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Abstract

We use the Tobin q value to derive the effective life of an investment determined by various factors including technological progress; we obtain four interesting results. First, anticipation of an imminent technological innovation or economic boom may postpone planned investment; investment cycle volatility increases when future expectations are positive. Second, given a continuous investment opportunity, investment inflow is accompanied by “scrapping” or depreciation; the depreciation rate and value of capital stock can be estimated using the book value; this reveals effective investments. Third, we show that the Perpetual Inventory Method (PIM) can be used to obtain the relationship between the price change of an investment and the depreciation rate, estimated using monetary information. Fourth, we apply the above methods to estimate sectoral depreciation rates in China, and obtain values close to those of the U.S.A.

JEL: D21, D24, E01, E22

Keywords: effective life, depreciation rate, investment, technological progress, value of capital, national wealth

1 Introduction

1.1 Background and research questions

At the micro level, fixed capital investment is indispensable for any corporation, and is an important component of any economy at the macro level. Investment booms or busts always reflect both the micro- and macro-economic situations. It is well known that investment is greatly influenced by technological progress. Does technological innovation extend the cycle of investment boom and bust? If so, how and why?

Fixed capital stock represents the accumulation of past investments, but also serves as a production input; production must be efficient. Thus, assessment of capital stock has been a fundamental topic in economics for a century. One of the most important works is that of Goldsmith (1951), applied in many later significant studies such as those of Wyckoff and Hulten (1979) and Hulten and Wyckoff (1981).[3] The basic idea is that an accounting rule[4] is in play, although the economic rationale remains unclear, even after Jorgenson (1965) built the micro foundation. For example, we do not yet really understand why inflow (newly installed equipment) and outflow (depreciation of that investment) co-exist. Applied studies require data on depreciation

[3] Wyckoff and Hulten (1979) estimated the depreciation rates of fixed assets based on equipment lifetime and asset structure.
[4] Goldsmith (1951) used depreciation expenses and the PIM to estimate capital depreciation rates. More recently, see Escriba-Perez et al. (2019).

rates and capital stocks; however, these are not always available. Therefore, the next question is: How can we estimate investment depreciation rates and capital values when the available information is limited, especially in developing countries? For example, we have only limited information on scrapping of past investments, the book values of fixed assets, and the prices of new investment. Furthermore, we generally have information not on real investments but, rather, on related monetary data. In such a situation, the Perpetual Inventory Method (PIM) can be used to estimate the depreciation rate if the effects of price changes on investment can be excluded, as shown by Christensen and Jorgenson (1973). However, as the prices of capital stock were lacking, such price changes were not well excluded in past empirical studies. The third question is: How can the PIM and investment price inflation or deflation be used to clarify depreciation rates when data are limited?

We initially answer the three questions from a theoretical viewpoint; we derive a structural model of effective life.[5] Next, we seek to apply our new method to estimate depreciation rates, especially in developing countries for which data are lacking. For example, Chinese data are far fewer than those available for the U.S.A. (Qiu and Wan

[5]Effective life is similar to the "economic life'' of the literature (Hotelling 1925; Preinreich 1940), but differs somewhat here because it is used to calculate investment timing, the depreciation rate, and the imputed value of capital stock. "Effective life" is used by the Australian Taxation Office: https://www.ato.gov.au/Business/Depreciation-and-capital-expenses-and-allowances/General-depreciation-rules---capital-allowances/Effective-life-of-an-asset/

2019).[6] The existing methodology deals with full datasets; it is difficult to apply to developing countries. Thus, the national wealth of China (the value of capital stock and land) remains unknown.

1.2 Our contribution

We build a model in which an investor chooses the timing of an investment to maximize profit even if the equipment to be replaced can still be used; the investor has thus ended the effective life of the item. Effective life is determined by various factors, including technological progress, measured using the Tobin *q*; innovation advances investment in time, because the opportunity cost of time is enhanced by innovation. In contrast, anticipation of an imminent economic boom or bust may postpone a planned investment; investment cycle volatility increases if future expectations are positive. As investment opportunities are always available, we simultaneously observe new investment inflow and past investment outflow at both the micro and macro levels; the depreciation rate and value of capital stock can then be estimated because the book value is that of the effective investments. We also derive the relationship between investment price change and the depreciation rate implied by the PIM; this can be used

[6] Chow (1995) and Chow and Li (2002) assume certain initial values of capital stock and then use investment flows to derive a time series of capital stock values in China. The assumed initial values are both very important and very controversial.

to estimate a depreciation rate based on only monetary information. We use these methods to estimate sector depreciation rates in China; the values are close to those of the U.S.A.

1.3 Organization of the paper

Section 2 models the endogenous effective life of capital investment, and discusses the impact of technological progress. Section 3 models the effective life, the depreciation rate, and the value of capital. Section 4 uses a new approach to obtain the relationship between investment price change and the depreciation rate implied by the PIM. In section 5, we apply the methods to estimate the effective life and the depreciation rate of the industrial and service sectors of China. Section 6 presents our conclusions and planned future work.

2 Effective life of an investment

2.1 Effective life of a discrete investment

We consider that an investor decides to make a new investment (replace equipment) after utilizing old equipment for some time (τ) (Figure 1A). The price or investment cost is I_0 at time zero and the interest rate (r) is assumed to be constant. The investment is assumed to be used for a long time; its marginal productivity or dividend (d) decreases with time at a constant rate δ because the initial investment I_0 (>0) is assumed to depreciate at the rate δ and the production function is assumed to be linear [$d = \alpha I_0, \alpha > 0, d_t = \alpha(I_0 e^{-\delta t})$]. Thus, the investor will obtain a discounted net profit [$\pi(\tau)$] at time zero:

$$\pi(\tau) = \int_0^\tau \alpha[I_0 e^{-\delta t}][e^{-rt}]dt - I_0$$

$$= \int_0^\tau [(\alpha I_0)e^{-\delta t}][e^{-rt}]dt - I_0$$

$$= \int_0^\tau [de^{-\delta t}][e^{-rt}]dt - I_0$$

$$= \frac{d}{r+\delta}\left[1 - e^{-(r+\delta)\tau}\right] - I_0. \tag{1}$$

To ensure a profit, Eq. (1) must be strictly positive; Eq. (1) is monotonic with respect to equipment use time τ. If the equipment is not replaced, its value converges to:

$$\lim_{\tau \to \infty} \pi(\tau) = \frac{d}{r+\delta} - I_0. \tag{2}$$

Note that $\frac{d}{(r+\delta)I_0} \left(= \frac{\alpha I_0}{(r+\delta)I_0} = \frac{\alpha}{r+\delta} = q\right)$ in Eq. (2) are the Tobin average and marginal q values (Tobin 1969) and that a ratio exceeding unity (q >1 iff $\alpha > r + \delta$) ensures a net positive profit. Here, it is assumed that $q > 1$, and the average q ($= \frac{\alpha I_0/(r+\delta)}{I_0}$) is equal to the marginal q ($= \frac{\alpha}{r+\delta}$) of Hayashi (1982) because the dividend is a linear homogeneous function of the net capital stock. In such a setting, the return on investment falls with time; the investor must thus consider when the equipment should be replaced. If the use time is too short, a positive net profit is not guaranteed. In contrast, if the use time is too long (for example $\tau \rightarrow \infty$), the marginal or net profit per unit time converges to zero. Hence, if the investor can maximize the net profit per period (τ), s/he will maximize the net profit over an infinite time horizon. The investor considers the following problem:

$$\max_{0<\tau<\infty} \pi(\tau) = \sum_{n=0}^{\infty} \int_{n\tau}^{(n+1)\tau} \{[de^{-\delta t}][e^{-rt}]dt - I_0\}$$

$$= \frac{1}{1-e^{-r\tau}} \left\{\frac{d}{(r+\delta)}[1 - e^{-(r+\delta)\tau}] - I_0\right\}, \quad (3)$$

where, n = 0, 1, 2, ….

The investor chooses the replacement time τ of Eq. (3). We obtain the following proposition (Figure 1B).

Proposition 1:

The investor can find an optimal replacement time (τ^*; "***effective life***"), which decreases when q is average or marginal; $\tau^* = \frac{1}{2r} \ln\left(\frac{q}{q-1}\right)$ for $r = \delta$.

Proof: See the Appendix; τ^*exists and is unique.

2.2 Effective life and technological progress

Next, we consider the impact of technological change on the optimal timing of replacement (τ^*). To express technological innovation in economic terms, we consider that innovation occurs, other things being equal, if dividend d increases.

Property 1:

The optimal timing of replacement τ^* decreases with the dividend d.

Proof:

$\frac{\partial \tau *}{\partial d} = \frac{\partial \tau *}{\partial q}\frac{\partial q}{\partial d} < 0$, since $\frac{\partial q}{\partial d} > 0$ and $\frac{\partial \tau *}{\partial q} < 0$ by *Proposition 1*. **Q.E.D.**

Next, we consider that the investment has been used for some time $[t' \in (0,\ \tau^*)]$ and an innovation occurs at t' (Figure 2A). The investor must decide whether and when to replace the old equipment. Will replacement be immediate or will it occur at time t'' $(t'' \in [t', \tau])$ (after a "wait-and-see" period) (Figure 2B). The replacement time will be chosen as follows.

Proposition 2:

The investor will advance replacement timing if new technology meets certain criteria.

Proof:

The net profit per unit time is $\frac{\int_0^{\tau'}[d'e^{-\delta t}][e^{-rt}]dt-I_0'}{\tau'}$ for new technology. When this is smaller than $de^{-\delta t''}$, the investor optimally engages in "wait-and-see". As depreciation increases over time, the investor replaces the equipment when $\frac{\int_0^{\tau'}[d'e^{-\delta t}][e^{-rt}]dt-I_0'}{\tau'} \geq de^{-\delta t''}$.

The optimal timing of replacement (t'') depends on:

$$de^{-\delta t''} \leq \frac{\int_0^{\tau'}[d'e^{-\delta t}][e^{-rt}]dt - I_0'}{\tau'},$$

where τ' ($\tau' < \tau^*$) is optimal for implementation of new technology. **Q.E.D.**

The implication of *Proposition 2* is that technological innovation will bring forward new investment to reduce opportunity costs; new technology raises the opportunity costs of using old technology even the old equipment is still functional. This is termed "creative destruction", and explains investment booms dictated by significant technological progress. However, for example, when the investor anticipates just before the end of the optimal period ($t = \tau^{*-}$) that new technology will be available in future ($t = \tau^* + \tau''', \tau''' < \tau^*$) (Figure 2C), the investment decision will be made using the following rule.

Theorem 1:

The investor will postpone replacement because s/he anticipates future technological progress or an economic boom.

***Proof**:*

As shown in Figure 2C, we set $\tau'' < \tau^*$, $\tau''' < \tau^*$, $\tau''' \leq \tau'''' \leq \tau''' + \tau''$. Then, the timing of replacement (τ^*) will be postponed to a time $t = \tau^* + \tau'''$ if

$$\int_0^{\tau''' + (\tau'''' - \tau''')} [d'' e^{-\delta t}][e^{-rt}]dt - I_0$$
$$\leq \int_0^{\tau'''} [de^{-\delta(\tau^* + t)}][e^{-rt}]dt + \int_{\tau'''}^{\tau''''} \frac{\int_0^{\tau''} [d'' e^{-\delta t}][e^{-rt}]dt - I_0''}{\tau''} [e^{-r(\tau''' + t)}]dt, \quad (4)$$

where τ'' ($\tau'' < \tau^*$) is the optimal period for installation of new technology. **Q.E.D.**

The implication of *Theorem 1* is that when new technology or an economic boom, which will raise the marginal profit of future investment is anticipated, the investor will optimally postpone past-planned investment. This is a novel point, being absent from (for example) Boucekkine et al. (2009); such behavior will render investment sluggish before a technological or economic boom, increasing investment cycle volatility.

3 Aggregation of investments

3.1 Capital stock

Assume that the investor has an identical investment opportunity at every point on an infinite time horizon (thus, continuous opportunity with a uniform distribution). Investment inflow and outflow will thus be simultaneously evident at all times at which other factors do not change. At any time *t*, the book value of capital stock can be expressed as described below.

Proposition 3:

The book value of capital stock equals $\tau^* I_0$, and the effective life can be imputed using this book value and investment, employing the accounting rules of straight-line or one-hoss shay decay of Hulten and Wykoff (1981, p. 89).

Proof:

$\int_{-\tau^*}^{0} I_0 dt = \tau^* I_0$, and then $\tau^* = \frac{\int_{-\tau^*}^{0} I_0 \, dt}{I_0}$. **Q.E.D.**

The implication of *Proposition 3* is that the book value of capital stock includes information on the effective life of capital investment. We next discuss the situation when investment inflow and depreciation outflow differ.

3.2 Effective life and depreciation rate when inflows and outflows differ

In reality, for a corporation (micro level) or a country (macro level), investment inflow always differs from investment outflow. For example, for a stable or growing (declining) corporation or country, the inflow is always the same as or higher (lower) than the outflow (Figures 3A, 3B, 3C, respectively). We make the following three definitions:

a = depreciation of plant assets (outflow);

b = newly increased fixed assets, or new investment (inflow);

c = book value of capital stock.

In such a case, the effective life can be imputed using the book value of capital stock and the inflow/outflow information.

Property 2:

Effective life can be expressed as:

$$\tau^* = \frac{c}{a} \tag{5}$$

using the accounting rule of straight-line decay, and as:

$$\tau^* = \frac{c}{a}\left(\frac{2a}{a+b}\right) = \frac{2c}{a+b} \tag{6}$$

using the accounting rule of one-hoss shay decay.

The key assumption of *Property 2* is that the speed of inflow growth is smooth. This assumption would not be very strong for an economy lacking dramatic technological progress. If we consider the price changes of capital goods (Figures 3D and 3E), the situation is as follows.

Property 3:

Effective life can be uniquely determined using information on investment price, capital stock, inflow, and outflow. Either the accounting rules of straight-line or one-hoss shay decay can be employed.

After we obtain the effective life of the capital investment, we can estimate the depreciation rate and imputed value.

Theorem 2:

The depreciation rate and imputed value of capital stock can be estimated using accounting tools.

Proof:

After estimating the depreciation rate using *Properties 2 and 3*, the imputed value of capital stock can be estimated as:

$$\int_{-\tau^*}^{0} I_t\left(1-\frac{1}{\tau^*}t\right)dt. \qquad \textbf{Q.E.D.}$$

We use *Theorem 2* to estimate capital stock by sector, and national wealth, especially when basic data on asset structure are very limited. The PIM is commonly used to estimate depreciation rates, capital stocks, and national wealth (Goldsmith 1951; Jorgenson 1965). However, it remains very difficult to deal with the price index of capital stock, even in the U.S.A., for which the most detailed data worldwide are available. To compare our method and the traditional PIM, we need to tackle the price index problem. Below, we analyze the depreciation rate yielded by the PIM when the investment price changes.

4 Depreciation rate derived using the PIM with price changes

4.1 The PIM

Following Jorgenson (1965), we use the PIM to obtain the transition of real net capital stock (K_t) undergoing depreciation at a rate (δ) derived using geometrically declining weights or the balance-declining method of accounting, as follows:

$$K_t = \sum_{\varphi = 0}^{\infty}\left[I_{t-\varphi} - \sum_{\omega = 1}^{\infty}(1-\delta)^{\omega}\ \delta I_{t-\omega}\right] \tag{7}$$

Eq. (7) features the past investment sequence of Eq. (3) and the Tobin *q* of *Proposition 1*; effective life contributes to the micro foundation of Eq. (7), which can be transformed into a well-known form of capital stock using the PIM:

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{8}$$

where I_t represents the investment at time *t*. We obtain the depreciation rate (δ_{pim}) implied by the capital stock series and investment:

$$\delta_{pim} = \frac{K_{t-1} + I_t - K_t}{K_{t-1}}. \tag{9}$$

Eqs. (8) and (9) can also be derived using the limited asset life method [the straight-line accounting rule of Goldsmith (1951)].

4.2 PIM with price change

We always lack information on real investment (I; *i.e.*, a machine) and real capital stock (K; *i.e.*, how many machines). We have only monetary information ($p_{it}i_t$ and $p_{kt}k_t$ respectively). We can observe the price of investment ($p_{it} = p_t$) but cannot observe p_{kt}. As shown by Christensen and Jorgenson (1973), p_{kt} is a function of investment price in the past and the future, capital, and the rental sequence cost, and is thus very complicated. Here, we assume that the capital stock price can be expressed as $\theta_t p_t (p_{kt} = \theta_t p_t)$, and that θ_t $(0 \ll \theta \ll +\infty)$ is not directly observable but indirectly estimable. Hence, we obtain $p_t i_t$ and $\theta_t p_t k_t$ at the end of a time *t*. Thus, the investment and capital stock may have different price deflators. For each of $\theta_t < 1$, $\theta_t = 1$, $\theta_t > 1$, the observed value of capital stock would be higher, equal to, or less than its market value, respectively. We obtain the relationship between price change and depreciation rate (as shown by the PIM) as follows:

Theorem 3:

$$\delta_t^* = \mathrm{f}(\delta_t(\delta_{pim-t})),\ \delta_t^* \geq (<)\ \delta_t\ for\ \theta_{t-1} \geq (<) 1, \text{ and:}$$

$$\delta_t = \frac{\delta_{pim-t} + \frac{p_t - p_{t-1}}{p_{t-1}} + \frac{p_t k_t}{p_{t-1} k_{t-1}} \left(\frac{\theta_t}{\theta_{t-1}} - 1 \right)}{p_{t/} p_{t-1}}$$

where:

$$\delta_{pim-t} \equiv \frac{\theta_{t-1} p_{t-1} k_{t-1} + p_t i_t - \theta_t p_t k_t}{\theta_{t-1} p_{t-1} k_{t-1}}$$

$$\delta_t \equiv \frac{k_{t-1} + \frac{i_t}{\theta_{t-1}} - k_t}{k_{t-1}}$$

$$\delta_t^* \equiv \frac{k_{t-1}+i_t-k_t}{k_{t-1}} \; for \; \theta_{t-t} = 1.$$

Proof: See Appendix.

Property 2: $\delta_t^* = \delta_t = \frac{\delta_{pim-t} + \frac{p_t - p_{t-1}}{p_{t-1}}}{p_t / p_{t-1}}$ for $\theta_{t-t} = \theta_t = 1$ by *Theorem 3*.

The Price Index for Investment in Fixed Assets (Purchase of Equipment and Instruments, Year of 1993 = 100) decreased from 100 in 1993 to 95 in 2016 [Figure 1 of Qiu and Wan (2019)]. In both sample periods (1993-2016 and 2001-2016), the investment price deflated slightly; the imputed value is the market value ($\theta_{t-t} = \theta_t = 1$). Thus, to derive a depreciation rate that excludes price changes, the deflation rate should be subtracted from the estimated $\delta_{\mathrm{pim-t}}$: δ_t^* should be a little less than the estimated $\delta_{\mathrm{pim-t}}$. The impact of price change is negligible if the change is small [Qiu and Wan (2019)].

5. Applications of effective life and capital stock values

5.1 Effective life and depreciation rate in China

We used data in the Statistical Yearbooks of China 2001-2016 to estimate the effective lives of 37 industrial sectors (Figure 4A). The average depreciation rate by effective life ($1/\tau^*$), defined using the straight line-decline method of Christensen and Jorgenson (1973), was significantly, (p=0.03) 1%, greater than that afforded by the PIM in the work of Qiu and Wan (2019), but close to the values in the U.S.A. [Wyckoff and Hulten (1979) and Hulten and Wyckoff (1981)].

We used the Yearbooks of the China Economic Censuses of 2004 and 2008 and all available China Statistical Yearbooks to estimate the effective lives and depreciation rates of five service sectors (Figure 4B); these are close to the U.S.A. values [Wyckoff and Hulten (1979) and Hulten and Wyckoff (1981)]. For example, the U.S.A. depreciation rate is 5% and the estimated value in China is about 5-8% for the education sector value in both countries. Notably, not the PIM but the approach of effective life developed in this paper can be used in service sectors in China because of the limited data.

5.2 Estimation of Chinese national wealth

The Economic Censuses of China conducted in 2004, 2008, 2013, and 2018 give the fixed assets of the first, second, and third industrial corporate sectors; the household sector; and the central and local governmental sectors. We calculated annual estimates of the various series from 2018 back to 1952 using investment inflow and depreciation outflow. The advantage of this approach is that we have detailed information on 2018 and thus do not need what we do not have (the national wealth in 1952). Thus, we calculated the national wealth in China; this has never been done before.

6. Conclusions and future research

We built a theory of effective life by modelling investor timing when replacing equipment on a continuous timeline. We found that effective life was determined by economic factors including technological change as measured by the Tobin *q*, and that innovation advanced new investment in time, because the opportunity cost of time increases as new technology becomes available. In contrast, we found that anticipation of an imminent economic or technological boom may postpone planned investment; investment cycle volatility is increased by positive future expectations.

If we assume that investment opportunities are both continuous and uniform, new investment inflow and past investment outflow occur simultaneously at both the micro and macro levels. Consequently, the depreciation rate and capital stock value can be estimated because the book value of such stock includes information on the effective life of capital investment.

We also developed a new method, using the PIM, that revealed the relationship between the investment price change and depreciation rate. To estimate the depreciation rate employing a PIM that excludes price inflation/deflation, we used the available information on investment price and the imputed value of capital stock (because we lacked data on stock price). In other words, we had only monetary information on both investment and capital stock.

Finally, we used the effective life approach to estimate sectoral depreciation rates in China, and obtained values close to those of the U.S.A. [Wyckoff and Hulten (1979) and Hulten and Wyckoff (1981)] and the Chinese industrial sector data of Qiu and Wan (2019). Our effective life methods and our PIM incorporating price changes can be used to estimate the depreciation rates and national wealth of other (especially developing) economies.

Appendix:

Proof of Proposition 1

The first order condition of Eq. (3) is:

$$e^{-r\tau}e^{-\delta\tau} - \frac{r+\delta}{r}\ e^{-\delta t} + \frac{d-(r+\delta)I_0}{d} = 0. \tag{A.1}$$

Set $Y = e^{-\delta t}$; then, Eq. (A.1) is transformed into:

$$Y = \frac{r}{r+\delta}\ Y^{\frac{r+\delta}{r}} + \frac{r}{r+\delta}\frac{d-(r+\delta)I_0}{d}. \tag{A.2}$$

As $Y = e^{-\delta t}$, we obtain $0< Y < 1$ for $0< t < +\infty$. The right

side of Eq. (A.2) increases with *Y*, and is:

$$0 < \frac{r}{r+\delta}\frac{d-(r+\delta)I_0}{d} < \frac{r}{r+\delta}\ Y^{\frac{r+\delta}{r}} + \frac{r}{r+\delta}\frac{d-(r+\delta)I_0}{d} < \frac{r}{r+\delta} + \frac{r}{r+\delta}\frac{d-(r+\delta)I_0}{d} < 1,$$

$$\text{for } \alpha(r-\delta) < r(r+\delta). \tag{A.3}$$

Thus, a unique Y^* $(0 < Y^* < 1)$ is the fixed point corresponding to a unique

$$\tau^* \ (0 < \tau^* < +\infty).$$

The impact of *q* on τ^* is as follows:

$$\frac{\partial\tau*}{\partial q} = \frac{\partial\tau*}{\partial Y^*}\frac{\partial Y^*}{\partial q} < 0, \text{ since } \frac{\partial Y^*}{\partial q} < 0 \text{ and } \frac{\partial\tau*}{\partial Y^*} > 0. \tag{A.4}$$

For the special case $r = \delta$, Eq. (A.2) is transformed into:

$$Y^2 - 2Y + \frac{d-(r+\delta)I_0}{d} = 0, \tag{A.5}$$

and we obtain:

$$Y^* = 1 - \left(\frac{2rI_0}{d}\right)^{0.5}, \tag{A.6}$$

and:

$$\tau^* = \frac{1}{2r}\ln\left(\frac{q}{q-1}\right). \tag{A.7}$$

Q.E.D.

Proof of Theorem 3

By Eq. (9) and the definitions of investment price and capital stock, we obtain:

$$\delta_{pim-t} \equiv \frac{\theta_{t-1}p_{t-1}k_{t-1} + p_t i_t - \theta_t p_t k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \frac{\theta_{t-1}p_{t-1}k_{t-1} + p_t I_t - \theta_{t-1}p_{t-1}k_t + (p_t - p_{t-1})I_t - (\theta_t p_t - \theta_{t-1}p_{t-1})k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \frac{k_{t-1} + \frac{I_t}{\theta_{t-1}} - k_t}{k_{t-1}} + \frac{(p_t - p_{t-1})I_t - (\theta_t p_t - \theta_{t-1}p_{t-1})k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \delta_t + \frac{(p_t - p_{t-1})I_t - (\theta_t p_t - \theta_{t-1}p_{t-1})k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \delta_t + \frac{(p_t I_t - p_{t-1}I_t) - (\theta_t p_t k_t - \theta_{t-1}p_{t-1}k_t)}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \delta_t + \frac{p_t I_t - p_{t-1}I_t - \theta_t p_t k_t + \theta_{t-1}p_{t-1}k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \delta_t + \frac{p_t I_t - \theta_t p_t k_t}{\theta_{t-1}p_{t-1}k_{t-1}} - \frac{I_t - \theta_{t-1}k_t}{\theta_{t-1}k_{t-1}}$$

$$= \delta_t + \frac{p_t I_t - \theta_t p_t k_t}{\theta_{t-1}p_{t-1}k_{t-1}} - \frac{\frac{I_t}{\theta_{t-1}} - k_t}{k_{t-1}}$$

$$= \delta_t + \frac{p_t I_t - \theta_t p_t k_t}{\theta_{t-1}p_{t-1}k_{t-1}} - (\delta_t - 1) \quad . \tag{A.8}$$

The second term on the right of Eq. (A.8) can be rewritten as:

$$\frac{p_t I_t - \theta_t p_t k_t}{\theta_{t-1}p_{t-1}k_{t-1}}$$

$$= \frac{\frac{p_t I_t}{\theta_{t-1} p_{t-1}} - \frac{\theta_t p_t k_t}{\theta_{t-1} p_{t-1}}}{k_{t-1}}$$

$$= \frac{\frac{p_t}{p_{t-1}}\left(\frac{I_t}{\theta_{t-1}} - \frac{\theta_t k_t}{\theta_{t-1}}\right)}{k_{t-1}}$$

$$= \frac{\frac{p_t}{p_{t-1}}\left(\frac{I_t}{\theta_{t-1}} - k_t\right) + \frac{p_t}{p_{t-1}}\left(k_t - \frac{\theta_t k_t}{\theta_{t-1}}\right)}{k_{t-1}}$$

$$= \frac{p_t}{p_{t-1}} \frac{\frac{I_t}{\theta_{t-1}} - k_t}{k_{t-1}} + \frac{p_t}{p_{t-1}} \frac{k_t}{k_{t-1}}\left(1 - \frac{\theta_t}{\theta_{t-1}}\right)$$

$$= \frac{p_t}{p_{t-1}}(\delta_t - 1) + \frac{p_t k_t}{p_{t-1} k_{t-1}}\left(1 - \frac{\theta_t}{\theta_{t-1}}\right) . \qquad \text{(A.9)}$$

Inserting Eq. (A.9) into Eq. (A.8), we obtain:

$$\delta_{pim-t} = \delta_t - \delta_t + 1 + \frac{p_t}{p_{t-1}}(\delta_t - 1) + \frac{p_t k_t}{p_{t-1} k_{t-1}}\left(1 - \frac{\theta_t}{\theta_{t-1}}\right)$$

$$= 1 + \frac{p_t}{p_{t-1}}\delta_t - \frac{p_t}{p_{t-1}} + \frac{p_t k_t}{p_{t-1} k_{t-1}}\left(1 - \frac{\theta_t}{\theta_{t-1}}\right). \qquad \text{(A.10)}$$

We obtain a solution of δ_t using Eq. (A.10):

$$\delta_t = \frac{\delta_{pim-t} + \frac{p_t - p_{t-1}}{p_{t-1}} + \frac{p_t k_t}{p_{t-1} k_{t-1}}\left(\frac{\theta_t}{\theta_{t-1}} - 1\right)}{p_t / p_{t-1}}. \qquad \text{(A.11)}$$

We also obtain:

$\delta_t^* \geq (<)\ \delta_t\ for\ \theta_{t-1} \geq (<) 1$ by the definition of δ_t^*.

(A.12)

Q.E.D.

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Figure 1A: Effective life of an investment

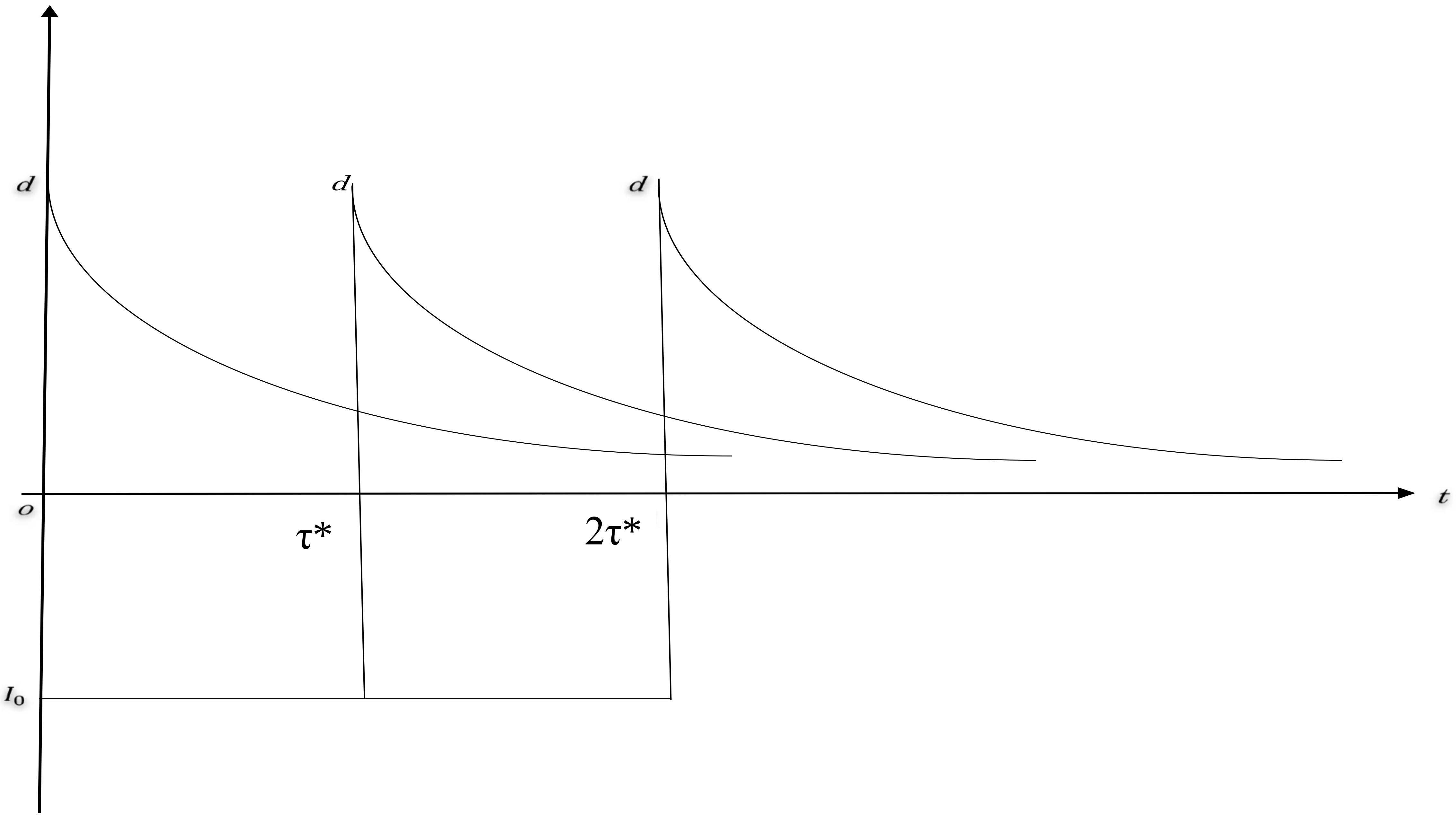


Figure 1B: Profit and effective life of an investment

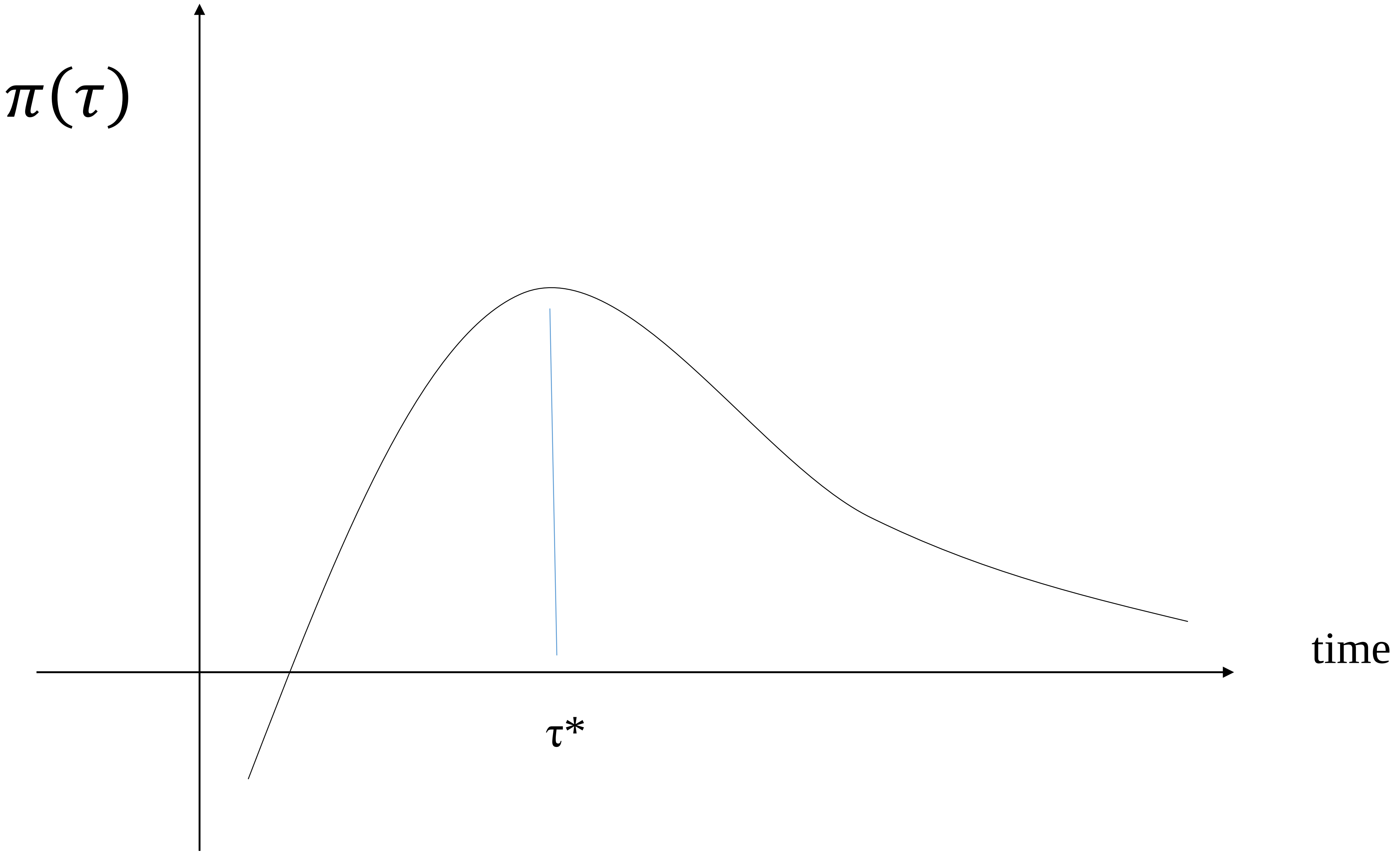


Figure 2A: Effective life and technological progress

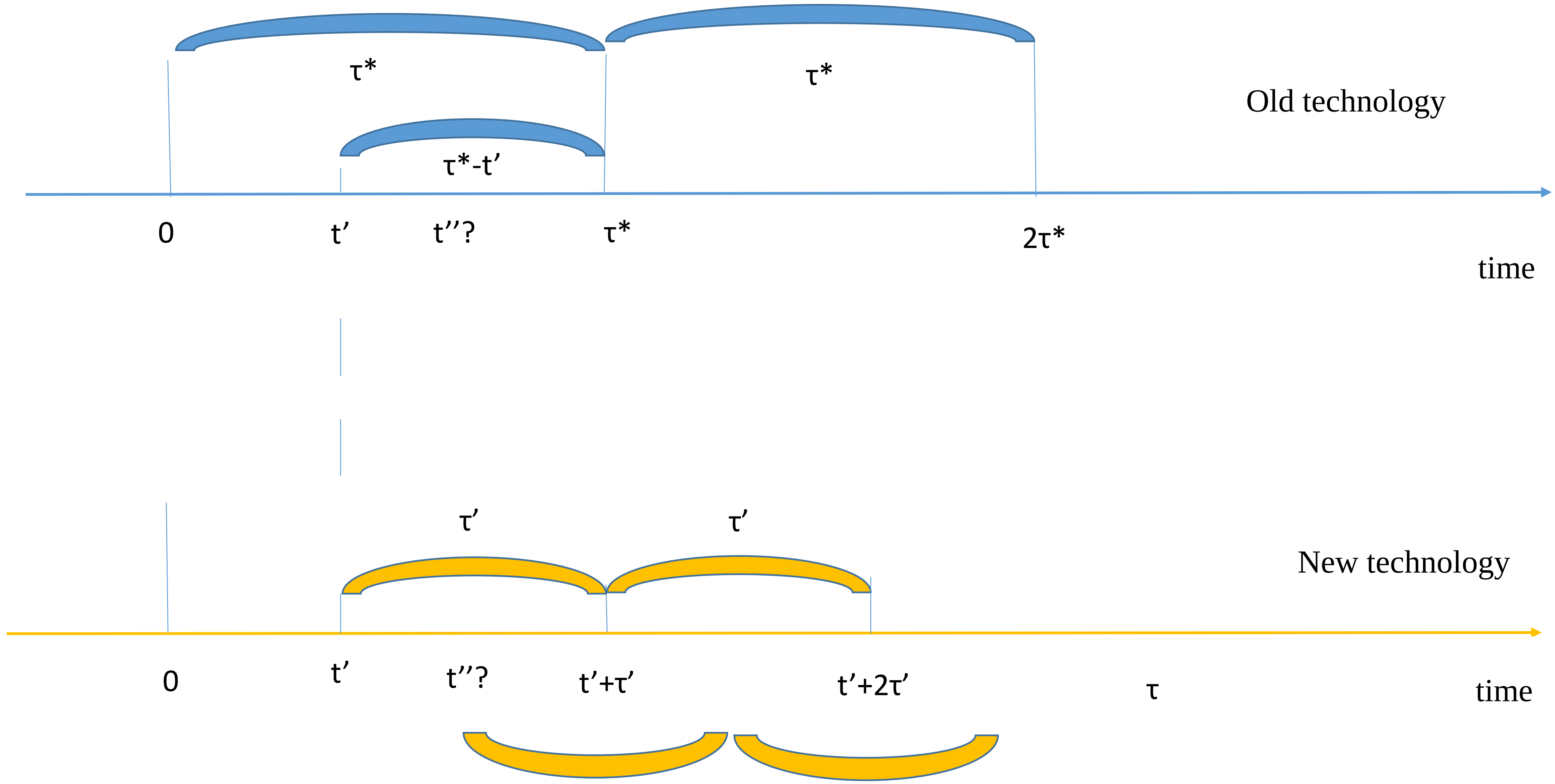


Figure 2B: Advancement of replacement timing

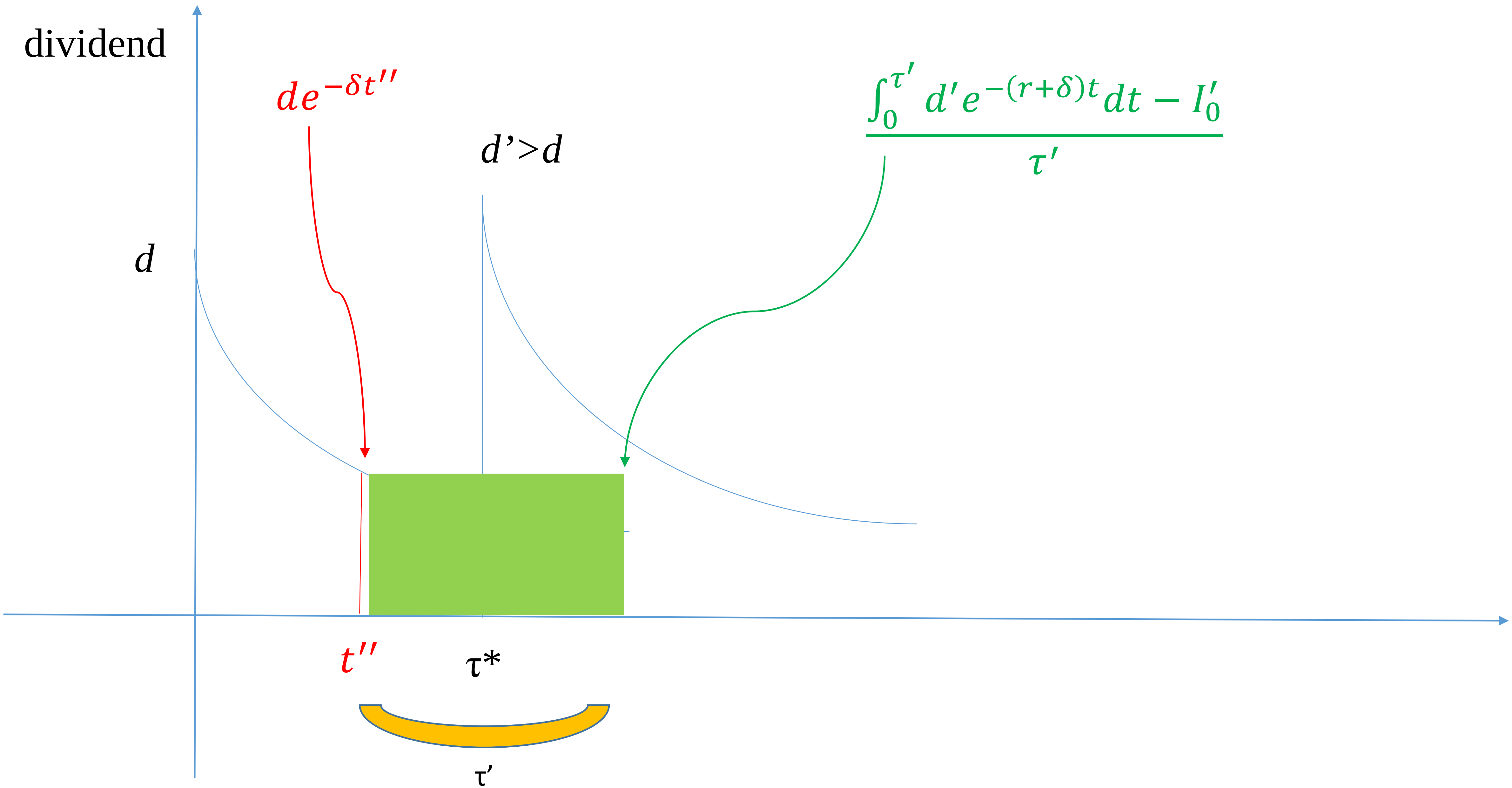


Figure 2C: Postponement of replacement timing

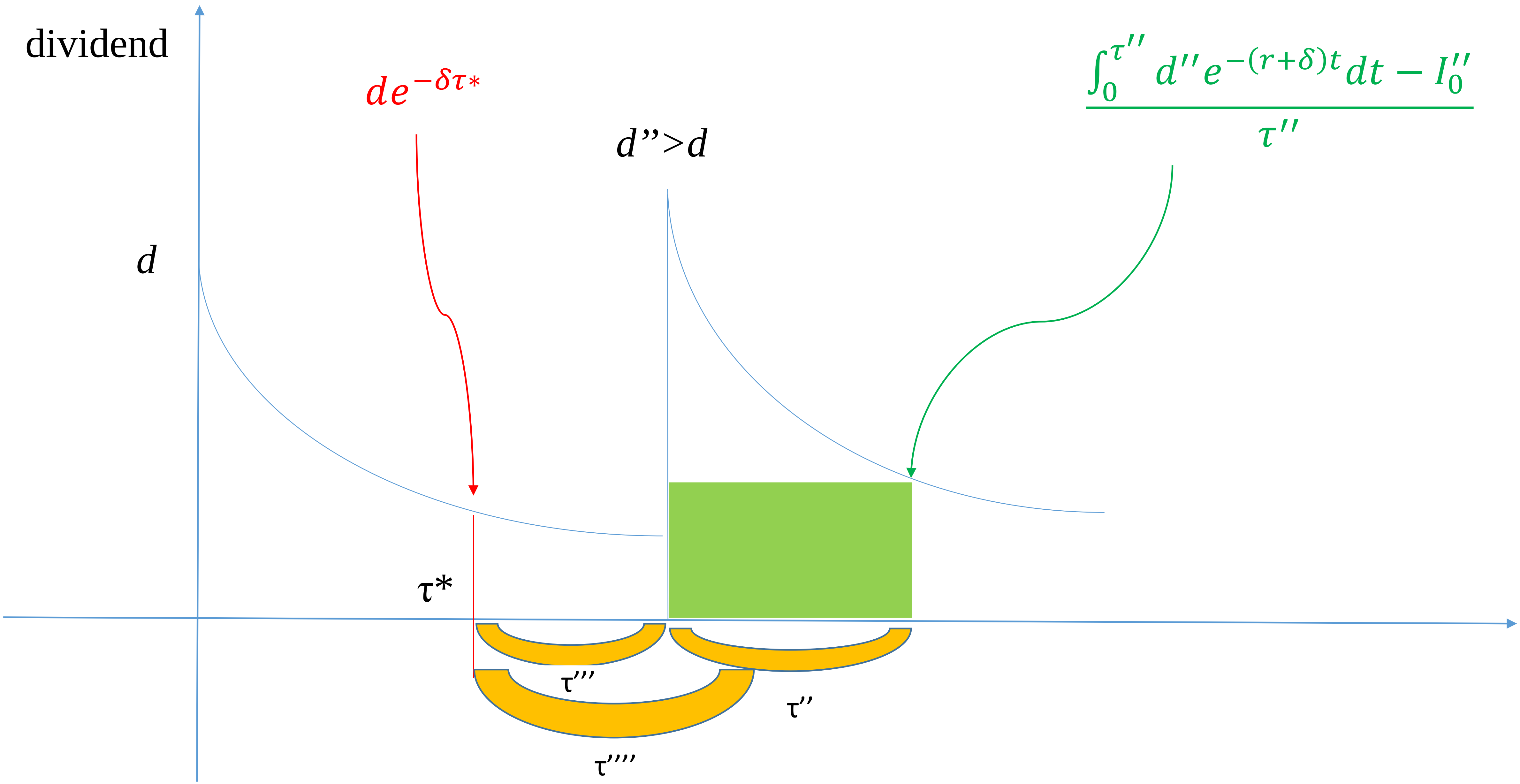


Figure 3A: Capital stock and effective life of a stable economy (inflows and outflows identical)

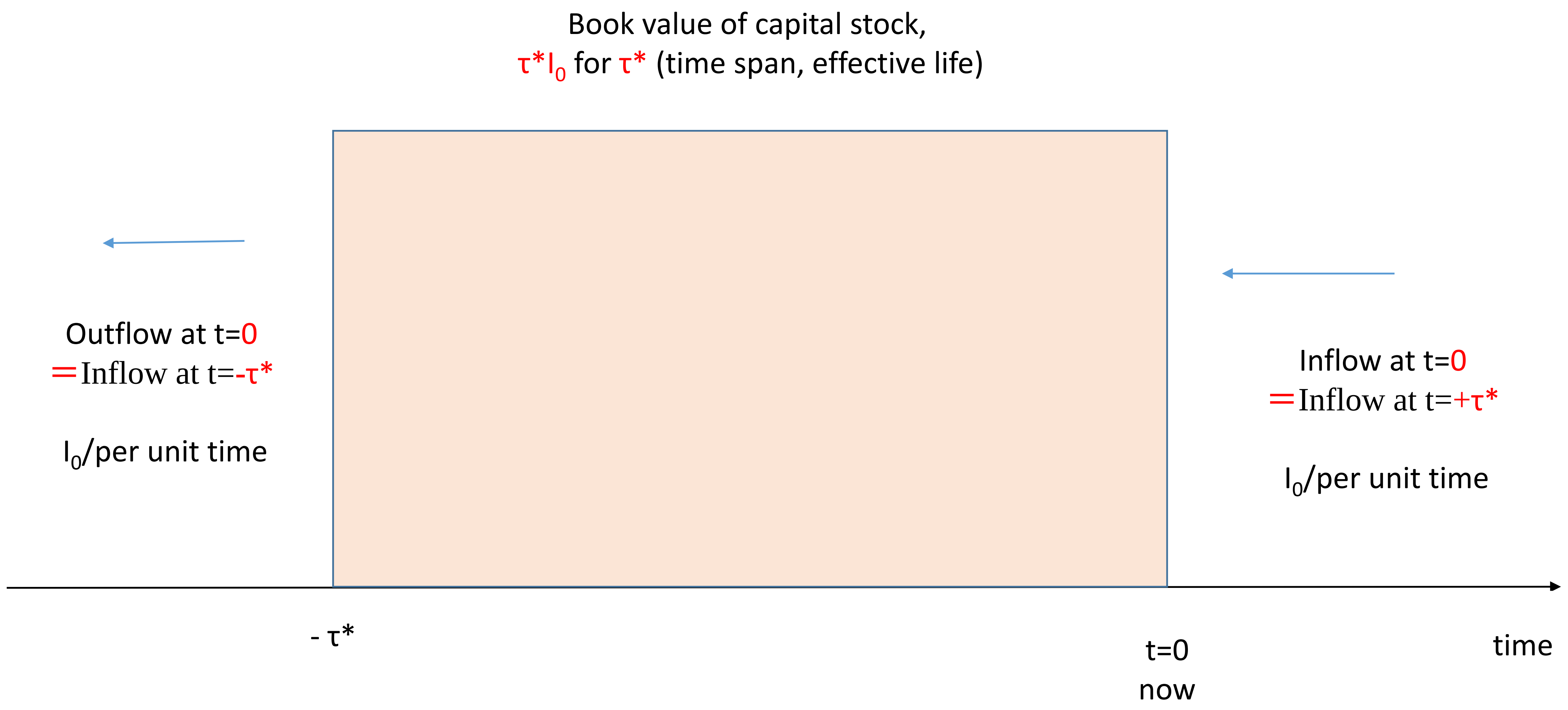


Figure 3B: Capital stock and effective life of a growing economy (large inflows but small outflows)

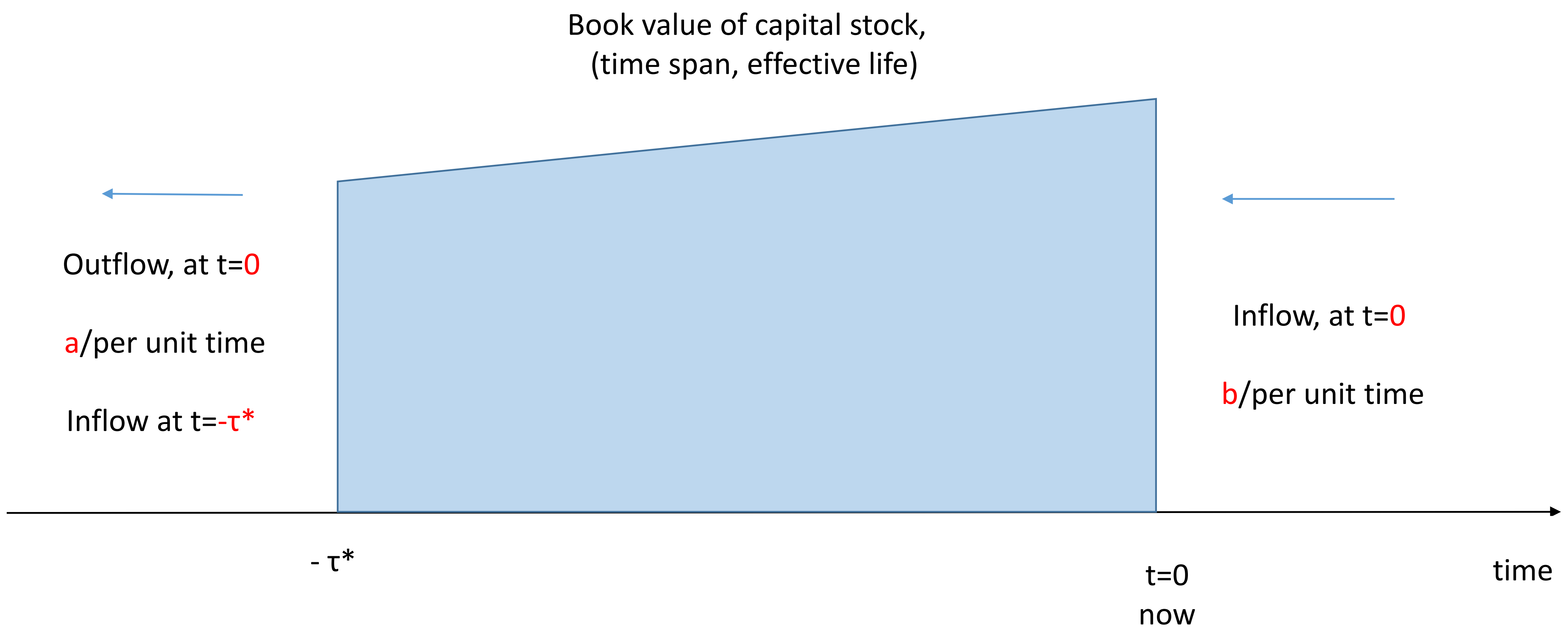


Figure 3C: Capital stock and effective life of a declining economy (small inflows but large outflows)

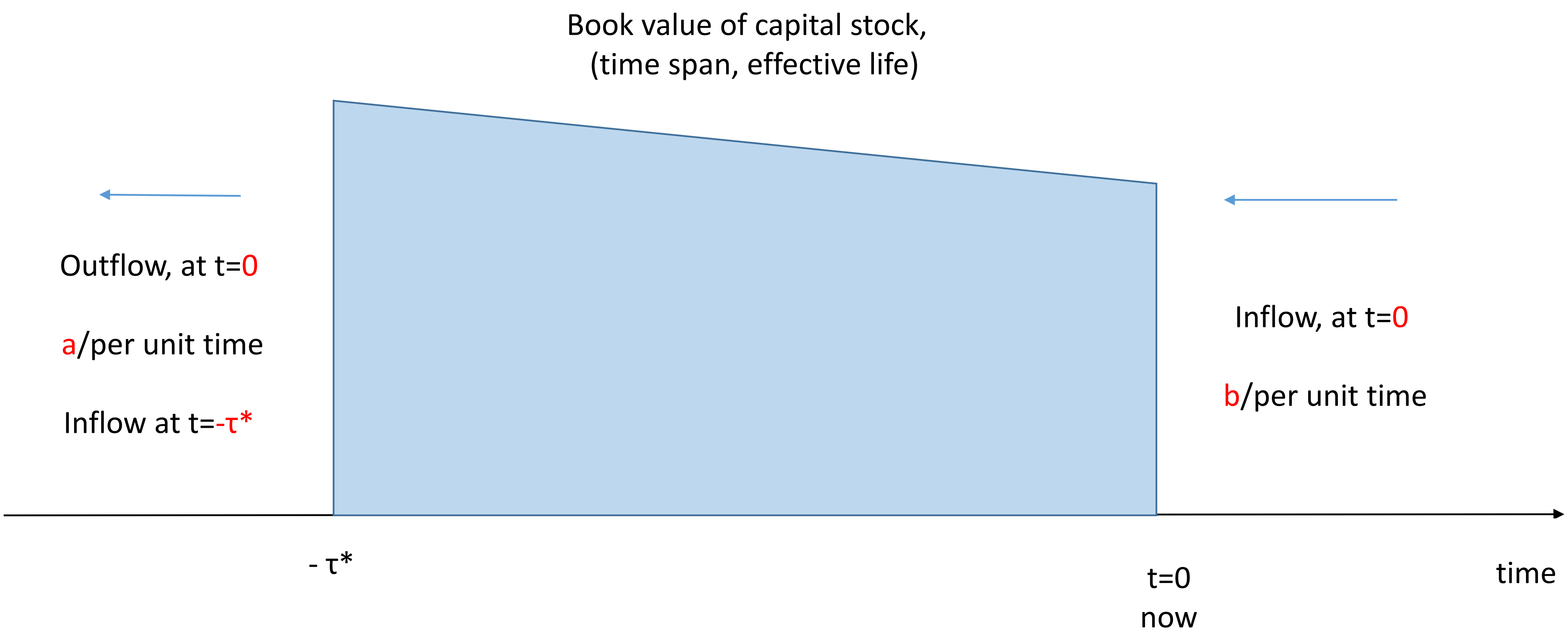


Figure 3D: Capital stock and effective life of a growing economy with price changes (large inflows but small outflows)

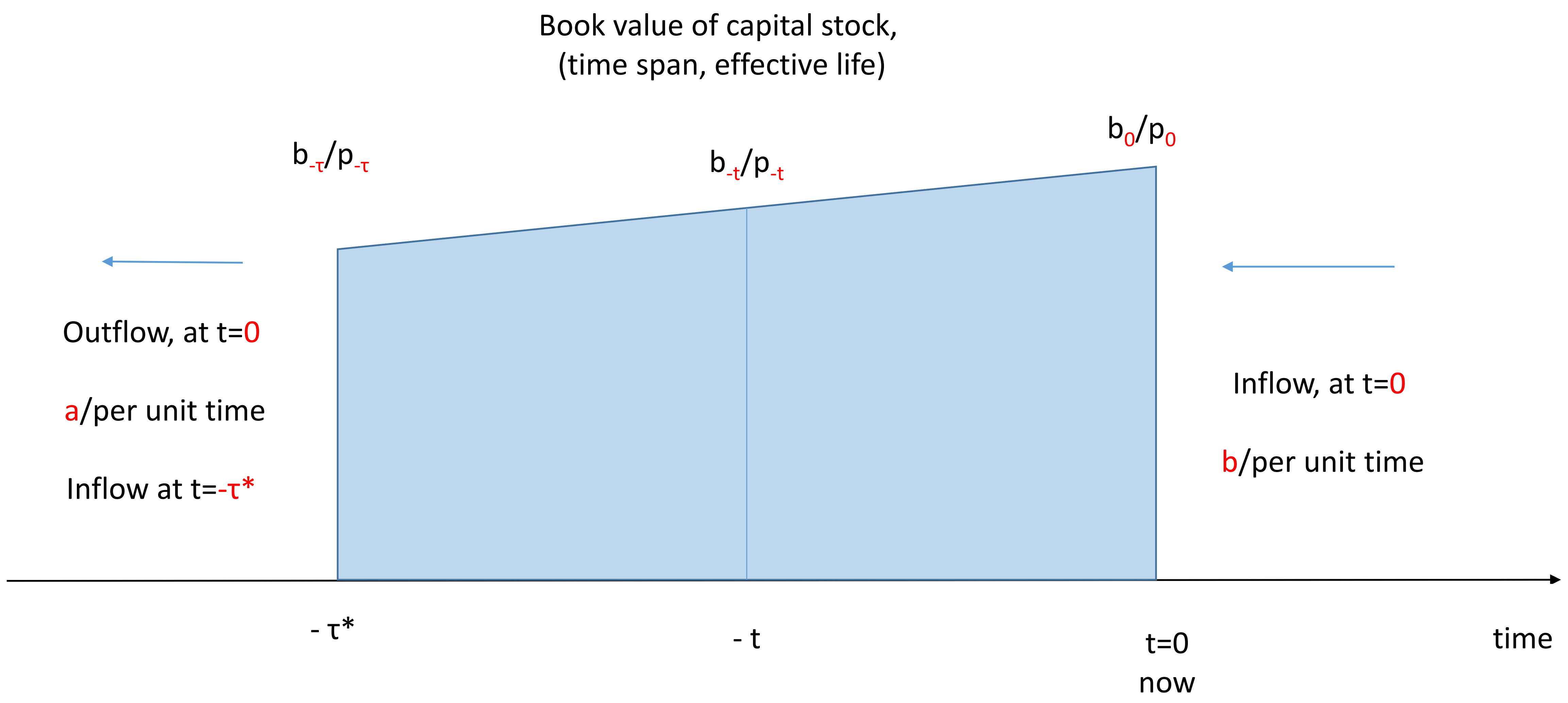


Figure 3E: Capital stock and effective life of a declining economy with price changes (small inflows but large outflows)

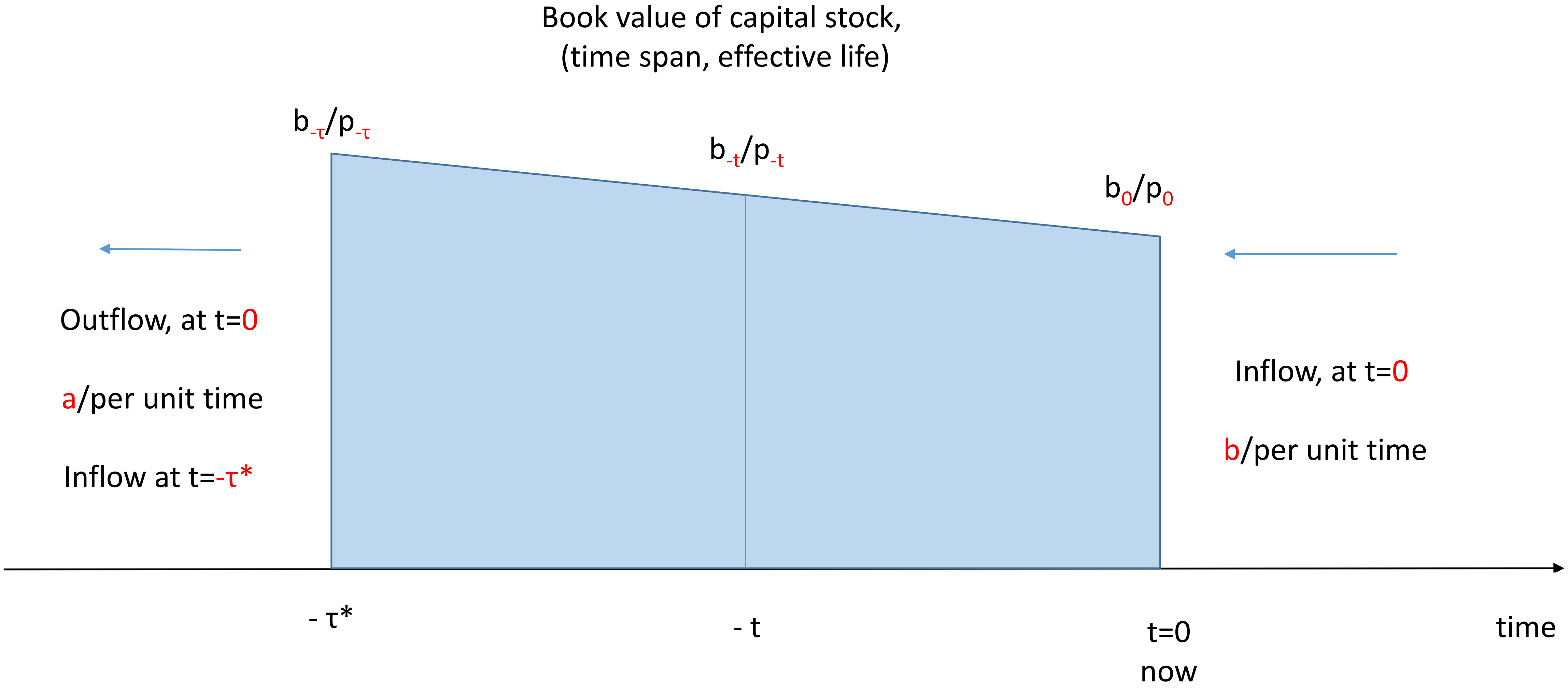
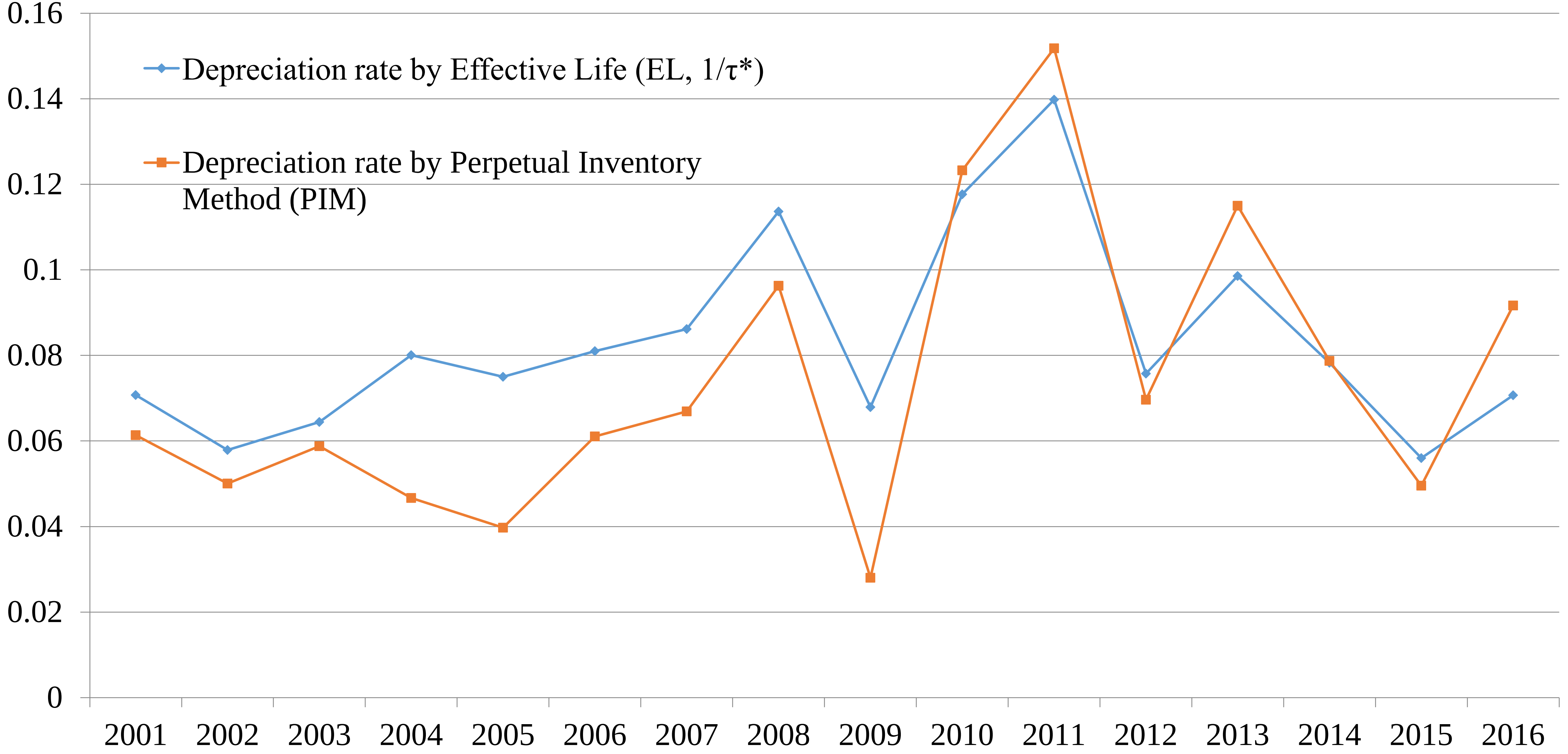
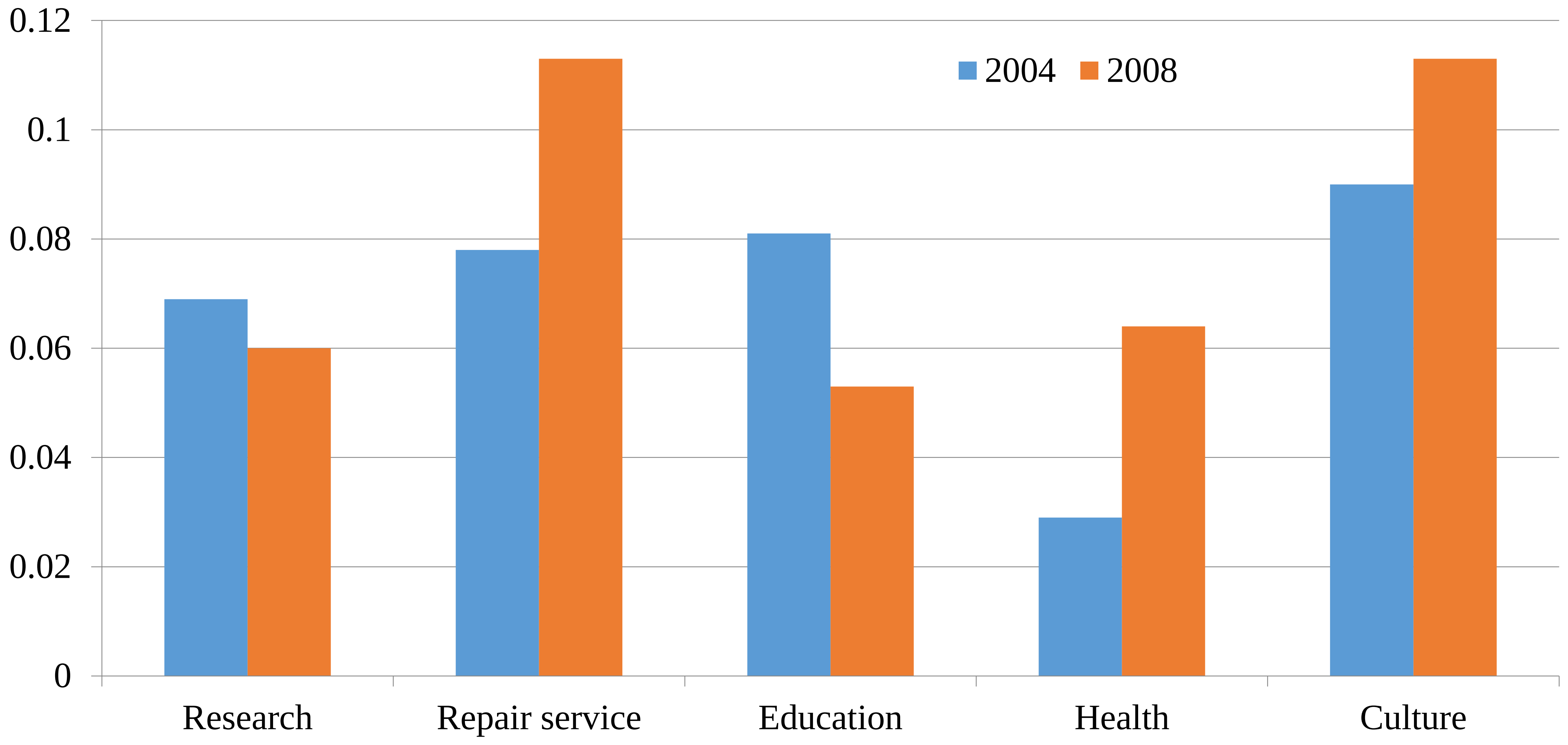


Figure 4A: Depreciation rates of 37 industrial sectors as indicated by the effective lives and the PIM [Qiu and Wan (2019)]



Source: Author's estimations based on data from the National Bureau of Statistics of China

Figure 4B: Depreciation rate of five service sectors by effective life, 2004, 2008



Source: Author's estimations based on Economic Census in China, 2004, 2008