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Endogenous cycles in a simple model with a lagged externality on production technology

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Abstract

This paper proposes a simple model in which the economy converges to its unique steady state through cyclical fluctuations. In the model, individuals face subsistence consumption need, and the human capital level in the previous generation has a positive externality on the current labor productivity. The investment to acquire human capital is countercyclical, which generates endogenous cycles until the economy reaches its steady state.

Keywords: Endogenous cycle; Externality; Human capital; Subsistence consumption

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1 Introduction

The aim of this study is to construct a model of endogenous cycles as simple as possible. The key elements of the model are (i) subsistence consumption, and (ii) a lagged externality of human capital on production technology. First, because of the subsistence consumption need, individuals devote a lot of time resource to acquire human capital in order to maintain a certain income when the economy-wide labor productivity is low and the wage rate is low. On the other hand, individuals afford to engage in an unproductive activity, called leisure, instead of the productive activity of human capital acquisition. Second, the level of human capital has a positive externality on productivity, but it takes time to exert the external effect. That is, the productivity in the current generation is determined by the human capital level of the previous generation. The two elements make human capital acquisition countercyclical and causes endogenous cycles. When the productivity is low because of the low level of human capital in the previous generation, individuals in the current generation spend a lot of time to acquire human capital, which makes the productivity in the next generation high. By contrast, when the productivity is high, individuals spend more time on leisure, which has a negative external effect on the productivity in the next generation.

There are some studies on endogenous cycles and human capital. Kaas and Zink (2007) investigate the interactions between human capital accumulation and productivity growth. Similarly to this study, they assume that human capital accumulation has a positive externality on future productivity growth, but they also consider that productivity growth increases the cost of education. This two-way interaction makes human capital investment countercyclical and causes endogenous growth cycles. Varvarigos (2016) considers an overlapping generations model and identifies parametric conditions in which human capital investment is countercyclical and the economy converges to limit cycles. This paper offers an alternative mechanism of endogenous cycles.

2 The model

Time is discrete and indexed by t that begins from 1 to infinity, $t = 1, 2, 3, ...$ The economy is represented by a non-overlapping generations model inhabited by homogeneous individuals who live for only one period. The population size of each generation is one. Different generations are connected by an intergenerational external effect of human capital. A single final good is produced, whose price is normalized to one.

2.1 Individuals

Individuals born in period t derive utility from consumption, c_t, and leisure, l_t, according to the following utility function:

$$u(c_t, l_t) = \alpha \log(c_t - \overline{c}) + (1 - \alpha) \log l_t, \tag{1}$$

where $\overline{c}$ is the subsistent consumption level. Each individual is endowed with n units of time resource, which is allocated between leisure and the acquisition of human capital. Leisure includes any unproductive activities that have private benefits, and human capital includes any knowledge and skills that contribute to the final good production. The level of human capital, h_t, is determined by

$$h_t = h(e_t), \tag{2}$$

where e_t is the amount of time spent on the acquisition of human capital such as education and training. I assume that $h(e)$ satisfies $h'(e) > 0$ and $h''(e) \leq 0$. The individual supplies human capital to the final good sector and receives the wage, w_t, per unit of human capital. The total wage income is $w_t h_t$, all of which is spent on consumption.

2.2 Production

A single final good is produced by competitive firms with the following technology:

$$y_t = A_t h_t, \tag{3}$$

where y_t is the output, h_t is human capital, and A_t is its productivity. The productivity, A_t, is positively affected by human capital acquired by the previous generations.[1] In other words, human capital has positive externalities on the productivity with one period lag:

$$A_t = A(h_{t-1}), \tag{4}$$

[1] Miller and Upadhyay (2000) provide the evidence that human capital generally has a positive effect on the total factor productivity.

where $A'(h_{t-1}) > 0$ and

$$A(0)h(n) > \bar{c}. \tag{5}$$

This inequality guarantees that the subsistence consumption is always satisfied.

3 Equilibrium analysis

Each individual maximizes utility (1) subject to

$$c_t = w_t h_t,\ \ e_t + l_t = n,$$

and the human capital production function (2), taking w_t as given. This maximization problem is reduced to choosing e_t to maximize

$$\alpha \log[w_t h(e_t) - \bar{c}] + (1-\alpha)\log(n - e_t), \tag{6}$$

and the solution is characterized by

$$[(1-\alpha)h(e_t) - \alpha h'(e_t)(n - e_t)]\, w_t = (1-\alpha)\bar{c}. \tag{7}$$

Since $h'(e) > 0$ and $h''(e) \leq 0$, the left-hand side is strictly increasing in both e_t and w_t. This implies the following lemma.

Lemma 1 *Time devoted to the acquisition of human capital, e_t, is strictly decreasing in the wage rate, w_t.*

Because $e_t = h^{-1}(h_t)$ and the equilibrium wage rate is $A(h_{t-1})$, the law of motion for h_t is written as follows:

$$\Psi(h_{t-1}, h_t) \equiv \left[(1-\alpha)\, h_t - \alpha h' \left(h^{-1}(h_t)\right) \left(n - h^{-1}\left(h_t\right)\right)\right] A\left(h_{t-1}\right) - (1-\alpha)\, \overline{c} = 0. \tag{8}$$

Because $w_t = A(h_{t-1})$ is strictly increasing in h_{t-1}, the following proposition immediately follows from lemma 1.

Proposition 1 *$\Psi(h_{t-1}, h_t) = 0$ yields a function $h_t = \eta(h_{t-1})$, which is strictly decreasing in h_{t-1}. If $\eta(h(n)) < h(n)$, there exists a unique steady state, h^*, where $h^* = \eta(h^*)$.*

The slope of $h_t = \eta(h_{t-1})$ is confirmed by applying the implicit function theorem:

$$\frac{dh_t}{dh_{t-1}} = -\frac{(1-\alpha)\overline{c}A'(h_{t-1})}{\left[1 - \alpha\frac{h''(e_t)}{h'(e_t)}(n - e_t)\right] A(h_{t-1})^2} < 0, \tag{9}$$

where $e_t = h^{-1}(h_t)$.

Hereafter, I specify the functional forms of $h(e_t)$ and $A(h_{t-1})$ as

$$h(e_t) = e_t^{\beta}, \quad \beta \in (0, 1], \tag{10}$$

$$A(h_{t-1}) = (\overline{A} + h_{t-1})^{\delta}, \quad \delta > 0, \tag{11}$$

and assume that

$$\overline{A} > \left[(1-\alpha)\overline{c}\delta\right]^{1/(\delta+1)}. \tag{12}$$

Inequality (12) ensures the stability of the steady state. Substituting (10) and (11) into (9) yields

$$\frac{dh_t}{dh_{t-1}} = -\frac{(1-\alpha)\bar{c}\delta}{\left[1+\alpha\left(1-\beta\right)e_t^{-1}\left(n-e_t\right)\right]\left(\overline{A}+h_{t-1}\right)^{\delta+1}}. \tag{13}$$

Because $e_t = h_t^{1/\beta}$ is decreasing in h_{t-1}, dh_t/dh_{t-1} is increasing in h_{t-1} and thus $\eta(h_{t-1})$ is strictly convex. Inequality (12) implies that $\overline{A}+h_{t-1} > [(1-\alpha)\bar{c}\delta]^{1/(\delta+1)}$, which is equivalent to $(\overline{A}+h_{t-1})^{\delta+1} > (1-\alpha)\bar{c}\delta$. For any h_{t-1}, $|dh_t/dh_{t-1}|$ satisfies

$$\begin{aligned}
\left|\frac{dh_t}{dh_{t-1}}\right| &= \frac{(1-\alpha)\bar{c}\delta}{\left[1+\alpha\left(1-\beta\right)e_t^{-1}\left(n-e_t\right)\right]\left(\overline{A}+h_{t-1}\right)^{\delta+1}} \\
&< \frac{(1-\alpha)\,\bar{c}\delta}{\left[1+\alpha\left(1-\beta\right)e_t^{-1}\left(n-e_t\right)\right](1-\alpha)\bar{c}\delta} < 1,
\end{aligned}$$

which implies that the steady state, h^*, is globally stable. Starting from any h_{t-1}, the economy converges to h^* through cycles. The obtained result is summarized in the following proposition.

Proposition 2 *Under the specifications of (10) and (11), $h_t = \eta(h_{t-1})$ is strictly convex and strictly decreasing in h_{t-1}. Moreover, with assumption (12) and $\eta\left(h(n)\right) < h(n)$, the economy starting from any h_{t-1} always converges to the unique steady state through cycles.*

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