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Disaster Prevention Capital as Social Common Capital

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Abstract

This paper rebuild the macreconomic dynamic equilibrium model with disaster prevention capital. The main diffrence of this paper is that we introduce the concept of social common capital, which is proposed by Hirofumi Uzawa. Disaster prevention capital as social common capital includes physical infrastructure, govermental system and people's knowledge for disaster prevention. And the capital could cause congestion which the concept of public goods never causes.

Now the economic growth rate is very low in Japan. Because of this, there is few necessity of disaster prevention investment so as to avoid congestion. Disaster probability and disaster seriousness, however, increase in Japan these days. From this point of view, we have to make large disaster prevention investment.

JEL classifications: E22, H12, P11.

Keywords: disaster prevention capital, social common capital, disaster prevention investment, disaster damage ratio, seriousness of disaster.

1. Introduction

So many disaster happens in Japan these days. We need consider disaster prevention from any direction. This paper focuses on disaster prevention from the view point of economics. This paper's model is based on the one in Segi-Ishikawa-Yokomatsu (2012). The two, however, differed from each other in just one point.

Uzawa (1972a, 1972b) proposed the concept of social common capital. This concept shares the characters with Samuelson's public goods. Both are supplied publicly and utilized collectively in normal cases. There are, however, quite a few differences. Social common capital is measured as a stock concept while public goods as a flow concept. The most important difference among them is that the former caused congestion while the latter is non-rivalrous in their utilization[1]. Social common capital includes not only physical infrastructure but also nature, institutions and knowledge.

Segi-Ishikawa-Yokomatsu's model treat only disaster prevention infrastructure. This paper's model expands the meaning of disaster prevention capital to governmental system of disaster prevention and people's knowledge on evacuation.

2. The Model of Disaster Prevention Capital

The economy has two types of capital. One is production capital K and the other is disaster prevention capital D. The economy use two types of production elements. One is production capital and the other is Labor N. The amount of labor is normalized as unity. Because of this normalization, production function is represented as a function of only production capital

$$Y_t = f(K_t) \tag{1}$$

Output Y is divided into consumption C, investment to production capital I^K and investment I^D.

$$Y_t = C_t + I_t^K + I_t^D \tag{2}$$

When δ represents depreciation rate, accumulations of capitals are as follows.

$$K_{t+1} = (1-\delta_K)K_t + I_t^K \tag{3}$$

$$D_{t+1} = (1-\delta_D)D_t + I_t^D \tag{4}$$

The representative individual's expectation of utility is written as follows using

[1] If social common capital is supplied infinitely in amount, any rivality will not happen. In this meaning, public goods show the ultimate case of social common capital.

discount rate β.

$$\mathrm{E}\left[\sum_{t=0}^{\infty} \beta u(C_t)\right] \quad (5)$$

$$\mathrm{u}'(\mathrm{C}) > 0, \mathrm{u}''(\mathrm{C}) < 0 \quad (6)$$

$$\lim_{C \to \infty} u'(C) = 0 \quad (7)$$

The representation of disaster is like these. The probability of disaster in the term t is p. A probability variable z is 1 in probability p and 0 in probability $1-p$. In the case of a disaster, production capital is damaged in the ratio of ψ. By using these symbols, the available amount of production capital in the term t is represented like this[2].

$$K_t - K_t \psi_t z_t \quad (8)$$

We now finished the establishment of the model. From these, a dynamic optimization problem can be deduced.

$$\max_{C_t, K_{t+1}, D_{t+1}} \mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t u(C_t)\right] \quad (9)$$

subject to

$$\mathrm{f}(K_t - K_t \psi z_t) + (1-\delta_K)(K_t - K_t \psi z_t) + (1-\delta_D)D_t = C_t + K_{t+1} + D_{t+1}$$

$$K_{t+1} - (1-\delta_K)(K_t - K_t \psi z_t) \geq 0, D_{t+1} - (1-\delta_D)D_t \geq 0$$

3. Solving the Problem

The Lagrangian of the optimization problem is like this. Here λ and μ are Lagrangian multipliers.

$$\begin{aligned} L_t = E\Bigg[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \{u(C_\tau) \\ &+ \lambda_\tau [f(K_\tau - K_\tau \psi_\tau z_\tau) + (1-\delta_K)(K_\tau - K_\tau \psi_\tau z_\tau) + (1-\delta_D)D_\tau - C_\tau - K_{\tau+1} \\ &- D_{\tau+1}] + \mu_\tau^K [K_{\tau+1} - (1-\delta_K)(K_\tau - K_\tau \psi_\tau z_\tau)] + \mu_\tau^D [D_{\tau+1} - (1-\delta_D)D_\tau]\}\Bigg] \end{aligned}$$

(10)

Kuhn-Tucker conditions are these five.

$$\lambda_t = u'(C_t) \quad (11)$$

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[2] In Segi-Ishikawa-Yokomatsu's model, disaster prevention capital is also damaged in the same ratio as production capital. Our model differs in this point because we define disaster prevention capital as including governmental system and people's knowledge as well as physical infrastructure.

$$\lambda_t - \mu_t^K = \beta E\left[\left\{\lambda_{t+1}[f'(K_{t+1} - K_{t+1}\psi_{t+1}z_{t+1}) + 1 - \delta_K]\left[1 - (\psi_{t+1} + K_{t+1}\frac{\partial \psi_{t+1}}{\partial K_{t+1}})z_{t+1}\right]\right\} - \left\{\mu_{t+1}^K(1-\delta_K)\left[1 - \left(\psi_{t+1} + K_{t+1}\frac{\partial \psi_{t+1}}{\partial K_{t+1}}\right)z_{t+1}\right]\right\}\right] \tag{12}$$

$$\lambda_t - \mu_t^G = \beta E\left[\left\{\lambda_{t+1}([f'(K_{t+1} - K_{t+1}\psi_{t+1}z_{t+1}) + 1 - \delta_K][-K_{t+1}\frac{\partial \psi_{t+1}}{\partial D_{t+1}}z_{t+1}] + 1 - \delta_D)\right\} - \left\{\mu_{t+1}^K(1-\delta_K)\left[-K_{t+1}\frac{\partial \psi_{t+1}}{\partial D_{t+1}}z_{t+1}\right]\right\} - \{\mu_{t+1}^D(1-\delta_D)\}\right] \tag{13}$$

$$\mu_t^K[K_{t+1} - (1-\delta_K)(K_t - K_t\psi_t z_t)] = 0 \tag{14}$$

$$\mu_t^D[D_{t+1} - (1-\delta_D)D_t] = 0 \tag{15}$$

We can interpret the conditions (12) and (13) like these. The left hand sides of both equations is the cost of 1 unit increase of production capital and disaster prevention capital. Lagransian multipliers mean shadow prices measured by utility. The left hand sides mean how much people lose utility by investing production capital or disaster prevention capital. The right hand side of the equations show the benefit from investment of 1 unit capitals. In the equilibrium path, the marginal cost and the marginal benefit must be equal.

4. Disaster Damage Ratio and Accumulation of Disaster Prevention Investment

As for disaster damage ratio ψ, we definite like this.

$$\psi_t = d \cdot \exp(-\theta \frac{D_t}{K_t}) \tag{16}$$

When D is 0, ψ is equal to d. Because of this, d is called seriousness of disaster. In equation (16), θ should be called efficiency of disaster prevention capital. This means technical progress of physical infrastructure and closeness of human relationship.

The larger D/K, the smaller disaster damage ratio becomes. Please pay attention to our use of the ratio of disaster prevention capital to production capital D/K not the level of disaster prevention capital D. If we use D, disaster prevention capital comes to have the same character as public goods. The effect of it is restricted by only the supply level. It doesn't depend on the level of human activity at all. In short, it doesn't have rivalrous character. In our case, however, the production activity level $f(K)$ increase as production capital is accumulated. If disaster prevention capital is given, D/K will decrease and the effect of disaster prevention capital becomes weaker. This means that our disaster prevention capital suffers from congestion.

Now we will paraphrase comparative statistics as to d, θ and disaster probability p. First of all, when d increases, disaster damage ratio increases. Because of this, the economy must invest more disaster prevention capital to reach the optimal path of economic growth. It, however, causes trade-off through income distribution among consumption and two types of investment. At the end, D increases more speedily than K. Then D/K increases and disaster damage ratio decreases again.

Secondly when θ increases, the necessity of disaster prevention investment becomes weaker. Then disaster prevention investment refrained and the resource will be directed to production capital investment. It makes congestion about disaster prevention capital. The necessity of disaster prevention investment will revive.

At the last of all, the effect of disaster probability p must be considered. In our model, disaster damage ratio does not include p. It affects the economy through probability variable z. The variable is 1 in the probability p and 0 in the probability $1-p$. Because of that, we can obtain the expectation of the amount of disaster damage like this.

$$E[K_t \psi_t z_t] = K_t dp \cdot \exp(-\theta \frac{D_t}{K_t}) \tag{17}$$

When p increases, the amount of disaster damage increases. This make the necessity of disaster prevention investment larger. As disaster prevention investment increases and production capital investment decreases, the congestion of disaster prevention capital become smaller. In the course of time, the expected amount of disaster damage will become small again. The change of proportion of investments will cease.

5. Conclusion

Nowadays the economic growth rate is very low. Because of this, the accumulation rate of production capital is also low. From this point of view, there is noly a few necessity of accumulation of disaster prevention capital, too.

Instead of that, the disaster serriousness and the disaster probability inceases dramatically in Japan now. We have to say that we need accumulate disaster prevention capital sppedily to reach the new optimal state of Japanese economy.

Disaster prevention investment need direct the resource from consumption and production capital investment. This will reduce the welfare of Japanese people temporarily. To avoid the heavy reduction of welfare, we need increse the efficiency of disaster prevention. This can be possible by using new technology of disaster prevention and strengthening human connection of lical habitants.

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