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Interpretation of Professor Schefold's Paper

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Abstract

Professor Schefold's trial to introduce random matrix into Marx's transformation problem is very ambitious and epoch-making. The input coefficient matrix, however, turns out to be trivial in order to make labor value and production price on average. But the idea that the two coincide on average is so suggestive that we can remember the history of labor value theory since Ricardo. Schefold also investigates Marx's treatment of differential calculus and shows that it was so operational. This discovery also has something in common with his approach to the transformation problem.

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1. Introduction

Needless to say, the transformation problem from labor value to production price has long been one of the most important question in Marxian economics. But it came to be thought as not being a real problem because we can deduce both labor value and production price from input coefficient matrices independently.

Professor Bertram Schefold published a new paper[1] and showed us another view point of the transformation problem. The approach use random matrices as mathematical tool. This paper tries to interpret the implication of the paper in mathematical and methodological aspects.

The paper of Schefold's argues Marx's treatment of differential calculus in the latter half. This paper also discusses it in comparison with Hegel's and Leibniz's views. Totally this paper will clarify characters implications of Professor Schefold's method.

2. Transformation Problem and Random Matrices

Scheflod's model on the transformation problem consists of Sraffian part and Marxian part. Sraffian part almost the same as the standard commodity system.[2] Sraffa vector means standard commodities. First of all, we consider a normal production system like equation (1).

$$y = Ay + b + s \qquad (1)$$

Here y, A, b and s are activity vector, input coefficient matrix, real wage vector and surplus production vector respectively.

$$y^{*} = (1 + R)Ay^{*} \qquad (2)$$

Here we consider eigenvalues and eigenvectors of input coefficient matrix A. In equation (2), R is the maximum profit rate that corresponds to Frobenius root. We can deduce Sraffa vector y^{*} as the right-hand-side eigenvector of input coefficient matrix. The word

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[1] Schefold, B., 'Profits equal surplus value on average and the significance of this result for the Marxian theory of accumulation,' *Cambridge Journal of Economics*, 40, 165-199, 2016.

[2] Sraffa, P., *Production of Commodities by Means of Commodities: Prelude to Critique of Economic Theory*, Cambridge University Press, 1960.

“Sraffa vector” is Schefold’s usage of words. As it, however, corresponds Sraffian standard commodities, the naming is appropriate.

Now we define the vector *m* whose elements are deviations of the real activity level from the standard one of Sraffa vector like equation (3).

$$\mathrm{m} = \mathrm{y} - y^{*} \qquad (3)$$

In contrast to Sraffa vector, Professor Schefold defines Marx vector, too. As Sraffa vector is the standard commodities of Sraffa’s, it is deduced from an amount equation. By contrast, Marx vector is deduced from a price equation like equation (4).

$$\mathrm{p} = (1 + \mathrm{r})\mathrm{pA} + wl \qquad (4)$$

Here p, r, w and l are production price vector, profit rate, wage rate and labor vector respectively. We also consider eigenvalues and eigenvectors of input coefficient matrix again.

$$p^{*} = (1 + R)p^{*}A \qquad (5)$$

Here p^{*} is the left-hand-side eigenvector of input coefficient matrix. Schefold calls this eigenvector Marx vector. It is, however, difficult to understand why this vector is named after Marx. In equation (5), maximum profit rate is valid. In other words, wage rate is zero here. This utterly contradicts Marx’s opinion. We, however, infer the reason of Schefold’s naming as follows; Marx concentrated his analysis to the relation between labor value and production price. This situation is almost similar with the approach in which Sraffa mainly considered the activity level of production. So we may call the standard prices Marx vector while we call the standard commodities Sraffa vector.

We define the vector v whose elements are deviations of labor input vector from Marx vector p^{*}like equation (6). This is, however, more difficult to understand than the case above. In equation (3), we defined the vector m. When the vector comes to be zero, the real activity level will coincide with the standard one.

$$\mathrm{v} = \mathrm{l} - p^{*} \qquad (6)$$

In contrast with that, equation (6) insists on the coincidence of labor input vector and the standard prices[3]. It, however, sounds very strange. When maximum profit rate is valid, it is not labor input vector but the real price vector that coincides with the standard prices. In the case of the vector m defined in equation (3), the maximum $\mu_1$profit rate actually makes the real activity of production and the standard commodities coincide.

We may suppose that Schefold aims at the coincidence of labor value and the standard price. Labor value is, however, the summation of living labor and dead labor. Whereas, labor input vector is only living labor. Anyway, a riddle remains. We will accept the definition by Schefold once and reconsider the riddle later after examining his mathematical explanation.

From equation (4), we can derive equation (7).

$$\mathrm{p} = wl[I - (1 + \mathrm{r})\mathrm{A}]^{-1} \qquad (7)$$

Using the mathematical formula, we can obtain equation (8). In equation (8), μ_i is eigenvalue. Only μ_1 is dominant. We mean that it is Frobenius root. Others are all non-dominant.

$$p = w \sum_i \frac{1}{1 - (1 + r)\mu_i} p_i \qquad (8)$$

Here $\mu_1=1/(1+R)$. If we suppose $\mu_2=\cdots=\mu_n=0$, we obtain equation (9). Here n is the dimension of vectors. The assumption is really crucial in Schefold's argument. The argument requires that input coefficient matrix should be a random matrix[4]. The elements are changing randomly in random matrices. And non-dominant eigenvalues disappear in average.

$$p = w \left[\frac{1}{1 - (1 + r)\mu_1} p_1 + p_2 + \cdots + p_n \right] \qquad (9)$$

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[3] Here we call the left-hand-side eigenvector of input coefficient matrix the "standard prices". Needless to say, we follow Sraffa in that he called the right-hand-side eigenvector the standard commodities.

[4] Random matrices are firstly introduced into statistics in the 1920s. E. P. Wigner investigated those in the 1950s for applications to nuclear physics. Random matrices have random numbers as those elements. E. F. Dyson built a Brown movement model using random matrices. The introduction of random matrices into quantum mechanics in the 1980s made mathematicians interested in eigenvalues of random matrices.

Because $p_1=p^*$ and $p_2+\cdots+p_n=v$ in equation (9), equation (10) follows finally.

$$p = \mathrm{w}\left[\frac{p^*}{1-\frac{1+r}{1+R}}+v\right] \tag{10}$$

Then we normalize production price vector and obtain equation (11).

$$1 = \bar{p}\mathrm{y} = \bar{\mathrm{w}}\left[\frac{p^* y^*}{1-\frac{1+r}{1+R}}+vm\right] \tag{11}$$

Bars over the letters p and w mean the normalization here. Because $cov(v, m)=0$, the equation below follows.

$$\mathrm{vm} = \mathrm{n}\bar{v}\bar{m} \tag{12}$$

Bars over the letters v and m mean averages here. We obtain equation (13) substituting equation (12) for (11).

$$\bar{w} = \frac{1}{\frac{p^* y^*}{1-\frac{1+r}{1+R}}+\mathrm{n}\bar{v}\bar{m}} \tag{13}$$

If we make an assumption the average value of v is 0, equation (14) follows. Here Schefold wrote the average value of v is 0. This assumption is, however, very crucial again. Though this assumption looks very simple and petty, it actually fixes the random matrix's essence. The matrix cannot avoid being very trivial one.

$$\bar{w} = \frac{1-\frac{1+r}{1+R}}{p^* y^*} \tag{14}$$

When $r=R$, normalized wage rate is zero.

Like wage rate, we can calculate total profit like equation (15).

$$\Pi = \bar{p}\mathrm{s} = \bar{w}\left[\frac{p^*}{1-\frac{1+r}{1+R}}+v\right]s = \frac{1-\frac{1+r}{1+R}}{p^*y^*}\left[\frac{p^*}{1-\frac{1+r}{1+R}}+v\right]s \tag{15}$$

$$= \frac{1}{p^*y^*}\left[p^*s+\left(1-\frac{1+r}{1+R}\right)vs\right]$$

Because *cov(v, s)=0*, equation (16) follows. Bars over the letters *v* and *s* mean averages here.

$$\mathrm{vs} = \mathrm{n}\bar{v}\bar{s} \tag{16}$$

We obtain equation (17) substituting equation (16) for (15).

$$\Pi = \frac{1}{p^*y^*}\left[p^*s+\left(1-\frac{1+r}{1+R}\right)\mathrm{n}\bar{v}\bar{s}\right] \tag{17}$$

We assume the average value of *v* is zero again and equation (18) follows.

$$\Pi = \frac{p^*s}{p^*y^*} \tag{18}$$

Even if the average value of *v* is not zero, equation (18) follows when *r=R* or *r=0*.

Transformation problem has been treated as gradual recalculation from labor value to production price. The substantial backbone of these formal procedures has been supposed to be confirmed by the three propositions: total value=total production price, total surplus value=total profit and total value product=total income. As well known, we can hold only one proposition of the three.

Professor Schafold's approach has a significance in that it can show the proposition of total surplus value=total profit is held on average. It can explore a new aspect of transformation problem. The approach tells not only that value system works in a deeper dimension than production price system but also that the two systems interact quantitatively at the same time.

We, however, have to say that Schefold's approach is not so successful in mathematical meaning. His conclusion depends on a special and trivial random matrices in fact. As for the riddle about using the vector *v* which means the difference between labor input vector and the standard prices. As we checked, the assumption of *v=0* is very crucial in Schefold's model.

The vector is necessary just for this assumption. The necessity comes from mathematics though the economic interpretation is difficult.

3. Differential Calculus in Marx

Schefold also referred to mathematical thought on differential calculus of G. W. Leibniz, G. W. F. Hegel and Marx in his paper. About the reason, Schefold wrote as follows[5];

> Marx starts from the assumption that the workers get a uniform wage and have the same working day, so that the value if labour power also is uniform and with it the rate of surplus value. Hence, surplus value is proportional to labour expended and independent of mass of constant capital used. The long way followed by Marx to get to his theory of prices of production is curiously compared with the way leading from elementary algebra to the idea that 0/0 could represent a 'real magnitude'. What Marx means has become comprehensible, since we have been introduced to his mathematical writings at least in provisional editions and since we have been able to compare them with modern mathematics on the one hand and with Hegel's philosophy of mathematics in his *Logics* on the other hand. Hence, if we want to comprehend the one and only significant hint Marx gave to explain the evident incompatibility between the uniform rate of surplus value and the uniform rate of profit in a theory of labour values, we must look at his understanding of mathematics, and his requires a comparison with Hegel. It turns out that both the mature Hegel and the mature Marx regarded the foundation of infinitesimal calculus as testing ground for their theories of dialectical logics.

Schefold discusses here Marx's approach to the transformation problem. It is a famous story that Marx liked mathematics, especially differential calculus. The mathematics had a very close connection with Marx's style of thinking. Schefold calls Marx's method a kind of dialectical logics. He also insists that we should compare the thoughts of Marx's and Hegel's on the infinitesimal.

Leibniz is said to be ambiguous on the concept of the infinitesimal. He, however, considered the antipathy of the contemporary scholars against such a new idea. Because of this consideration, he defined dx or dy as finite quantities and described them less than any given quantities in the same Latin paper. Leibniz tried to persuade and make people

[5] Schefold [2016], 183.

understand the new idea. It is Hegel that extended the idea of infinitesimal in the philosophical field. Schefold wrote about Hegelian thought in his paper like this[6].

> Hegel here sees an 'intermediate state', which the mathematicians do not perceive, between 'being and nothing'. For him, the unity of being and nothingness is not a state, but it is 'the becoming, is alone the truth'. The infinitesimals therefore are for him only in a transition. 'Becoming' was between 'Being' and 'Nothingness', as in Greek philosophy. He believes that the idea of differential calculus cannot be determined more correctly, than 'as Newton stated it'. 'It may be objected that vanishing magnitudes do not have a *final ratio*, because any ratio before the magnitudes vanish cannot be the last, and once vanished, there is no ratio any more.' But the relationship of vanishing magnitudes is to be understood as occurring, 'not *before* or *after* they vanish, but the ratio *with which* they vanish (*quacum evanescent*)'. The 'final ratios' in differentiating are limits, to which the magnitudes diminishing without limit are closer than any given finite difference. Small magnitudes (we should say, of second or higher order of smallness) were often left aside in Newtonian and Leibnizian calculus, and this was justified because 'correct' derivatives could thus be obtained. But Hegel criticises the comparison of the omission of small magnitudes with empirical approximations, of which, unfortunately, Wolff was guilty. He has objections to Euler and also Laplace, because as soon as 'the terms of a ratio are quantitatively a zero', there is in Laplace (according to Hegel) no conceptual understanding of this ratio. Hence again the suggestion to interpret the differentials as qualities. In consequence, the intrusion of the qualitative into mathematics is affirmed with other examples, beginning with the natural numbers; they how in the beginning only and external quantitative development, but qualitative moments result, for instance, from the musical harmonies.

Hegel added to this mathematical concept a very philosophical meaning. He treated the infinitesimal as something between being and nothing. He called it an intermediate state. However, Marx's understanding is not so philosophical. He was particular about the fact that $\varDelta y/\varDelta x=dy/dx$ in the linear case while $\varDelta y/\varDelta x\neq dy/dx$ in the non-linear case. Marx also picked Leibniz rule up.

$$\frac{dyz}{dx}=z\frac{\mathrm{dy}}{dx}+y\frac{dz}{dx} \tag{19}$$

[6] Schefold [2016], 184.

In equation (19), the right-hand-side no longer defines the left-hand-side. If we eliminate all *dx* from the both side, we get equation (20).

$$\mathrm{dyz} = z\mathrm{dy} + ydz \qquad (20)$$

Watching equation (20), we cannot regard *dx*, *dy* and *dz* as an 'intermediate state' between 'being and nothing'. They have to be interpreted nothing but a 'real magnitude'. However, it does not require another metaphysics. Professor Schefold interprets that Marx took a distance from Hegel by regarding the differentials not as magnitudes of the infinitesimal but as operators. Marx wrote that the equation is thus only a symbolic indication of the operations to be performed. Schefold wrote like this[7].

> As long as the Marxian mathematical manuscripts are not edited completely, our judgements about his approach to mathematics must remain speculative to some extent. My impression is that he seems to have kept his distance from the axiomatic method. Mathematics in Marx do not appear like construct, resulting from different conceptions defined by axioms, but it is a coherent realm of objects, to be analysed by experience and research—hence the 'intermediate terms', which are there to connect algebra and the infinitesimal calculus. Ways for their representation are discovered and operation introduced, by which new mathematical objects are engendered, like the derivation from functions, as if the functions were to be worked on. The lack of an axiomatic build-up does not mean that Marx did not formulate his hypotheses also in the mathematical realm to analyse their consequences. But as his realism always made him look for plausible assumptions, close to reality, in economics, he examined in his mathematics usable functions, capable of orderly differentiation. He was not after 'pathological' constructs such as a non-Euclidean geometry or a function, which would be continuous, but, in the origin, not differentiable, such as $y=x\sin(1/x)$.

Because Marx intrinsically tried to analyze economic reality, his mathematics did not swerve into 'pathological' cases of details in mathematics. His method of mathematics was healthy, operational and operable.

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[7] Schefold [2016], 186.

4. Conclusion

We have discussed how we can connect the different dimensions of labor value and production price in the transformation problem. Originally Marx tried to solve the problem by calculating endlessly. It is also a challenge to the infinity in the opposite direction of the infinitesimal.

However, connecting the two thing in the different dimension quantitatively is logically impossible. Professor Schefold tried to make this possible by considering an average state of changing realities. He utilized the mathematics of random matrices. We have to say the result has turned to be trivial case finally.

We, nonetheless, can appreciate Professor Schefold's proposal. Saying that the two systems coincide on average is almost the same as to say that labor value system approximates production price system. In retrospect, this statement has been repeated since Ricardo's days. Labor value system is much simpler than production system. We could even say that labor value system is "fundamental" to production price system in this meaning. This logic may suit Marx's operable and pragmatic usage of mathematics.