$) for the null hypothesis $\mathsf{H}(r)$ against the alternative hypothesis $\mathsf{H}(p)$ by using simulation-based knowledge of the statistic's non-standard asymptotic distribution; see Nielsen (1997) and Doornik (1998) for further details. Once r is determined, we are able to explore economic interpretations of α and β^* by examining various hypotheses to which standard limiting χ^2-based inferences are applicable. See Johansen (1996, Chs. 6 - 8) for further details.

3 Testing hypotheses about inflation expectations

This section addresses the issue of how to reveal the underlying roles of inflation expectations in the dynamics of actual inflation. The suggested procedure involves three hypotheses under $r = 2$. Suppose that the CVAR model with no deterministic terms is, for the sake of simplicity, applicable to data under study and a likelihood-based empirical analysis of them indicates $r = 2$ or $\mathsf{H}(2)$; thus, α and β are expressed as

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \vartheta_1 & \vartheta_2 \end{pmatrix} \qquad \text{and} \qquad \beta = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \\ \gamma_1 & \gamma_2 \end{pmatrix},$$

where ϑ_i and γ_i for $i = 1, 2$ denote $(p-2)$-dimensional vectors of parameters. Introduce a normalization matrix

$$L = \begin{pmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{pmatrix}^{-1} = \begin{pmatrix} \beta^{(11)} & \beta^{(21)} \\ \beta^{(12)} & \beta^{(22)} \end{pmatrix},$$

and apply this to β:

$$L\beta' = \begin{pmatrix} 1 & 0 & \beta^{(11)}\gamma_1' + \beta^{(21)}\gamma_2' \\ 0 & 1 & \beta^{(12)}\gamma_1' + \beta^{(22)}\gamma_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\gamma_1^{*\prime} \\ 0 & 1 & -\gamma_2^{*\prime} \end{pmatrix}.$$

The first hypothesis is formulated to find evidence for the existence of a set of common factors for the long-run dynamics of actual and expected inflation. The hypothesis is given as

$$\mathsf{H}_0^{(1)} : \gamma_1^* = \gamma_2^*,$$

which means that there is a common coefficient vector for W_t in the cointegrating relationships. Under this hypothesis W_{t-1} acts as a class of common factors for π_t and π_t^e; see Hunter and Burke (2011) for some analogies to $\mathsf{H}_0^{(1)}$ in $L\beta'$ above. In order to maintain the value of the product $\alpha\beta'$ under $r = 2$, it is also necessary to examine

$$\alpha L^{-1} = \begin{pmatrix} \alpha_{11}\beta_{11} + \alpha_{12}\beta_{12} & \alpha_{11}\beta_{21} + \alpha_{12}\beta_{22} \\ \alpha_{21}\beta_{11} + \alpha_{22}\beta_{12} & \alpha_{21}\beta_{21} + \alpha_{22}\beta_{22} \\ \vartheta_1\beta_{11} + \vartheta_2\beta_{12} & \vartheta_1\beta_{21} + \vartheta_2\beta_{22} \end{pmatrix} = \begin{pmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{22}^* \\ \vartheta_1^* & \vartheta_2^* \end{pmatrix}. \tag{5}$$

The second hypothesis is then formulated as

$$\mathsf{H}_0^{(2)} : \alpha_{12}^* = -\alpha_{11}^*, \quad \alpha_{21}^* = 0 \quad \textit{and} \quad \vartheta_1^* = 0.$$

The implication of the joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(2)}$ is made clear by rotating α and β. Let us give an expression of $\alpha\beta' X_{t-1}$ under this joint hypothesis as

$$\alpha\beta' X_{t-1} = \begin{pmatrix} \alpha_{11}^* & -\alpha_{11}^* \\ 0 & \alpha_{22}^* \\ 0 & \vartheta_2^* \end{pmatrix} \begin{pmatrix} 1 & 0 & -\gamma_1^{*\prime} \\ 0 & 1 & -\gamma_1^{*\prime} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi_{t-1}^e \\ W_{t-1} \end{pmatrix}. \tag{6}$$

Introduce a transformation matrix M:

$$M = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix},$$

so that (6) is re-expressed as

$$\begin{aligned} \alpha\beta' X_{t-1} &= \begin{pmatrix} \alpha_{11}^* & -\alpha_{11}^* \\ 0 & \alpha_{22}^* \\ 0 & \vartheta_2^* \end{pmatrix} M^{-1} M \begin{pmatrix} 1 & 0 & -\gamma_1^{*\prime} \\ 0 & 1 & -\gamma_1^{*\prime} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi_{t-1}^e \\ W_{t-1} \end{pmatrix} \\ &= \begin{pmatrix} \alpha_{11}^* & 0 \\ 0 & \alpha_{22}^* \\ 0 & \vartheta_2^* \end{pmatrix} \begin{pmatrix} \pi_{t-1} - \pi_{t-1}^e \\ \pi_{t-1}^e - \gamma_1^{*\prime} W_{t-1} \end{pmatrix}, \end{aligned}$$

which is in agreement with the structure of the ICM in the previous section. In order to ensure the empirical validity of this joint hypothesis, we also intend to cross-examine the following hypothesis:

$$\mathsf{H}_0^{(3)} : \alpha_{12}^* = 0, \quad \alpha_{21}^* = -\alpha_{22}^* \quad and \quad \vartheta_2^* = 0.$$

Thus, under the joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$,

$$\alpha\beta' X_{t-1} = \begin{pmatrix} \alpha_{11}^* & 0 \\ -\alpha_{22}^* & \alpha_{22}^* \\ \vartheta_1^* & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\gamma_1^{*\prime} \\ 0 & 1 & -\gamma_1^{*\prime} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi_{t-1}^e \\ W_{t-1} \end{pmatrix}.$$

Introduce a transformation matrix N:

$$N = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix},$$

and the joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ thus implies

$$\begin{aligned} \alpha\beta' X_{t-1} &= \begin{pmatrix} \alpha_{11}^* & 0 \\ -\alpha_{22}^* & \alpha_{22}^* \\ \vartheta_1^* & 0 \end{pmatrix} N^{-1} N \begin{pmatrix} 1 & 0 & -\gamma_1^{*\prime} \\ 0 & 1 & -\gamma_1^{*\prime} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi_{t-1}^e \\ W_{t-1} \end{pmatrix} \\ &= \begin{pmatrix} 0 & \alpha_{11}^* \\ \alpha_{22}^* & 0 \\ 0 & \vartheta_1^* \end{pmatrix} \begin{pmatrix} \pi_{t-1}^e - \pi_{t-1} \\ \pi_{t-1} - \gamma_1^{*\prime} W_{t-1} \end{pmatrix}. \end{aligned}$$

That is, $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ means that the rate of actual inflation, not inflation expectations, is moored by the set of economic fundamentals represented by W_t, a property which is in contrast to the ICM above.

It is possible to summarize the above argument as follows: if the first joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(2)}$ is not rejected in the likelihood-based testing of the data while the second one $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ is rejected, this can be seen as solid statistical evidence in favor of the ICM. In other words, inflation expectations play the role of a long-run connection of economic fundamentals and the rate of actual inflation. The forward-looking nature of monetary policy may underlie this structure. If, however, $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(2)}$ is rejected while $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ fails to be rejected, the actual inflation rate plays the long-run coupling role instead. As a practical issue, it is also important to adopt an appropriate multiple testing procedure in examining these joint hypotheses; see Hendry and Nielsen (2007, Ch.7) for further details.

4 Empirical illustration

This section provides an empirical illustration of the above testing procedure by analyzing New Zealand's macroeconomic time series data. Section 4.1 checks diagnostic tests for an empirical VAR model and determines its cointegrating rank. Section 4.2 applies the testing procedure to the data to make sure that the inflation expectations play the role of a long-run coupler combining economic fundamentals and actual inflation.

4.1 Checking the cointegrating rank

The parameters of an unrestricted VAR(4) (that is, $k = 4$) model is estimated using New Zealand's quarterly data for the following set of variables:

$$X_t = (\pi_t, \pi_t^e, W_t)' = \left(\pi_t, \pi_t^e, m_t - p_t, y_t, i_t^l - i_t^s\right)',$$

where π_t is the rate of realized annual inflation, π_t^e is the rate of expected annual inflation, $m_t - p_t$ is the log of real broad money, y_t is the log of real output and $i_t^l - i_t^s$ is a spread between long-term and short-term interest rates. The period for estimation runs from the fourth quarter in 1992 to the first quarter in 2015, which is expressed as 1992.4 - 2015.1 hereafter. There is some evidence suggesting the existence of autocorrelation in residuals of the π_t equation, so the adjustment of short-run dynamics is made by following Kurita and Nielsen (2009); that is, lagged second-order differenced series, $\Delta^2\pi_{t-4}$ and $\Delta^2\pi_{t-5}$, have been added to the VAR(4) system unrestrictedly, with a view to solving the problem of autocorrelation without influencing the asymptotic theory for cointegration analysis. In addition, an outlier presumably due to the Asian financial crisis in 1997, which was found in the residual series of the π_t equation, has been handled by introducing a dummy variable holding 1 in 1997.3 and 0 otherwise.

	π_t	π_t^e	$m_t - p_t$	y_t	$i_t^l - i_t^s$
$F_{AR}(5,60)$	1.74 [0.14]	1.04 [0.40]	1.49 [0.21]	1.03 [0.41]	1.71 [0.15]
$F_{ARCH}(4,82)$	0.67 [0.61]	1.36 [0.25]	0.11 [0.98]	1.16 [0.33]	0.62 [0.65]
$F_{HET}(46,42)$	1.55 [0.08]	1.43 [0.12]	1.36 [0.16]	0.89 [0.65]	1.32 [0.18]
$\chi^2_{ND}(2)$	0.16 [0.92]	4.77 [0.09]	0.16 [0.92]	1.06 [0.59]	2.25 [0.33]

Note. Figures in square brackets are p-values.

Table 1: Mis-specification tests for the VAR system

Table 1 provides residual-based diagnostic tests for each equation in the VAR system. Most of them are formulated as $F_j(k, \cdot)$, which represents an approximate F test against a hypothesis specified by j: kth-order autocorrelation (F_{AR}: see Godfrey, 1978, Nielsen, 2006), kth-order autoregressive conditional heteroskedasticity or ARCH (F_{arch}: see Engle, 1982), heteroskedasticity (F_{het}: see White, 1980). The table also includes a chi-square test for normality (χ^2_{nd}: see Doornik and Hansen, 2008). There is no clear evidence indicating mis-specification problems, so it is justifiable to employ the VAR model for the purpose of a likelihood-based study of the underlying cointegrating rank r.

Table 2 presents a set of log-likelihood ratio test statistics for r; $\log LR(r)$ is a standard trace test statistic (see Johansen, 1996, Ch.6) while $\log LR(r)^S$ is a $\log LR(r)$ subject to a small-sample correction (see Johansen, 2002). The former tests indicate $r = 2$ at the 1% level, and the latter ones also support $r = 2$ at the 5% level. Bearing in mind size-distortion problems in small samples, we are justified in concluding that the cointegrating rank is 2 and proceeding to a further study of empirical roles of inflation expectations in the next sub-section.

	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$	$r\leq 3$
$\log\ LR$	112.57[0.00]*	78.27[0.00]*	47.25[0.02]	27.00[0.03]	9.82[0.14]
$\log\ LR^S$	95.00[0.02]	68.33[0.02]	42.40[0.06]	23.80[0.09]	8.49[0.22]

Note. Figures in square brackets are p-values.
$*$ denotes significance at the 1% significance level.

Table 2: Determination of the cointegrating rank

4.2 Checking the role of inflation expectations

Given $r=2$, it is now possible to check various hypotheses developed in Section 3. First, we will examine whether or not the hypothesis $\mathsf{H}_0^{(1)}$ defined in the previous section is empirically valid. By checking a set of unrestricted estimates for α and (β',ρ), we have obtained a rough idea of the underlying structure of these parameters and thus arrived at the following acceptable restricted estimates:

$$\widehat{\alpha}\left(\widehat{\beta}',\widehat{\rho}\right)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} = \begin{pmatrix} \underset{(0.13)}{-0.42} & \underset{(0.26)}{0.41} \\ \underset{(0.06)}{0.02} & \underset{(0.13)}{-0.17} \\ \underset{(0.41)}{-0.36} & \underset{(0.86)}{2.67} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -0.13 & 0 & -0.5 & 0.0015 \\ 0 & 1 & -0.13 & 0 & -0.5 & 0.0015 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi^e_{t-1} \\ m_{t-1}-p_{t-1} \\ y_{t-1} \\ i^l_{t-1}-i^s_{t-1} \\ t \end{pmatrix}, \tag{7}$$

where a set of restrictions under $\mathsf{H}_0^{(1)}$ are imposed upon (β',ρ), together with those zero restrictions on α which correspond to weak exogeneity (see Engle, Hendry and Richard, 1983; Johansen, 1992) of y_t and $i^l_t - i^s_t$ with respect to the cointegrating parameters. The calculated $\log LR$ against no restriction (apart from $r=2$) is 16.71[0.16], where the figure in square brackets is a p-value according to $\chi^2(12)$; hence, the overall null hypothesis is not rejected at a standard 1% level, a finding allowing us to proceed to a further study of inflation expectations. Note that coefficients in the first row of $\widehat{\alpha}$ above, -0.42 and 0.41, corresponding to α^*_{11} and α^*_{12} in Eq.(5), appear to be subject to the constraint of $\alpha^*_{12}=-\alpha^*_{11}$ under $\mathsf{H}_0^{(2)}$. This encourages us to investigate the empirical validity of a joint hypothesis corresponding to $\mathsf{H}_0^{(1)}\cap\ \mathsf{H}_0^{(2)}$.

In the following we report updated estimates of the parameters, on which additional restrictions under $\mathsf{H}_0^{(2)}$ are placed:

$$\widehat{\alpha}\left(\widehat{\beta}',\widehat{\rho}\right)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} = \begin{pmatrix} -0.45 & 0.45 \\ 0 & \underset{(0.06)}{-0.15} \\ 0 & \underset{(0.43)}{2.04} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -0.13 & 0 & -0.5 & 0.0015 \\ 0 & 1 & -0.13 & 0 & -0.5 & 0.0015 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \pi^e_{t-1} \\ m_{t-1}-p_{t-1} \\ y_{t-1} \\ i^l_{t-1}-i^s_{t-1} \\ t \end{pmatrix}, \tag{8}$$

where the additional restrictions are reflected in the estimates $\widehat{\alpha}$; see Eq.(6) as the basis for Eq.(8). The $\log LR$ test statistic for Eq.(8) against the hypotheses embodied in Eq.(7) is

1.04[0.90], in which the figure in square brackets is a p-value based upon $\chi^2(4)$. The joint null hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(2)}$ is not rejected at the standard 1% level, implying that inflation expectations play the role of a long-run coupling connecting the economic fundamentals W_t and actual inflation π_t.

With a view to verifying the validity of this finding, the other joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ is also under investigation. As a trial, we introduce a subset of the restrictions under $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$, that is, $\alpha_{12}^* = 0$ and $\vartheta_2^* = 0$ only, not including the restriction of $\alpha_{21}^* = -\alpha_{22}^*$. The resultant restricted estimates are

$$\widehat{\alpha}\left(\widehat{\beta}', \widehat{\rho}\right)\begin{pmatrix} X_{t-1} \\ t \end{pmatrix} = \begin{pmatrix} \underset{(0.07)}{-0.25} & 0 \\ \underset{(0.06)}{0.06} & \underset{(0.12)}{-0.27} \\ \underset{(0.23)}{0.76} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & -0.13 & 0 & -0.5 & 0.0015 \\ 0 & 1 & -0.13 & 0 & -0.5 & 0.0015 \end{pmatrix}\begin{pmatrix} \pi_{t-1} \\ \pi_{t-1}^e \\ m_{t-1} - p_{t-1} \\ y_{t-1} \\ i_{t-1}^l - i_{t-1}^s \\ t \end{pmatrix}, \tag{9}$$

where the estimates $\widehat{\alpha}$ have been changed as a result of the newly introduced restrictions. The log LR test statistic for Eq.(9) against the alternative hypotheses underlying Eq.(7) is 16.12[0.00]*, in which the figure in square brackets is a p-value based upon $\chi^2(2)$. Hence, the joint null hypothesis for Eq.(9) is judged to be rejected at the 1% level; it is thus unlikely that $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ is a valid representation of the data. Indeed, the second row of $\widehat{\alpha}$ appears to be against the equality $\alpha_{21}^* = -\alpha_{22}^*$, indicating rejection of the complete joint hypothesis $\mathsf{H}_0^{(1)} \cap \mathsf{H}_0^{(3)}$ including this equality.

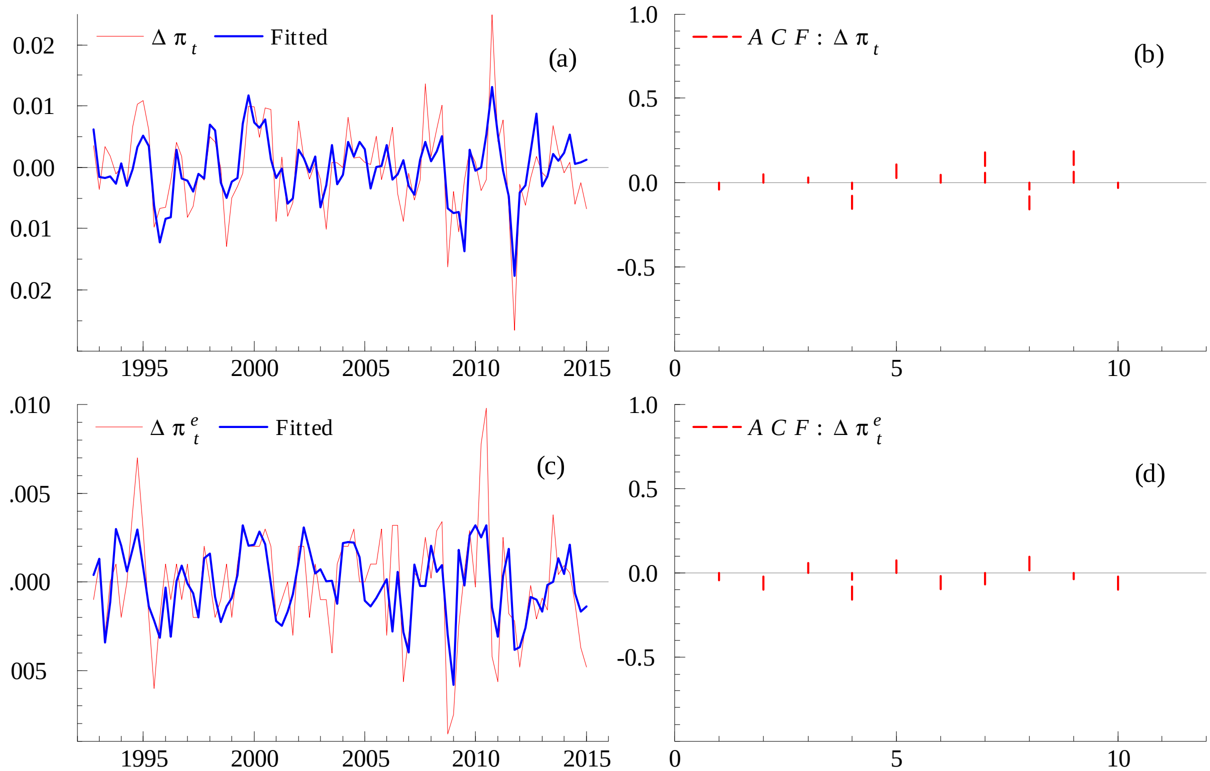
Given the non-rejection of the hypotheses in Eq.(8) while those in Eq.(9) are rejected, it is possible to claim that inflation expectations play the long-run coupler role and to rotate Eq.(8) so as to find the following expression:

$$\begin{aligned}\begin{pmatrix} \Delta\pi_t \\ \Delta\pi_t^e \\ \Delta\left(m_t - p_t\right) \\ \Delta y_t \\ \Delta\left(i_t^l - i_t^s\right) \end{pmatrix} &= \begin{pmatrix} -0.45 & 0 \\ 0 & -0.15 \\ 0 & 2.04 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \pi_{t-1} - \pi_{t-1}^e \\ \pi_{t-1}^e - \widehat{\eta}' W_{t-1} - 0.00155t \end{pmatrix} \\ &\quad + \sum_{i=1}^{k-1} \widehat{\Gamma}_i \begin{pmatrix} \Delta\pi_{t-i} \\ \Delta\pi_{t-i}^e \\ \Delta\left(m_{t-i} - p_{t-i}\right) \\ \Delta y_{t-i} \\ \Delta\left(i_{t-i}^l - i_{t-i}^s\right) \end{pmatrix} + \widehat{\mu} + \widehat{\Phi} D_t + \widehat{\varepsilon}_t, \end{aligned} \tag{10}$$

where

$$\widehat{\eta}' W_{t-1} = 0.13\left(m_{t-1} - p_{t-1}\right) + 0.5\left(i_{t-1}^l - i_{t-1}^s\right),$$

and system (10) is consistent with the hypothetical ICM (3) discussed in Section 2. This evidence indicates that New Zealand is seen as an economy in which inflation expectations are acting as a coupler connecting the set of economic fundamentals and actual inflation. The policy of inflation targeting in the presence of forward guidance presumably underlies the revealed structure of system (10). Finally, Figure 4.2 (a) and (c) present a set of actual values for $\Delta\pi_t$ and $\Delta\pi_t^e$, together with their fitted values derived from a vector



ECM corresponding to system (10) free of the set of weak exogeneity restrictions. Their residual autocorrelation functions (ACFs) are also recorded in Figure 4.2 (b) and (d). The fitted values track the actual ones well, and there is no clear evidence for the problem of residual autocorrelation. Thus, system (10) can be judged to be a reliable representation of the underlying process for $\Delta\pi_t$ and $\Delta\pi_t^e$.

5 Summary and Conclusion

This note has provided an operational CVAR-based testing procedure to contribute to a better comprehension of empirical macroeconomic dynamics centering on inflation expectations. A couple of hypothetical ECMs were introduced to set the stage for the testing procedure. The ECMs were expressed as an ICM nested in a general CVAR system, so that various likelihood-based tests were discussed in the CVAR framework with a view to specifying adjustment and cointegrating parameters. Furthermore, New Zealand's macroeconomic time series data were analyzed as an empirical illustration. This analysis demonstrated the usefulness of the procedure in applied macroeconomic research. The suggested procedure in this study can be seen as a route towards the DGP that underlies the interaction between the rates of actual and expected inflation. As future research, it will be worthwhile to examine various other countries and regions by using the proposed testing procedure, in terms of the validity of the hypothetical ICM as an empirical representation of actual and expected inflation.

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Appendix:

(Details of the data)

Data sources:

<1> International Financial Statistics CD-ROM (IFS, September, 2015) published by International Monetary Fund.

<2> Website of the Reserve Bank of New Zealand "M14 Survey of expectations - Annual CPI growth - 1 year out"
http://www.rbnz.govt.nz/statistics/M14 (Accessed on October 5, 2015).

Data definitions (quarterly data):

π_t: 4th-order difference of the log of the consumer price index at time t,
i.e. $\Delta^4 p_t$ (CPI, all groups), Source <1>. IFS code: 19664...ZF...

π_t^e: Expected four-quarter-ahead $(t+4)$ CPI inflation at time t (in decimal fraction) based on a survey of business managers, Source <2>.

$m_t - p_t$: The log of broad money (M3) $-$ the log of the CPI (all groups),
Source <1>. IFS codes: 19659MCBZF.... and 19664...ZF...

y_t: The log of real gross domestic product, Source <1>. IFS code: 19699BVRZF...

$i_t^l - i_t^s$: Government bond yield $-$ 3-month T-bill rate (new issue rate), Source <1>.
IFS codes: 19661...ZF... and 19660C..ZF...

Note:

Each component in $i_t^l - i_t^s$ is defined as $i_t^s = ln(1 + I_t^s/100)$ and $i_t^l = ln(1 + I_t^l/100)$, in which I_t^s and I_t^l represent the corresponding original series obtained from the IFS.

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