CAES Working Paper Series

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WP-2018-001

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12 January 2018
(Preliminary Version)

Abstract

This study aims to explore the dynamics of the real yen-dollar rate and tradable-goods inflation rates by analysing Japan-US monthly economic data in recent years. A real interest rate differential (RID) derived from international parity conditions plays a critical role in the empirical exploration. A cointegration analysis of the data reveals evidence which supports the RID hypothesis as a long-run economic relationship, distinct from the existing evidence in literature. The relationship also paves a way for the regime-switching analysis that explicitly allows for influences of the two countries' monetary policies. The non-linear dynamic nature of the real yen-dollar rate is clarified in this analysis.

Keywords: Real yen-dollar rate, International parity conditions, Real interest rate differential, Cointegration, Regime-switching analysis.
JEL classification codes: C32, C50, E31, F31.

1 Introduction

International parity relationships, such as uncovered interest rate parity (UIP), constitute the core concepts of international finance and thus play critical roles in the theory of exchange rate determination. Reflecting such importance they have been under thorough empirical investigation in literature. However, a consensus has not yet been reached regarding the answer to the question of whether those parity relationships hold precisely as empirical long-run linkages among economic and financial variables. There exist a number of econometric studies that have found discouraging evidence about the empirical validity

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of international parity conditions; see Baxter (1994), Edison and Pauls (1993), Obstfeld and Rogoff (1996, Ch.9; 2000), *inter alia*. See also Chinn (2012) for a comprehensive survey of recent literature along this line.

In order to address this difficulty and illuminate the underlying complex relationships between exchange rates and various economic variables, a joint modelling approach adopted by Juselius and MacDonald (2000) seems to indicate a promising research direction we should follow. In their study this approach was embodied in the pursuit of general-to-specific econometric modelling (see Hendry, 1995) within the framework of a cointegrated vector autoregressive (CVAR) system by Johansen (1988); see Section 3 of the present paper for further details. Juselius and MacDonald (2000) explored various interpretable long-run relationships by analysing monthly time series data from Germany and the US. They succeeded in revealing, in particular, a very important long-run linkage form the data in a theory-consistent manner — a linkage between the real exchange rate, inflation rates and interest rates. In their analysis a combination of inflation and interest rates turned out to be a real interest rate differential (RID). The importance of a differential between two countries' real interest rates in exchange rate modelling was demonstrated by Frankel (1979) in the spirit of a mixture of the neoclassical and the Keynesian-Dornbush (see Dornbush, 1976) models. Frankel (1979) presented elegant economic theory based on the RID and also conducted an econometric analysis centering on the German mark - US dollar exchange rate data. Extensive empirical studies have since been performed (see Obstfeld and Rogoff, 1996, Ch.9 and references therein). Although Issac and De Mel (2001) provided updated evidence contradicting the RID model, Juselius and MacDonald (2000) is still counted as one of the notable studies that have been successful in revealing this crucial relationship from Germany-US data.

Upon application to Japan-US data, however, it does not appear that the RID model has established its empirical validity. MacDonald and Nagayasu (1998) conducted detailed quantitative analysis based on the RID model. It is remarkable that they found two long-run linkages consistent with the model, but the exact form of Frankel (1976)'s theoretical equation was not revealed from the analysis. Juselius and MacDonald (2004) also reported a cointegrating combination indicating the importance of an interest-rate term spread between Japan and the US; this finding itself is notable but, again, this cointegrating combination does not coincide with the theoretical counterpart by Frankel (1979). Kurita (2007) also adopted the cointegration method and attained a data-congruent econometric system for Japan-US quarterly data, but failed to reveal the Frankel-type long-run relationship. Thus, establishing empirical validity of the RID model remains an unsolved task in the field of the yen-dollar exchange rate modelling.

With regard to the Japanese economy in recent years, large-scale financial deregulation was implemented in the late 1990s, during which the Japanese economy still suffered the aftermath of the collapse of an asset-price bubble economy over the late 1980s. This regulatory reform, called the Japanese big bang, included removal of control over foreign exchange transactions, hence contributing to facilitation of international capital movements. The economy continued to stagnate as a result of bad loans accumulated after the bubble burst. Government-led disposals of bad loans were then advanced and the Japanese economy finally turned the corner around 2002 (see OECD, 2005). The economy took off from the long-lasting stagnation and was on a steady growth path in 2003, although the

US financial crisis in 2008 put the economy back in recession again. Thus, an econometric analysis focusing on monthly observations from 2003 onwards may be advantageous in terms of immunity from ramifications of the post-bubble stagnated economy. Moreover, this analysis is surely benefited from the financial deregulation enhancing international capital mobility. As a result, it is likely, as compared with the previous studies using data in the late 20th century, that we will be able to find evidence consistent with the RID model; we may be successful, furthermore, in revealing that long-run relationship between the yen-dollar rate and inflation dynamics which exactly matches the Frankel-type RID equation. In addition to the long-run cointegration analysis, this paper aims to shed light on the underlying time-varying influences on monetary policy on the real yen-dollar rate dynamics. Regime-switching methodology by Hamilton (1989) plays an important role in pursuit of this objective.

The structure of the rest of this study is as follows. Sections 2 and 3 briefly review the RID model and CVAR econometric methodology, respectively. Section 4 then performs a detailed CVAR analysis of monthly observations from Japan and the US, with a view to revealing a long-run relationship exactly matching Frankel (1979)'s equation. Furthermore, Section 5 conducts a regime-switching analysis of the real yen-dollar rate by focusing on monetary policy influences. Concluding remarks are provided in Section 6. As a notational convention, a process integrated of order d is denoted by $I(d)$, so that a mean-zero stationary process is represented by $I(0)$. All econometric analyses in this note were conducted using *PcGive* (Doornik and Hendry, 2013; Doornik, 2013a).

2 International parity conditions

Following Frankel (1976) as well as MacDonald and Nagayasu (1998), this section introduces a two-country model consisting of home and foreign countries, which are assumed to be economically interdependent. We intend here to derive a theoretical equation connecting a real exchange rate and inflation dynamics based on several international parity conditions. First, the idea of purchasing power parity (PPP) allows us to give the real exchange rate q_t as

$$q_t = s_t + p_t^* - p_t, \tag{1}$$

where s_t is the log of the spot exchange rate defined as the price of a unit of foreign currency in terms of home currency, p_t^* is the log of the foreign price level of tradable goods while p_t is the log of the home price level of tradable goods.

Next, we review the Frankel-type formulation for exchange rate expectations. Introduce a target anchor for s_t as $\overline{s}_t = \mu - p_t^* + p_t$; that is, the anchor value is determined by the PPP value augmented with an intercept μ, which corresponds to the mean of the real exchange rate. The expected rate of change in the exchange rate is then formulated as

$$s_{t+1}^e - s_t = -\phi(s_t - \overline{s}_t) + \pi_{t+1}^e - \pi_{t+1}^{e*} \quad \text{for} \quad 0 < \phi < 1, \tag{2}$$

where the superscript e indicates the market expectation at $t+1$ of a variable in question based on information at t, while π_t and π_t^* are inflation rates for tradable goods in the home and the foreign countries, respectively. The idea underlying (2) is that the expected exchange rate adjusts to the disequilibrium error represented by $s_t - \overline{s}_t$ by the fractional

degree of ϕ, and it is also influenced by the expected inflation differential between the two countries. In particular, when s_t takes the anchor value or $s_t = \overline{s}_t$, the expected rate of change in the exchange rate matches the differential between the two countries' expected inflation rates. The expectations formation (2), introduced by Frankel (1979), generated numerous exchange rate studies incorporating real interest rate differentials; see Issac and De Mel (2001). By assuming random-walk type expectations with respect to the inflation rates, it is justifiable to replace π^e_{t+1} and π^{e*}_{t+1} in (2) with π_t and π^*_t, respectively, for the rest of this study.

The final condition to be introduced here is uncovered interest rate parity (UIP), which is expressed as

$$s^e_{t+1} - s_t = i_t - i^*_t, \tag{3}$$

where i^s_t and i^*_t denote a home and foreign interest rates, respectively. That is, the expected rate of change in the exchange rate coincides with two countries' interest rate differential. Note that PPP is a goods-market parity condition, while UIP is a parity relationship in the capital market. Thus, in the absence of capital controls and significant transaction costs, risk-neutral speculation can yield the UIP as the equilibrium condition.

By combining (1), (2) and (3) with the target $\overline{s}_t = \mu - p^*_t + p_t$, we arrive at

$$q_t = \mu - \frac{1}{\phi}\left[(i_t - \pi_t) - (i^*_t - \pi^*_t)\right] \quad \text{for} \quad 0 < \phi < 1. \tag{4}$$

This expression is seen as a reduced form equation for the real exchange rate, incorporating a differential between two countries' real interest rates as a critical explanatory variable. Equation (4) is referred to as a RID equation, which has been derived above as a result of a combination of several parity conditions. In the context of multivariate time series econometrics, this equation may be interpreted as a candidate for the underlying mean-zero stationary combination of non-stationary variables. Given this interpretation, it is justifiable to rewrite this as

$$q_t - \frac{1}{\phi}\left[\pi_t - \pi^*_t - (i_t - i^*_t)\right] - \mu \sim I(0) \quad \text{for} \quad 0 < \phi < 1, \tag{5}$$

That is, the RID equation lays the foundation for the conceivable cointegrating linkage (5) between the real exchange rate and the inflation rates, augmented with the nominal interest rate differential. Given the interdependent nature of all the variables in (5), it is sensible to adopt a joint econometric-modelling approach to real-life data for them. This approach is embodied in building a well-formulated CVAR system. Before launching this empirical exploration, the next section presents a brief review of CVAR methods.

3 Cointegrated vector autoregressive methodology

In this section we review likelihood-based CVAR methodology for modelling $I(1)$ non-stationary time series data. See Johansen (1988, 1996), Bårdsen, Eitrheim, Jansen and Nymoen (2005) and Juselius (2006) as references for this methodology. An unrestricted VAR(k) model for a p-dimensional process $X_{-k+1}, \cdots, X_T$ is introduced as

$$\Delta X_t = (\Pi, \Pi_c)\begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \quad \text{for} \quad t = 1, ..., T, \tag{6}$$

where $\varepsilon_1, \cdots, \varepsilon_T$ represent innovations distributed as independent and identical normal $IN(0, \Omega)$ conditional on a set of starting values $X_{-k+1}, ..., X_0$, and the parameters Π, Γ_i, $\Omega \in \mathbf{R}^{p \times p}$ and Π_c, $\mu \in \mathbf{R}^p$ all vary freely, along with the property that Ω is positive definite. In order for us to define a CVAR model nested in the above VAR model (6), we need to provide the following three regularity conditions based on Johansen (1996, Ch.4):

1. Introduce the following characteristic equation based on (6):

$$\det \left\{ (1-z) I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_j (1-z) z^i \right\} = 0.$$

   The roots of this equation fulfill either $z = 1$ or $|z| > 1$ in the complex plane.

2. $(\Pi, \Pi_c) = \alpha(\beta', \delta)$, where $\alpha, \beta \in \mathbf{R}^{p \times r}$ are of full column rank $r < p$ and $\delta \in \mathbf{R}^r$.

3. $\operatorname{rank}(\alpha_\perp' \Gamma \beta_\perp) = p - r$, where $\alpha_\perp, \beta_\perp \in \mathbf{R}^{p \times p-r}$ are orthogonal complements defined in such a way that $\alpha' \alpha_\perp = 0$ and $\beta' \beta_\perp = 0$ with $(\alpha, \alpha_\perp)$ and $(\beta, \beta_\perp)$ being of full rank p, while $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$.

The first condition allows us to affirm that the process X_t is neither explosive nor seasonally cointegrated. The second is known as a reduced rank condition, which means that at least $p - r$ common stochastic trends are present in the process, so that the phenomenon of cointegration realises when $r \geq 1$ and r is thus called cointegrating rank. By introducing notation $\beta^{*\prime} = (\beta', \delta)$ and $X_{t-1}^* = \left(X_{t-1}', 1\right)'$, we refer to $\beta^{*\prime}$ and $\beta^{*\prime} X_{t-1}^*$ as cointegrating parameters and cointegrating relationships or combinations, respectively; hence α is interpretable as representing adjustment parameters. The final condition is recognised as a full rank condition, which prevents the process from being of higher order than $I(1)$. As a consequence of these conditions, we are able to find the following constant-restricted CVAR model as a sub-model of (6):

$$\Delta X_t = \alpha \beta^{*\prime} X_{t-1}^* + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \quad \text{for} \quad t = 1, ..., T, \tag{7}$$

Using this CVAR model, we are exploring in the next section various empirical questions associated with the dynamics of the real yen-dollar rate and inflation measures.

According to Johansen (1996, Ch.6), reduced rank regression (see Anderson, 1951) enables us to maximise a log-likelihood function for (7) by solving a generalised eigenvalue problem. A log-likelihood ratio ($\log LR$) test statistic for the null hypothesis of the rank r, $\mathsf{H}(r)$, against the alternative $\mathsf{H}(p)$, is then utilised to make a choice of the underlying rank. Johansen (1996, Ch.15) presents various quantiles of the non-standard limit distribution of this statistic; see also Nielsen (1997) as well as Doornik (1998) for gamma approximation methods to compute the quantiles. By selecting r for the system (7), we can move on to a position to seek valid restrictions on β^* with a view to revealing the underlying data-generating structure consistent with theories of macroeconomics. Cointegrating combinations subject to such restrictions are interpretable as equilibrium correction mechanisms built in the CVAR system.

4 Multivariate econometric modelling

This section examines whether or not the hypothetical relationship, (5), holds as an empirical cointegrating relationship revealed from the analysis of real-life time series observations. A class of economic variables to be modelled is as follows:

$$X_t = (q_t, \pi_t, \pi_t^*, i_t - i_t^*)', \tag{8}$$

whose members have already been introduced in Section 2. In this empirical study Japan is the home country while the US is the foreign country, so that the real exchange rate q_t corresponds to the real yen-dollar exchange rate. A set of time series data corresponding to (8) is thoroughly studied in this section. Note that the two countries interest rates (i_t and i_t^*) are given as annual rates, so that their inflation rates (π_t and π_t^*) are also measured as annual or year-on-year rates by taking 12th-order differences of price indices. See the Appendix for further details of the data, along with their graphs. The effective observation period is from January 2003 to December 2016, expressed as 2003.01-2016.12 henceforth. Thus, the number of observations available for estimation in 168. See the Introduction for the justification of selecting this sample period in our study.

4.1 Determination of the cointegrating rank

The maximum likelihood estimation of an unrestricted VAR model for (8) is performed here in order to determine the cointegrating rank. The lag order for the VAR model is set at 4, judging from F test statistics for lagged variables's significance. This VAR model should represent the data well in terms of residual diagnostics so that likelihood-based inferences can be justified. A preliminary regression analysis found some problems in terms of 12th-order residual autocorrelation, presumably due to multiplicative seasonality, in the equations for both countries' inflation rates. Hence, an adjustment suggested by Kurita and Nielsen (2009) is made to the model so as to mitigate the problems without impairing asymptotic theory for cointegration, *i.e.* adding a pair of lagged second-order differenced terms, $\Delta^2\pi_{t-12}^*$ and $\Delta^2\pi_{t-12}$, to the VAR model in an unrestricted manner.

Furthermore, the model's residuals indicate several outliers, due to the US financial crisis in 2008.10, the downgrading of the US credit-rating in 2011.08, and an increase in Japan's consumption tax rate in 2014.04 and a resultant drop in the rate of Japan's inflation in 2015.04. In response to the first two outliers, a set of dummy variables is inserted in the model, $D_{fin\_2008,t}$ and $D_{cred\_2011,t}$, which hold 1 in 2008.10 and 2011.08 respectively, holding 0 otherwise; a dummy variable for the third is also introduced and denoted by $D_{tax\_2014,t}$, which holds 1 in 2014.04 and -1 in 2015.04 respectively, holding 0 otherwise. These dummy variables are all contained in the VAR model in an unrestricted fashion, according to Doornik, Hendry and Nielsen (1998). Figure 1 conducts a graphic diagnostic analysis of residuals from this adjusted VAR model; the first, second and third columns display scaled residuals, residual correlogram, and residual quantile-quantile plots according to the standard normal distribution, respectively. No strong evidence is found in this figure contradicting the assumption of no serial correlation, although it doesn't seem the assumptions of normality and homoscedasticity are completely satisfied; in particular, autoregressive conditional heteroscedasticity (ARCH) effects appear to be present in some

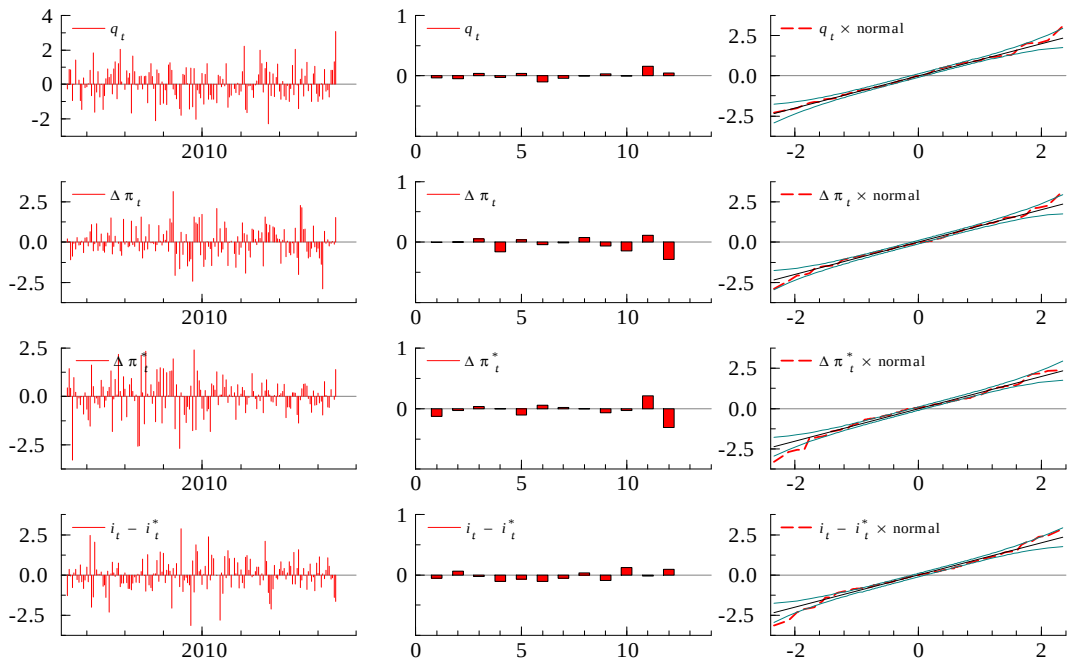

Figure 1: Graphic analysis of the residuals

of the residual series. It is known that, in the absence of residual serial correlation, a standard CVAR analysis can be justified in the context of a quasi maximum likelihood analysis; see Cheung and Lai (1993), Gonzalo (1994), Hendry and Juselius (2001), Rahbek, Hansen and Dennis (2002) and Kurita (2013) for the robustness of the CVAR methods to residual diagnostic problems such as ARCH. This argument allows us to proceed to the determination of cointegrating rank using this VAR model.

			$r = 0$		$r \leq 1$		$r \leq 2$	$r \leq 3$
$-2\log LR\{\mathrm{H}(r)\,\vert\mathrm{H}(4)\}$			56.892[0.026]*		31.725[0.113]		8.812[0.754]	2.978[0.593]
mod $(r = 1)$	1	1	1	0.918	0.918	0.642	0.642	0.536

Note. Figures in square brackets are p-values. The symbol $*$ denotes 5% level of significance.

Table 1: Testing for the cointegrating rank

The first panel in Table 1 reports a class of $\log LR$ test statistics for the selection of cointegrating rank (see Johansen, 1996, Ch.6). The test results show that the null hypothesis of $r = 0$ is rejected at the 5% significance level, whereas the hypotheses of $r \leq 1$ and greater values fail to be rejected at this level. Thus, the selection of $r = 1$ seems to be appropriate. With a view to reinforcing this selection, we also check moduli (denoted by *mod*) of the eight largest eigenvalues of a companion matrix for the CVAR

model evaluated under $r = 1$. See the second panel in Table 1. These moduli represent the reciprocal values of the roots of the CVAR characteristic equation, and the selection of $r = 1$ imposes three unit roots ($p - r = 3$) on the equation. The other eigenvalues seem to be significantly smaller than 1, indicating that $r = 1$ is an appropriate restriction in the sense that all the remaining roots appear to be stationary. The overall evidence recorded in Table 1 supports the restriction of $r = 1$ imposed on the VAR model, which is to be utilised for further empirical investigations.

4.2 RID as the long-run economic relationship

The determination of $r = 1$ enables us to test for several hypotheses associated with β^*. For this purpose, let us recall the candidate for the cointegrating linkage (5) at $t - 1$:

$$q_{t-1} - \frac{1}{\phi}\left[\pi_{t-1} - \pi^*_{t-1} - \left(i_{t-1} - i^*_{t-1}\right)\right] - \mu \sim I(0) \quad \text{for} \quad 0 < \phi < 1.$$

This linkage is used as a reference when testing various hypotheses. First, we are checking the restriction that π_{t-1} and π^*_{t-1} hold the same long-run coefficient but with opposite sign. The parameter estimates (denoted by the hat symbol), along with the corresponding standard errors in brackets, are given as

$$\widehat{\alpha}\widehat{\beta}^{*\prime} X^*_{t-1} = \begin{pmatrix} \underset{(0.014)}{-0.030} \\ \underset{(0.0024)}{0.00017} \\ \underset{(0.0085)}{-0.031} \\ \underset{(0.001)}{0.0016} \end{pmatrix} \begin{pmatrix} \underset{(-)}{1} \\ \underset{(-)}{-5.371} \\ \underset{(0.898)}{5.371} \\ \underset{(4.612)}{7.891} \\ \underset{(0.102)}{-5.149} \end{pmatrix}' \begin{pmatrix} q_{t-1} \\ \pi_{t-1} \\ \pi^*_{t-1} \\ i_t - i^*_t \\ 1 \end{pmatrix}, \tag{9}$$

for which the log LR statistic against the alternative of no restrictions is 1.654[0.199], and the figure in square brackets is a p-value according to $\chi^2(1)$. The null hypothesis is not rejected at the 5% level, and thus the inflation differential $\pi_{t-1} - \pi_{t-1}$ is a valid restriction on the cointegrating relationship. The signs of the estimated coefficients for π_{t-1} and π^*_{t-1} in (9) are negative and positive respectively, which is seen as evidence comfortable to the theoretical relationship (5). In addition, it should be noted that the absolute value of these estimates is greater than 1, consistent with $1/\phi$ for $0 < \phi < 1$, as shown in (5). These findings encourage us to explore a further restriction in line with (5).

Next, we are testing for the hypothesis that the coefficient for the interest rate spread $i_{t-1} - i^*_{t-1}$ is the same as that for π^*_{t-1}. Introducing this restriction results in the following estimates and standard errors:

$$\widehat{\alpha}\widehat{\beta}^{*\prime} X^*_{t-1} = \begin{pmatrix} \underset{(0.013)}{-0.030} \\ \underset{(0.0023)}{0.00006} \\ \underset{(0.008)}{-0.029} \\ \underset{(0.001)}{0.0017} \end{pmatrix} \begin{pmatrix} \underset{(-)}{1} \\ \underset{(-)}{-5.412} \\ \underset{(0.926)}{5.412} \\ \underset{(-)}{5.412} \\ \underset{(0.029)}{-5.206} \end{pmatrix}' \begin{pmatrix} q_{t-1} \\ \pi_{t-1} \\ \pi^*_{t-1} \\ i_t - i^*_t \\ 1 \end{pmatrix}, \tag{10}$$

and the log LR statistic for the overall restrictions is 1.908[0.385], with its p-value based on $\chi^2(2)$. Again, the null hypothesis is not rejected at the 5% level, so that one can see that the long-run equilibrium correction mechanism built in this system is

$$ecm_{t-1} = q_{t-1} - 5.412\left[\pi_{t-1} - \pi^*_{t-1} - \left(i_{t-1} - i^*_{t-1}\right)\right] - 5.206 \sim I(0) \quad \text{for} \quad \widehat{\phi} = 0.185,$$

which coincides with the RID-based long-run linkage (5) at $t-1$, predicted in Section 2. This cointegrating combination can be seen as empirical evidence in support of the RID model describing Japan-US economic interactions in recent years. Note that the adjustment coefficients for q_{t-1} and π^*_{t-1} in (10) are judged to be significant, according to their relatively small standard errors in brackets. Thus, the CVAR model can be viewed as an equilibrium correction system accounting for $\Delta\pi^*_t$ as well as Δq_t. It is noteworthy, in particular, that the RID relationship plays a critical role in the dynamics of the US inflation rate. This finding also conveys important information for macroeconomic analysis of the US economy: for the purpose of modelling the US inflation dynamics, it is important to take into account various spillover influences of international goods and financial markets, an example of which has been demonstrated above as the RID relationship incorporating the yen-dollar exchange rate, Japanese inflation and interest rates.

5 A further study: Markov-switching modelling

As demonstrated in the previous section, the RID-based cointegrating relationship plays a key role in the multivariate system for Japan and the US. Focusing on the objective of modelling the real yen-dollar rate dynamics may justify exploring a different scenario, albeit within the RID framework used so far. The idea is to allow for the possibility that monetary policy influences on the exchange rate dynamics can be significant but regime-dependent, varying from one regime to another. In view of this objective, it is essential to adopt a Markov-switching-type model pioneered by Hamilton (1989, 1990); we also refer to Taylor (2004), Frőmmel, MacDonald and Menkhoff (2005), and De Grauwe and Vansteenkiste (2007), Kurita (2016), *inter alia*, for various illustrations of the Markov-switching methodology in the modelling of financial data.

Before proceeding to the empirical study, we give a brief review of Markov-switching methodology based on Hamilton (1994, Ch.14) and Doornik (2013a, b). Let us introduce u_t, which is an unobservable random variable corresponding to a state or regime at time t; this random variable can hold an integer from 0 to $G-1$ so $u_t \in \{0, \ldots, G-1\}$. A G-regime Markov chain for this random variable is

$$p_{m|l} = \Pr\left(u_{t+1} = m \,|u_t = l\right) \quad \text{for} \quad l, m = 0, \cdots, G-1,$$

where $p_{m|l}$ denotes the transition probability of regime m following regime l. A class of these transition probabilities is subject to the constraint that $\sum_{m=0}^{G-1} p_{m|l} = 1$ and $p_{m|l} \geq 0$ for $l, m = 0, \cdots, G-1$. Given this chain of transition probabilities, a Markov-switching variance model for the dependent variable Δq_t is presented as

$$\Delta q_t = \gamma' Y_t + \sum_{i=0}^{j} \zeta_i\left(u_t\right)' Z_{t-i} + \eta\left(u_t\right) + \sigma\left(u_t\right)\xi_t \quad \text{for} \quad t = 1, ..., T, \tag{11}$$

where Y_t and γ are a vector of contemporaneous and lagged explanatory variables and a vector of the corresponding fixed parameter, respectively; Z_{t-i} and $\zeta_i(u_t)$ for $i = 0, ..., j$ are a vector of explanatory variables and a vector of the corresponding regime-dependent parameter, respectively. The term $\eta(u_t)$ is a regime-dependent intercept, while ξ_t is a set of error terms following independent and identical normal distributions with zero mean and unit variance, so the variance of $\sigma(u_t)\xi_t$ is switching according to regimes. See also Kim, Nelson and Startz (1998) and Bhar and Hamori (2003) for examples of switching variance specifications.

In order to describe an estimation procedure for (11), we let ϕ denote a vectorisation of the overall unknown parameters for (11). This parameter vector can be estimated using maximum likelihood under the above Markov-chain constraints on transition probabilities. Next, let Σ_T^1 represent the information set available from observations for (11); Kim (1994)'s algorithm enables us to estimate the probabilities $\Pr\left(u_t = m \middle| \Sigma_T^1, \widehat{\phi}\right)$ of a regime being categorised as m for $m = 0, \cdots, G-1$, given the information set Σ_T^1 and the estimates $\widehat{\phi}$. These probabilities are referred to as smoothed probabilities; they possess time-varying properties, allowing us to make inferences for which category of states the model is likely to fall into over the observation period.

In this Markov-switching analysis we focus on three conceivable regimes: (i) a regime in the absence of significant influences from the two countries' monetary policies, (ii) a regime primarily characterised by effects from the US monetary policy and (iii) a regime which is mainly characterised by influences from Japan's monetary policy. For this reason we add Δmb_t^*, Δmb_t and their lagged series to a set of explanatory variables composed of contemporaneous and lagged first-order differences of variables in (8); mb_t^* is the log of US monetary base while mb_t is the log of Japan's monetary base. See the Appendix for their further details. These variables are viewed as representing the two countries' monetary policy stances. Let us also note that ecm_{t-1} is included in the set of lagged explanatory variables. In addition, Δmb_t^*, Δmb_t and their lagged series, along with ecm_{t-1}, are treated as regressors holding regime-dependent parameters in the initial model setting.

Our research interest lying in the three types of regimes allows us to fit a general three-regime Markov-switching model to the data and move on further to a specific model with a reduced number of parameters; see Hendry (1995) for further details of a general-to-specific modelling approach. The specific Markov-switching model we've arrived at is

$$\Delta q_t = \gamma \Delta (i_t - i_t^*) + \zeta(u_t)' Z_t + \eta(u_t) + \sigma(u_t)\xi_t, \tag{12}$$

for $u_t \in \{0, 1, 2\}$ and

$$Z_t = (\Delta mb_t^*, \Delta mb_{t-1}, ecm_{t-1})'.$$

Diagnostic test results for (12) are reported in Table 2, while a set of the model's estimated coefficients, along with their regime classifications, is recorded in Table 3.

Table 2 presents three types of diagnostic tests for the preferred model (12): a test for normality (Doornik and Hansen, 2008), first-order and third-order portmanteau tests for serial correlation (Box and Pierce, 1970), first-order and third-order ARCH tests (Engle, 1982). A test for linearity (see Doornik, 2013a, p.56) is also reported in the table. The diagnostic test results indicate no mis-specification problems at 5%, with the linearity

Normality test	$\chi^2\{2\}:$	0.096[0.953]
ARCH (1) test	$F\{1,147\}:$	0.946[0.332]
ARCH (3) test	$F\{3,143\}:$	1.008[0.391]
Portmanteau (1) test	$\chi^2\{1\}:$	0.044[0.834]
Portmanteau (3) test	$\chi^2\{3\}:$	3.375[0.337]
Linearity test	$\chi^2\{13\}:$	33.619[0.001]**

Notes: Figures in curly and square brackets represent degrees of freedom and p-values, respectively. The symbol ** denotes 1% level of significance.

Table 2: Diagnostic statistics for the reduced Markov-switching model

	u_t				u_t		
$\Delta(i_t - i_t^*)$	-	−5.045	(0.802)**				
Δmb_t^*	0	0.078	(0.058)	ecm_{t-1}	0	−0.008	(0.008)
	1	−0.299	(0.061)**		1	0.008	(0.012)
	2	−0.145	(0.176)		2	−0.040	(0.019)*
Δmb_{t-1}	0	0.118	(0.073)	1	0	−0.0008	(0.0027)
	1	−0.252	(0.137)		1	0.0016	(0.0030)
	2	0.769	(0.160)**		2	0.0098	(0.0054)

Note: Figures in brackets denote standard errors. The symbols * and ** represent 5% and 1% levels of significance, respectively.

Table 3: Estimated coefficients for the reduced Markov-switching model

hypothesis being rejected significantly. These findings allow us to judge that the reduced model is a satisfactory representation of the data from the statistical viewpoints.

Next, let us check the parameter estimates reported in Table 3. Before examining the coefficient for ecm_{t-1}, we check the estimates of regime-dependent parameters for Δmb_t^* and Δmb_{t-1}, which indicate differences in influences of the two countries' monetary policies on the real yen-dollar rate dynamics. According to the table, the coefficient for Δmb_t^* in regime 1 is negative and highly significant, suggesting that the US monetary expansion in this regime contributes to an immediate depreciation of the US dollar against the Japanese yen in real terms. In contrast, its coefficients in the other two regimes are far from significant. Thus, regime 1 is regarded as an economic state marked by the presence of significant influences from the US monetary policy. The lagged regressor Δmb_{t-1} has a 77% positive impact on a change in the real yen-dollar rate in regime 2, while it has no significant influences on the exchange rate in the other two regimes. That is, Japan's monetary relaxation in regime 2 leads to a substantial decrease in the real value of the yen in terms of the US dollar. Combined with the evidence for Δmb_t^*, regime 2 is judged to be primarily characterised by significant effects from Japan's monetary policy, in the absence of the US monetary policy influences. The set of the insignificant coefficients for Δmb_{t-1} also gives weight to the preceding interpretation that regime 2 is indeed highlighted by monetary policy influences from the US. The equilibrium correction term, ecm_{t-1}, works significantly only in regime 2. Thus, feedback effects from the equilibrium correction term on the real yen-dollar rate arise in the regime marked by the significance of Japan's

monetary policy. In addition, the estimates for $\widehat{\sigma}(u_t)$ according to the above three states are $\widehat{\sigma}(0) = 0.008(0.002)$, $\widehat{\sigma}(1) = 0.021\,(0.002)$ and $\widehat{\sigma}(2) = 0.016\,(0.003)$ respectively, where the figures in brackets represent standard errors.

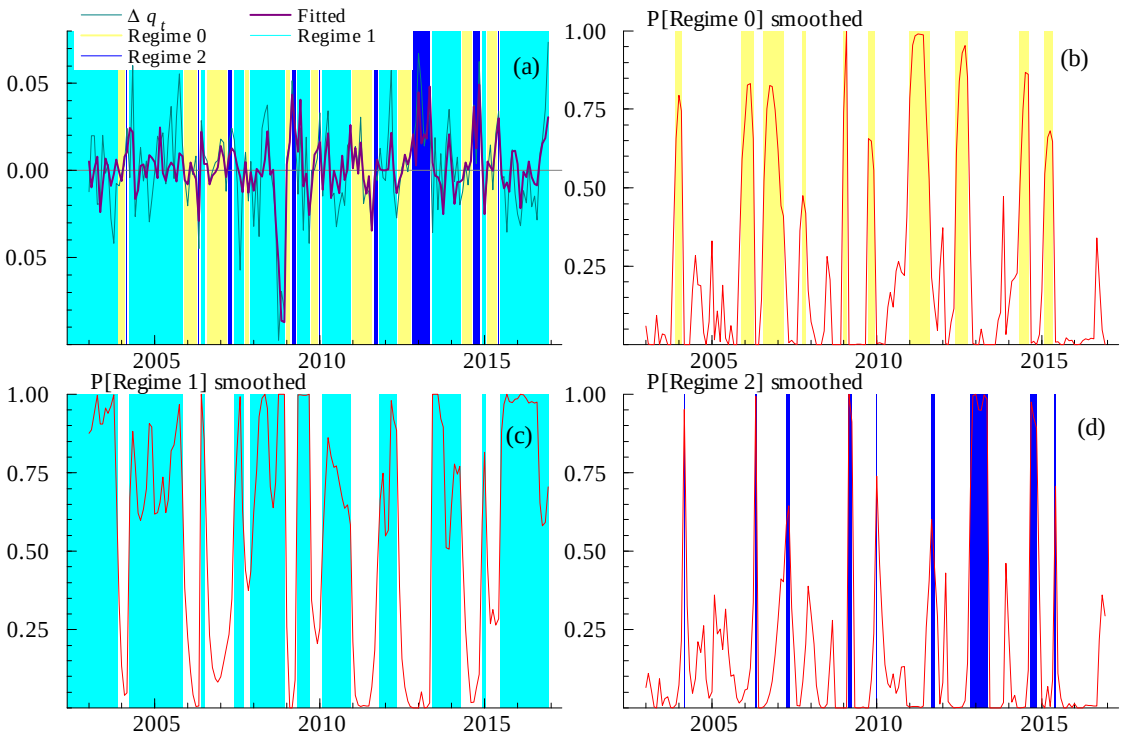

Figure 2: Regime classifications

Finally, a graphic analysis is performed with regard to the specific model. Figure 2 (a) displays a series of actual values Δq_t, along with that of fitted values $\Delta\widehat{q}_t$ obtained from (12), while Figures 2 (b), (c) and (d) present smoothed probabilities for regimes 0, 1 and 2, respectively. As shown in the figure, regime 1 is judged to be the most frequent and persistent among the three categorised regimes, indicating that the US monetary policy influences on the real yen-dollar rate dynamics are substantial. In addition, the period of November 2012 - May 2013 is classified as regime 2 in Figure 2 (d), roughly corresponding to the introduction of a package of new monetary policy measures by the Bank of Japan (see OECD, 2013). Figure 2 (d) indicates that the impacts of the monetary policy package, including unconventional policies such as Quantitative and Qualitative Monetary Easing (QQE) initiated in April 2013, are captured by our regime-switching model.

6 Concluding remarks

This study has investigated the dynamics of the real yen-dollar exchange rate and tradable-goods inflation rates by analyzing a set of recent monthly economic data from Japan and the US. With regard to the yen-dollar rate, the literature failed to provide strong empirical

evidence in support of the RID hypothesis derived from combining international parity conditions. In order to address this issue, we focus on an econometric study of monthly data starting in the early 2000s, which can be advantageous from the viewpoint of the financial deregulation enhancing international capital mobility, along with a recovery from the post-bubble stagnated economy. A multivariate cointegration study of the data has successfully revealed a long-run economic relationship, which can be interpreted as an empirical representation of the RID theory. This revealed relationship has then led to a regime-switching study of the dynamics of the yen-dollar rate, illuminating various time-varying influences of the two countries monetary policy on the exchange rate dynamics. This paper allows us to have a deeper comprehension of the two countries' economic interactions in recent years. It also indicates the possibility of a revival of econometric studies of real exchange rates and tradable-goods inflation rates based on international parity conditions.

Acknowledgements:

T. Kurita gratefully acknowledges financial support from JSPS KAKENHI (Grant Number: 26380349) for this work.

Appendix:

(Data definitions and sources)

Data definitions:

s_t : the log of the spot yen-dollar exchange rate; Source <1>.

p_t : the log of the Japanese producer price index for all commodities; Source <1>.

p_t^* : the log of the US producer price index for all commodities; Source <2>.

q_t : the real yen-dollar exchange rate defined as $s_t + p_t^* - p_t$.

π_t : 12th-order difference of p_t, *i.e.* $\Delta^{12} p_t$.

π_t^* : 12th-order difference of p_t^*, *i.e.* $\Delta^{12} p_t^*$.

i_t : the Japanese long-term interest rate in decimal form; Source <3>.

i_t^* : the US long-term interest rate in decimal form; Source <3>.

m_t : the log of the Japanese monetary base (seasonally adjusted); Source <4>.

m_t^* : the log of the US monetary base (seasonally adjusted); Source <5>.

Sources:

<1> Bank of Japan Time Series Data Search (www.stat-search.boj.or.jp/index_en.html).

<2> US Bureau of Labor Statistics, retrived from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/PPIACO).

<3> OECD Main Economic Indicators, OECD iLibrary.

<4> Bank of Japan website (www.boj.or.jp/en/statistics/boj/other/mb/index.htm/).

<5> Federal Reserve Bank of St. Louis, St. Louis Adjusted Monetary Base, retrived from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/AMBSL).

The sources for $s_t,\ p_t,\ p_t^*,\ i_t$ and i_t^* were accessed on 31 Jan. 2017, while those for m_t and m_t^* were accessed on 27 Feb. 2017.

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