Endogenous timing in tax competition and tax revenue orientation

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WP-2019-006

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March 22, 2019

Abstract
This paper models a timing game in tax competition where the objective function of government in one of two countries is a weighted sum of social welfare and tax revenue. We find parameter values that the governments choose to play a sequential-move game, even if mobile capital is fully owned by the individual in the economy. The reason is that when the government in a country is more strongly in favor of its revenue maximizing, that country has an incentive to raise its tax rate to raise its tax revenue. The interaction of this tax inventive with the terms-of-trade effect is an important factor to determine the equilibrium of timing game.

Keywords: Tax competition, Endogenous timing, Revenue orientation

Journal of Economic Literature Classification Numbers: H77, H73, F21

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*I thank Yukihiro Nishimura, Nobuo Akai, and Shingo Ishiguro for valuable comments as well as participants at International Symposium of Urban Economics and Public Economics and the 56th Annual Meetings of Public Choice Society. This work was supported by JSPS KAKENHI Grant Number 18H00866.

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1 Introduction

In the presence of interjurisdictional capital mobility, strategic interactions between governments exist. In this situation, not only tax rates on capital but also their timing are the government’s decision. Recent studies on the tax competition relax the assumption that competing governments choose their strategic variables simultaneously. For instance, Kempf and Rota-Graziosi (2010) apply a model with a two-country setting and a pre-stage in which the governments commit to act early or late before in selecting capital tax rates, and show that the sub-game perfect equilibria (hereafter, SPNEs) are two Stackelberg outcomes. More recently, it has been pointed out that, as long as capital is owned by individuals, the main factor in determining the equilibrium of the timing game is the position of the individual as a capital importer or a capital exporter. The government of the country with a capital importer has an incentive to lower capital price by raising its capital tax rate, while the government of the country with a capital exporter has an incentive to increase the capital price by decreasing its capital tax rate. The conflict of interest makes both governments be first movers. Indeed, Ogawa (2013) demonstrates that the Stackelberg situation is not commitment robust under a setting of full ownership. Moreover, Hindriks and Nishimura (2017) show that, under the existence of a degree of ownership, the SPNE of the timing game changes the two Stackelberg outcomes to a simultaneous move outcome.

The tax incentive to control capital price is based on the view that governments are benevolent and aim to maximize social welfare. Edwards and Keen (1996) and Wilson (2005) consider a case where governments maximize their net tax revenues to increase government size; in other words, the government acts as a Leviathan. Several empirical studies provide evidence of the relationship between the size of the public sector and economic variables. For example, Nelson (1987) and Zax (1989) measure the effect of decentralization in governments on the size of the public sector. On the other hand, we construct a model where governments’ objectives and production technology vary between two countries. The goal of the government of one of the countries is to maximize social welfare, while its objective function in the other country is the weighted sum of social welfare and tax revenue. Following Ogawa (2013), we assume that capital, which is a mobile factor, is fully owned by the individuals in the economy. We obtain parameter values so that the Stackelberg outcome is the SPNE of the timing game when the objective of the government of at least one country is not to maximize social welfare. A higher degree of tax revenue orientation in the objective function makes the government raise its tax rate to increase its tax revenue. This tax incentive can offset or reinforce the incentive to control the price of capital. The interaction between these

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Theoretically, Pal and Sharma (2013) and Kawachi et al. (2018) construct a model of interregional competition for mobile capital, with a stage where the governments choose their objective functions. They show that, for sequential move competition, the leader chooses to be benevolent, while the follower chooses to be a Leviathan. These mean that not only the asymmetric production function but also the governments’ objectives can be seen as a source of heterogeneity between countries.
incentives is a novel factor in determining the equilibrium of the timing game. Kemp and Rota-Graziosi (2015) provide numerical examples to examine how the incentive to be a second-mover is affected by the tax revenue orientation degree in the objective function. While in their analysis, the government’s objectives are assumed identical among countries, our results provide intuitive explanations on the effect of the government’s objective of a second-mover incentive and show that the Stackelberg equilibrium emerges if the objective of the government of at least one country is not to maximize social welfare.

The rest of this paper is organized as follows. Section 2 presents a setup of model. Section 3 analyzes the equilibrium and discusses results. Section 4 concludes.

2 Model

Consider two countries, country 1 and 2. In each country, a single private good is produced. The private good can either be consumed as a consumption good or used as input for the provision of a local public good. The production function of country $i$ is denoted by $f_i(k_i, l_i)$, where $k_i$ is the amount of mobile capital and $l_i$ the amount of immobile factors, such as labor and land ($i = 1, 2$). The production function exhibits constant returns to scale. Assume an equal endowment with the immobile factor for each country normalized to 1, that is, $l_1 = l_2 = 1$. Therefore, the production function is given as follows:

$$f_i(k_i) = (\gamma_i - \frac{1}{2}k_i)k_i, \quad i = 1, 2$$

where $\gamma_i$ is strictly positive. Hereafter, we call country $i$ that has a higher value of $\gamma_i$ than country $j$ the large country and country $j$ the small country.

The governments of each country decide their tax rates $t_i$ on mobile capital $k_i$ to provide a local public good. The net tax revenue of country $i$ is as follows:

$$NT_i = t_i k_i, \quad i = 1, 2.$$ 

A lump-sum transfer from the government to an individual lived in its country is financed by the tax revenue.

Suppose that the capital market is perfectly competitive, and the capital is perfectly mobile between countries. The market clearing conditions are as follows:

$$r = \gamma_i - k_i - t_i, \quad i = 1, 2,$$

$$k_1 + k_2 = 1.$$
Note that $\gamma_i$ represents the vertical intercept of the demand for mobile capital of country $i$ and $r$ is the price of capital. An individual in either country has $\frac{1}{2}$ units of capital as initial endowment. Thus, the return to the immobile factor of country $i$ is as follows:

$$IR_i = \frac{1}{2}k_i^2 + \frac{r}{2}, \quad i = 1, 2$$

The utility function of a representative agent in country $i$ is assumed to be linear in private consumption and include a lump-sum transfer from the government. The social welfare of country $i$ is as follows:

$$SW_i = \frac{1}{2}k_i^2 + \frac{r}{2} + t_ik_i, \quad i = 1, 2.$$ 

Following Edwards and Keen (1996), the objective function for country $i$ is defined as:

$$O_i = \alpha_i SW_i + (1 - \alpha_i) NT_i, \quad i = 1, 2,$$

where $\alpha_i \in [0, 1]$. If $\alpha_i = 1$, then, country $i$’s objective is to maximize social welfare. We call country $i$ fully social welfare oriented country. By contrast, if $\alpha_i = 0$, the government of country $i$ is a tax revenue maximizing Leviathan, or fully tax revenue oriented. We assume that country 1 is fully social welfare oriented, but country 2 is not, that is, $\alpha_1 = 1$ and $\alpha_2 = \alpha$. The parameter $\alpha$ captures the degree of tax revenue orientation, in other words, the asymmetry in the governments’ objectives between the two countries.

We consider the timing game introduced by Hamilton and Slutsky (1990). This game consists of two stages. At the first stage, each country simultaneously decides whether to set its tax rate “Early” or “Late.” These countries’ commitment is assumed to be perfect. The second stage corresponds to the tax competition game, whose timing is based on the outcomes of the first stage: (i) game $G^N$, where both countries choose $t_i$ early or late; (ii) game $G^i$, where country $i$ chooses $t_i$ early while country $j$ chooses $t_j$ late ($i \neq j$). The game can be expressed in the following normal form:

<table>
<thead>
<tr>
<th></th>
<th>Early</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early</td>
<td>$W_1^N$, $W_2^N$</td>
<td>$W_1^L$, $W_2^F$</td>
</tr>
<tr>
<td>Late</td>
<td>$W_1^F$, $W_2^L$</td>
<td>$W_1^N$, $W_2^N$</td>
</tr>
</tbody>
</table>

Table 1: Payoff matrix

In table 4.1, $W_i^N$ is the indirect utility of country $i$ in game $G^N$, $W_i^L$ is the indirect utility of country $i$ in game $G^i$, and $W_i^F$ is the indirect utility of country $i$ in game $G^j$. 4
3 Characterizing the tax competition

To insure interior solutions, we introduce an additional assumption. Let $\Delta \equiv \gamma_1 - \gamma_2$.

**Assumption 1.**

$$\Delta \in (\Delta, \bar{\Delta}), \quad \Delta \equiv -\frac{7-3\alpha}{2} \text{ and } \bar{\Delta} \equiv \frac{3+\alpha}{2}$$

To further understand the model structure, we introduce the following two concepts: plain and strategic interaction. The game exhibits plain complements (substitutes) for country $i$, if for any pair of tax rates, $\frac{\partial O_i}{\partial t_j} > 0$ ($< 0$). Let $\Delta_2 \equiv -1 + \alpha$ and $\Delta_3 \equiv \frac{(3-\alpha)(1-\alpha)}{2}$. It is easy to verify that variable $\Delta_2$ is negative but variable $\Delta_3$ is positive due to $\alpha \in (0,1)$. Subsequently, these are ranked as follows: $\Delta_2 < \Delta_3$.

Evaluating the plain interaction at game $G^N$ gives:

$$\frac{\partial O_1}{\partial t_2} \geq 0 \Leftrightarrow \Delta \geq \Delta_2, \quad (1)$$

$$\frac{\partial O_2}{\partial t_1} \geq 0 \Leftrightarrow \Delta_3 \geq \Delta. \quad (2)$$

This means that the outcome of plain interaction depends on parameter values. Here, the reaction functions $BR_i(t_j) \equiv \arg \max_{t_i} O_i(t_i, t_j)$ are written as

$$BR_1(t_2) = \frac{1}{3}(\Delta + t_2), \quad (3)$$

$$BR_2(t_1) = \frac{1}{4 - \alpha}(2(1 - \alpha) + (2 - \alpha)(\Delta + t_1)). \quad (4)$$

Reaction functions (3) and (4) mean that country $i$ regards tax rates as strategic complement.

By combining equations (3) and (4), we can obtain tax rates, mobile capital, and indirect utilities in game $G^N$:

$$t_1^N = \frac{(1 - \alpha) + \Delta}{5 - \alpha}, \quad t_2^N = \frac{3(1 - \alpha) - (2 - \alpha)\Delta}{5 - \alpha},$$

$$k_1^N = \frac{(7 - 3\alpha) + 2\Delta}{2(5 - \alpha)}, \quad k_2^N = \frac{(3 + \alpha) - 2\Delta}{2(5 - \alpha)},$$

$$W_1^N = \frac{1}{8(5 - \alpha)^2} \left[ 12\Delta^2 + 124\gamma_1 - 24\gamma_2 + \alpha^2(11 + 4\gamma_1) - 2\alpha(7 + 32\gamma_1 - 12\gamma_2) - 13 \right],$$

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Under the two-country setting, where capital is fully owned by individuals, when an individual in country $i$ is a capital importer ($k_i > \frac{1}{2}$), the individual in country $j$ is a capital exporter ($k_j < \frac{1}{2}$). Indeed, for game $G^N$, we have:

$$k_1^N - \frac{1}{2} \geq 0 \text{ and } k_2^N - \frac{1}{2} \leq 0 \iff \Delta_2 \leq \Delta. \quad (5)$$

For game $G^1$, tax rates, mobile capital, and indirect utilities are, respectively:

$$t^L_1 = \frac{(3 - \alpha)(1 - \alpha) + \Delta}{7 - 2\alpha}, \quad t^F_1 = \frac{(5 - \alpha)(1 - \alpha) - (2 - \alpha)\Delta}{7 - 2\alpha},$$

$$k^L_1 = \frac{(9 - 4\alpha) + 2\Delta}{2(7 - 2\alpha)}, \quad k^F_1 = \frac{5 - 2\Delta}{2(7 - 2\alpha)},$$

$$W^L_1 = \frac{1}{8(7 - 2\alpha)} \left[ 4\Delta^2 + 36\gamma_1 - 8\gamma_2 - 2\alpha(3 + 8\gamma_1 - 4\gamma_2) + 4\alpha^2 - 3 \right],$$

$$W^F_2 = \frac{1}{8(7 - 2\alpha)^2} \left[ 4(5 - 2\Delta)^2 + 8\alpha^4 + (-76 + 8\gamma_1 + 8\gamma_2)\alpha^3 - 4(-62 + 15\gamma_1 + 13\gamma_2)\alpha^2 
+ \alpha\{ -305 + 132\gamma_1 - 4\gamma_1^2 + 8(8 + \gamma_1)\gamma_2 - 4\gamma_2^2 \} \right].$$

For game $G^2$, tax rates, mobile capital, and indirect utilities are, respectively:

$$t^F_1 = \frac{3(1 - \alpha) + 2\Delta}{2(6 - \alpha)}, \quad t^L_2 = \frac{9(1 - \alpha) - 2\Delta(3 - \alpha)}{2(6 - \alpha)},$$

$$k^F_1 = \frac{(9 - 4\alpha) + 2\Delta}{2(6 - \alpha)}, \quad k^L_2 = \frac{(3 + 2\alpha) - 2\Delta}{2(6 - \alpha)},$$

$$W^F_1 = \frac{1}{8(-6 + \alpha)^2} \left[ 2\alpha^2(13 + 2\gamma_1) - 6\alpha(7 + 14\gamma_1 - 6\gamma_2) 
+ 3\{ -3 + 4\gamma_1^2 + 4\gamma_1(15 - 2\gamma_2) - 4\gamma_2(3 - \gamma_2) \} \right],$$

$$W^L_2 = \frac{1}{8(6 - \alpha)} \left[ (3 - 2\Delta)^2 + 2\alpha^2(5 - 2\gamma_1) + 8\alpha(-3 + 2\gamma_1 + \gamma_2) \right].$$

As per Appendix A, we can compare the equilibrium tax rates between games $G^N$ and $G^i \ (i = 1, 2)$ as follows.
Lemma 1. When the objective function of only the government if country 2 is the weighted sum of social welfare and tax revenue, there are three levels of equilibrium tax rates:

- For $\Delta \in (\Delta, \Delta_2)$, we have $(t^L_2, t^F_1) < (t^N_2, t^N_1)$ and $(t^F_2, t^L_1) > (t^N_1, t^N_2)$.
- For $\Delta \in (\Delta_2, \Delta_3)$, we have $(t^L_2, t^F_1) > (t^N_2, t^N_1)$ and $(t^F_2, t^L_1) > (t^N_1, t^N_2)$.
- For $\Delta \in (\Delta_3, \tilde{\Delta})$, we have $(t^L_2, t^F_1) > (t^N_2, t^N_1)$ and $(t^F_2, t^L_1) < (t^N_1, t^N_2)$.

Equations (1) and (2) mean that if $\Delta \in (\Delta, \Delta_2)$ or $\Delta \in (\Delta_3, \tilde{\Delta})$, the small country suffers from a negative external effect, while the large country does not. For $\Delta \in (\Delta_2, \Delta_3)$, country 2 is the small country and prefers a lower tax rate. Then, the pair of tax rates in game $G^2$ is lower than in game $G^N$. For $\Delta \in (\Delta, \Delta_2)$, since the small country is country 1, tax rates in game $G^1$ is lower than game $G^N$. As such, for $\Delta \in (\Delta_2, \Delta_3)$, there are positive spillovers for both countries. Then, tax rates in game $G^N$ are always lower than in games $G^1$ and $G^2$.

To characterize the SPNE, we define the first-mover incentive (second-mover incentive): country $i$ has a first-mover incentive (second-mover incentive) if its indirect utility function in game $G^i$ ($G^j$) is higher than in the game $G^N$. It is easy to check

$$W^L_1 - W^N_1 = \frac{(2 - \alpha)^2((1 - \alpha) + \Delta)^2}{2(5 - \alpha)^2(7 - 2\alpha)} > 0,$$

except at $\Delta = \Delta_2$; as well

$$W^L_2 - W^N_2 = \frac{(3 - \alpha)(1 - \alpha) - 2\Delta)^2}{8(5 - \alpha)^2(6 - \alpha)} > 0,$$

except at $\Delta = \Delta_3$. On the other hand, whether both countries have a second-mover incentive depends on parameter values. We define variables $\Delta_1 \equiv \frac{(1-\alpha)(27-5\alpha)}{2(11-2\alpha)}$ and $\Delta_4 \equiv \frac{(1-\alpha)(23-2\alpha(7-\alpha))}{3(1-\alpha)}$. Because of $\alpha \in (0, 1)$, $\Delta_1$ is negative but $\Delta_4$ is positive, and they are ranked as $\Delta_1 < \Delta_4$. Applying the equilibrium outcomes to the definition of the second-mover incentive yields:

$$W^F_1 - W^N_1 = -\frac{3(11 - 2\alpha)}{2(6 - \alpha)^2(5 - \alpha)^2} [\Delta_1 - \Delta] [\Delta_4 - \Delta]$$

$$W^F_2 - W^N_2 = -\frac{3(2 - \alpha)(4 - \alpha)^2}{2(7 - 2\alpha)^2(5 - \alpha)^2} [\Delta_2 - \Delta] [\Delta_4 - \Delta]$$

In Appendix B, we show that $\Delta_1 < \Delta_2 < 0 < \Delta_3 < \Delta_4$. The results of the timing game are characterized as follows. When the asymmetry in productivities is sufficient, ($\Delta \in (\Delta, \Delta_1)$ or $\Delta \in (\Delta_4, \tilde{\Delta})$), no country has a second-mover incentive. Playing
“Early” is the strictly dominant strategy for the governments of both countries, and the pair of strategies (Early, Early) is the SPNE. These results are consistent with Ogawa (2013). When the asymmetry in productivities is in an intermediate range, \((\Delta_2(\Delta_1, \Delta_4))\), at least one government has a second-mover incentive. For \(\Delta_2(\Delta_3, \Delta_4)\), the government of country 2 has a second-mover incentive, while the government of country 1 does not. Then, choosing “Early” is the dominant strategy for country 1, so that (Early, Late) is the SPNE. For \(\Delta_2(\Delta_2, \Delta_3)\), the country that has a second mover incentive is reversed, and then (Late, Early) is the SPNE. For \(\Delta_2(\Delta_2, \Delta_3)\), as both countries have a second-mover incentive, the two sequential move equilibria are the SPNEs. These can be summarized as follows:

**Proposition 1.** When the objective function of only the government of country 2 is the weighted sum of social welfare and tax revenue, we can derive the SPNEs of timing game.  

**Case 1** For \(\Delta \in (\Delta_1, \Delta_3)\), there is one simultaneous-move equilibrium (Early, Early).

**Case 2** For \(\Delta \in (\Delta_1, \Delta_2)\), there is one sequential move equilibrium (Late, Early).

**Case 3** For \(\Delta \in (\Delta_2, \Delta_3)\), there are two sequential move equilibria (Early, Late) and (Late, Early).

**Case 4** For \(\Delta \in (\Delta_3, \Delta_4)\), there is one sequential move equilibrium (Early, Late).

**Case 5** For \(\Delta \in (\Delta_4, \Delta)\), there is one simultaneous-move equilibrium (Early, Early).

Figure 1 presents graphically the equilibrium classification. The area surrounded by dashed lines represents parameters values under Assumption 1. The sequential-move outcome is derived when the degree of tax revenue orientation is higher (low \(\alpha\)) and the asymmetry in productivities among countries is small. On the other hand, the simultaneous-move outcome is derived when the degree of tax revenue orientation is lower (high \(\alpha\)) and the asymmetry in productivities among countries is large.

To clarify the interaction of tax incentives between countries, we start with the case where the governments of both countries are fully social welfare oriented (\(\alpha = 1\)). In this case, variables \(\Delta_i\) \((i = 1, 2, 3, 4)\) are equal to zero. For example, when country 1 is the large country (\(\Delta \in (0, \Delta_1)\)), equation (5) implies that country 1 is a capital importer and country 2 is a capital exporter in game \(G_N\).  

The government of country 1 has the incentive to lower the price of capital by raising its tax rate, whereas the government of country 2 has the incentive to increase the capital price by reducing its tax rate. These results lead to \((t_1^L, t_2^F) > (t_1^N, t_2^N)\) and \((t_1^F, t_2^L) < (t_1^N, t_2^N)\), which are further confirmed by evaluating the outcomes of Lemma 1 at \(\alpha = 1\). These tax incentives can be interpreted

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2 We disregard the borderline cases of \(\Delta = \Delta_i\) for all \(i = 1, 2, 3, 4\).

3 When \(\alpha = 1\) and \(\Delta = 0\), we have \(t_i^N = t_i^L = t_i^F\) for all \(i = 1, 2\). Then, we consider the case of \(\Delta \neq 0\).
as “the terms-of-trade effect.” As a result, the governments in both countries have an incentive to be the first mover, and we then obtain the simultaneous-move outcome.

Next, we turn to the case where the government of a country is not fully social welfare oriented \((\alpha \in (0,1))\). In this case, there is the novel tax incentive, “the revenue orientation effect,” which is an incentive for the government of country 2 to increase its tax rate. This is because a decrease in \(\alpha\) raises the gradient of the reaction function in country 2 (see equation (3)) and induces the government of that country to choose a higher tax rate. Intuitively, the government of country 2 benefits from increasing its tax rate because an incremental unit increase of the tax rate increases its net tax revenue. Equation (5) means that, for \(\Delta \in (\Delta_2, \Delta)\), country 2 is a capital exporter in game \(G^N\). According to the terms-of-trade effect, the government of that country has an incentive to lower its tax rate. However, this tax incentive can be offset by the revenue orientation effect. Lemma 4.1 shows that, for \(\Delta \in (\Delta_2, \Delta_3)\), the tax rates of games \(G^1\) and \(G^2\) are always higher than that of game \(G^N\). This implies that variable \(\Delta_3\) is a switching point, below which the revenue orientation effect dominates the terms-of-trade one. Therefore, for \(\Delta \in (\Delta_2, \Delta_3)\), both governments raise their tax rates and two sequential move equilibria are the SPNEs. By contrast, equation (5) implies that, for \(\Delta \in (\Delta, \Delta_2)\) where country 2 has a higher productivity of capital \((\gamma_2 > \gamma_1)\), country 2 is a capital importer in game \(G^N\), and its government has an incentive to increase its tax rate due to the terms-of-trade effect. Moreover, the revenue orientation effect reinforces this incentive.
4 Conclusion

This chapter contributes to the literature on endogenous timing in tax competition by introducing a novel factor, the asymmetry in governments’ objectives into the model. In country 1, the government chooses its tax rate to maximize social welfare. By contrast, the objective function of the government of country 2 is the weighted sum of social welfare and net tax revenue. The value of the weight captures how the government of country 2 evaluates the importance of its tax revenue and generates an incentive to raise its tax rate. The parameter values show that the tax incentive can offset the incentive to control capital price. Subsequently, the simultaneous move outcome cannot prevail as equilibrium outcome even if capital is owned by individuals.

Bárcena-Ruiz (2007) analyzes the endogenous timing of price competition under a mixed duopoly, and shows that both a profit-maximizing private firm and a welfare-maximizing public firm choose “Early” at the SPNE. Intuitively, the public firm is concerned about consumer surplus and wants to increase market competition by lowering its price, while the private firm seeks to reduce market competition by raising its price. Because there is a conflict of interest among two firms, both prefer to be price-leaders. According to the asymmetric objectives among players, our model is similar to a situation under mixed duopoly. However, we show that sequential move outcomes can be the SPNE. The difference in the results comes from the interaction between the revenue orientation and the terms-of-trade effects. For example, consider the case that the individual in country 1 is a capital importer and that in country 2 is a capital exporter. If the revenue orientation effect dominates the terms-of-trade effect, the government of country 2 has an incentive to raise its tax rate (see Case 3 in Figure 1). As the government of country 1 has the same incentive to lower the price of capital, the conflict of interest disappears.
Appendix

Appendix A

Taking the differences between the tax rates of game $G^i$ for $i = 1, 2$ and game $G^N$ yields:

$$t_1^L - t_1^N = - \frac{(\Delta_2 - \Delta)(4 - \alpha)(2 - \alpha)}{(5 - \alpha)(7 - 2\alpha)} \tag{A.1}$$

$$t_1^F - t_1^N = \frac{(\Delta_3 - \Delta)}{(5 - \alpha)(6 - \alpha)} \tag{A.2}$$

$$t_2^L - t_2^N = \frac{3(\Delta_3 - \Delta)}{(5 - \alpha)(6 - \alpha)} \tag{A.3}$$

$$t_2^F - t_2^N = - \frac{(\Delta_2 - \Delta)(2 - \alpha)^2}{(5 - \alpha)(7 - 2\alpha)} \tag{A.4}$$

Because of $\alpha \in (0, 1)$, the signs of equations (A.1) – (A.4) depend on $\Delta_2$ and $\Delta_3$. For $\Delta \in (\Delta, \Delta_2)$, equations (A.1) and (A.4) are negative but (A.2) and (A.3) are positive. For $\Delta \in (\Delta_2, \Delta_3)$, equations (A.1) – (A.4) are all positive. For $\Delta \in (\Delta_3, \Delta)$, equations (A.1) and (A.4) are positive but (A.2) and (A.3) are negative. These yield the results in Lemma 1. \hfill \Box

Appendix B

Variables $\Delta_1$ and $\Delta_2$ are both negative. Then, taking these differences yields:

$$\Delta_1 - \Delta_2 = - \frac{(5 - \alpha)(1 - \alpha)}{22 - 4\alpha} \tag{B.1}$$

Because of $\alpha \in (0, 1)$, equation (B.1) is negative. Then, $\Delta_1$ is smaller than $\Delta_2$. Variables $\Delta_3$ and $\Delta_4$ are both positive. Taking these differences yields:

$$\Delta_3 - \Delta_4 = - \frac{(5 - \alpha)(2 - \alpha)(1 - \alpha)}{6(4 - \alpha)} \tag{B.2}$$

Because of $\alpha \in (0, 1)$, equation (B.2) is negative. Then, $\Delta_3$ is smaller than $\Delta_4$. Overall, these variables can be ranked as follows: $\Delta_1 < \Delta_2 < \Delta_3 < \Delta_4$. \hfill \Box
References


