CAES Working Paper Series

Bubble Occurrence and Landing

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July 19, 2021

WP-2021-009

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[1] This research was partially supported by the China National Natural Science Foundation - Peking University Data Center for Management Science (grant no. 2016KEY05) and a JSPS KAKENHI Grant (no. 16K03764). The author gratefully acknowledges the support of these funds.

[2] The author thanks Masahiro Abiru, Charles Yuji Horioka, Takuma Kunieda, Justin Yifu Lin, Ko Nishihara, Masao Ogaki, Kazuo Ogawa, Masaya Sakuragawa, Eden Siu Hung Yu, and other meeting participants for their useful comments and encouragement when this paper was presented at the Japanese Economic Association 2018 Spring Meeting at Hyogo University, the Asia-Pacific Economic Association 2018 Annual Meeting at the University of South California; and at various universities including the Central University of Economics and Finance, Chu Hai College of Higher Education, Ming Chuan, Fukuoka, Kansai, Kyushu, Osaka, Peking, and Tokyo. Any remaining errors are the author's responsibility. Correspondence: Nanakuma 8-19-1, Jounan Ward, Fukuoka City, Fukuoka 8140180, Japan; (e-mail) wan@fukuoka-u.ac.jp; (tel.) +81-92-871-6631 (ext. 4208); (fax) +81-92-864-2904.

Highlights

1. The lending rate to South Sea stock investors increased as the stock price rose and was significantly higher than the interest rate imposed on merchants. The South Sea bubble revealed by the bubble test serves as a quasi-natural experiment that identifies the difference between the lending rate and the normal interest rate as the bubble premium.

2. Over finite time or zero time (i.e., timelessness), the necessary and sufficient conditions for a rational bubble are a bubble premium and sufficient investors. Asset-holding taxes (e.g., property taxes) cannot prevent such bubbles.

3. Repeatedly, rational bubbles grow faster than do risk-free assets because bubble premiums are required by risk-neutral investors. Capital gains taxes and the Tobin tax can prevent bubbles and guide existing bubbles to hard or soft landings.

4. The bubble premium solves the equity premium puzzle associated with a rational bubble framework and a risk-neutral investor.

Abstract

First, the bubble test reveals that a bubble was indeed present with respect to the monthly stock prices of the South Sea Company from 1718 to 1722. The bank lending rate to an investor significantly increased with the stock price; it was significantly higher than the lending rate to merchants. The South Sea Bubble serves as a quasi-natural experiment that identifies the difference between the two lending rates as the bubble premium. Second, over finite time or zero time (i.e., timelessness), rational bubbles occur; the necessary and sufficient conditions are bubble premiums and sufficient numbers of investors. Asset-holding taxes (e.g., property taxes) cannot prevent such bubbles. Third, over various periods, a rational bubble grows faster than does a risk-free asset because a bubble premium is required by a risk-neutral investor. Capital gains taxes and the Tobin tax can prevent bubbles and guide existing bubbles to hard or soft landings. Finally, the bubble premium explains the equity premium puzzle associated with a rational bubble framework and a risk-neutral investor.

JEL classification: D46, D84, G18

Keywords: Bubble premium, South Sea Bubble, Rational bubble, Bubble prevention, Tax, Equity premium puzzle, Timelessness

1 Introduction

It is unclear why housing bubbles occur in the U.K. and Singapore where most land is leasehold (Barrell, Kirby, and Riley 2004; Zhou and Sornette 2003; Jiang, Phillips, and Yu 2015). By their nature, leases end. Conventional asset-pricing theory predicts the absence of rational bubbles. Here, I theoretically explain this puzzling fact by showing that within a finite time or zero time (i.e., when timelessness applicable), rational bubbles occur. The necessary and sufficient conditions are a bubble premium and sufficient investors. This work contributes to recent debates regarding whether the housing markets of the U.K. and Singapore exhibit rational bubbles. Giglio, Maggiori, and Stroebel (2016) compared the prices of leasehold properties (with finite maturities) and freehold properties (with infinite maturities); they found no evidence of failure concerning the transversality condition. Thus, they concluded that the U.K. and Singapore housing markets lacked classic rational bubbles. However, Domeij and Ellingsen (DE, 2020) commented that the leasehold property bubble rule cannot be violated; if the prices of the two land types do not differ, no classic rational bubble can occur. Giglio, Maggiori, and Stroebel (2020) replied: "*DE's note highlights a central problem of rational bubble theory, namely that it requires assumptions strong enough that the bubble cannot arise on finite-maturity assets; it does not pose a problem to our test of that theory*." Hence, there is a need to model the bubble of an asset with finite maturity (such as leasehold land) when seeking to solve the central problem of rational bubble theory. That is my topic here.

The literature contains extensive debate on bubbles. A bubble has been well-defined theoretically (Samuelson 1958; Diamond 1965; Harrison and Kreps; 1978; Tirole

1982; 1985; Weil 1987; Diba and Grossman 1988; Santos and Woodford 1997; Abreu and Brunnermeier 2003). However, a bubble is empirically viewed in two ways. For example, Shiller (1981) found a bubble in the U.S. stock market, whereas Fama (2014, line 41 on p.1,475) argued "*All this is consistent with an efficient market in which the term "bubble," at least as commonly used, has no content*." Nonetheless, Fama and French (1987) reported that the market portfolios of U.S. stocks yielded negative returns. Thus, the empirical definition of a bubble is not settled.

Returning to the South Sea Company event, a bubble must have content. *"For a further example of an outside destabilizing speculator who bought high and sold low, there is the edifying history of a great Master of the Mint, Isaac Newton, a scientist and presumably rational. In the spring of 1720 he stated: "I can calculate the motions of the heavenly bodies, but not the madness of people." On April 20, accordingly, he sold out his shares in the South Sea Company at a solid 100 percent profit for ₤7,000. Unhappily, a further impulse later seized him, an infection from the mania gripping the world that spring and summer. He reentered the market at the top for a larger amount and ended up losing ₤20,000. In the irrational habit of so many of us who experience disaster, he put it out of his mind and never, for the rest of his life, could he bear to hear the name South Sea* (Kindleberger 1978, p. 31)." Therefore, asset market bubbles cannot be excluded in the real world.

In theory, a bubble is undesirable because it compromises economic stability and efficiency (Xiong 2013; Hirano, Inaba, and Yanagawa 2015; Miao, Wang, and Zhou

2015).[3] There is a need to prevent new bubbles and "burst" existing bubbles (Wan 2018a). Although Wan (2018a) stated that infinite asset usage was a necessary precondition for a rational bubble, serious housing bubbles have arisen in Hong Kong (Yiu, Yu, and Jin 2013; Huang and Shen 2017) and mainland China (Wan 2015; 2018b), despite fixed periods of allowed land use in these locations. A new theory is therefore required to explain bubbles associated with time-constrained assets.

I will extend Wan (2018a) to a framework that accepts finite time or timelessness (i.e., the extreme case). First, the rational bubble crash risk described by Blanchard (1979) and Weil (1987) is explicitly modelled as a bubble premium, thus providing market evaluation of a loss default. After empirical confirmation that the monthly stock price of the South Sea Company exhibited a bubble, I use the difference between the lending rates to South Sea investors and merchants to empirically identify the bubble premium. Second, I shorten the time span used by Wan (2018a) to timelessness and increase the number of investors to infinity. By incorporating the bubble premium into intra-temporal (or inter-investor) arbitrage, I model a rational bubble in finite time or timelessness; I also show that the necessary and sufficient conditions for a bubble are a strictly positive bubble premium and a sufficient number of investors. Third, I discuss whether bubbles can be prevented. Finally, I consider the relationship between the bubble premium and the equity premium puzzle.

My theory differs from the theory associated with the option value of resale

[3] Bubbles harm the balance sheets of firms, banks (Ogawa 2009; Wan 2018b; Martin, Moral-Benito, and Schmitz 2021; Sakuragawa 2021), and households (Ogawa and Wan 2007; Wan 2015, 2021).

(Xiong 2013; Bordalo, Gennaioli, Kwon, and Shleifer 2020); this does not readily address timeless bubble growth. Because my bubble has a finite time horizon, my theory also differs from the theories published by Conlon (2004) (who introduced asymmetric information) and Doblas-Madrid (2014) (who required multidimensional uncertainty). My model could be viewed as a rational Ponzi game as defined by O'Connell and Zeldes (1988), with additional considerations of timelessness and a bubble premium. Perhaps the descriptor "Ponzi premium model" is appropriate. My model assumes that risk-neutral investors have identical information regarding the speculative investment.

The rest of the paper is organized as follows. Section 2 describes the South Sea Bubble and empirically identifies the bubble premium. Section 3 presents the basic model of a rational bubble in finite time or timelessness, then discusses bubble prevention and landing. Section 4 explores the relationship between the bubble premium and the equity premium puzzle. Section 5 contains concluding remarks.

2 South Sea Bubble vs. Bubble Premium

2.1 The Bubble Test

I obtained monthly data regarding South Sea stock from 1718 to 1722 from Temin and Voth (2006), then plotted the data in Figure 1. Because the South Sea dividend was fixed at approximately 5%, the ratio of the stock price to the dividend should move in a manner consistent with the stock price. Hence, the stock price can be used to directly search for a bubble. In accordance with the approach used by Phillips, Shi and Yu (2015), I performed a bubble test and present the empirical results in Table 1. The monthly stock prices of the South Sea Company exhibited a bubble.

[Insert Figure 1 here.]

[Insert Table 1 here.]

2.2 Identifying the Bubble Premium

I obtained data regarding the lending rates to South Sea stock investors imposed by Hoare's Bank from Temin and Voth (2004, Table 7, and description at p. 1665); the lending rate, loan value, market price, and discount rate data were available. I obtained data concerning the assets of Hoare's Bank from Temin and Voth (2006, Figure 2). The bank invested a peak of 14% of its assets in South Sea stock on June 24 1720; the proportions invested in other years were much lower. The government bond rate was 5%; the mean interest rate imposed by Hoare's Bank from 1702 to 1713 was approximately 6% and fell to approximately 5% from 1714 to 1722 (Temin and Voth, 2006). The Hoare's Bank lending rates to South Sea stock investors and the mean interest rates imposed on merchants from 1719 to 1720 are shown in Figure 2.

[Insert Figure 2 here.]

It is clear that the lending rate for South Sea investors increased with the stock price. In accordance with the approach used by Wan (2018a), I define the difference between the lending rate to the bubble investor and the risk-free interest rate as the bubble premium; this is fundamentally based on the bubble burst risk described by Blanchard (1979) and the loss after default. The South Sea bubble, as well as the different lending rates imposed by Hoare's Bank, serve as a quasi-natural experiment that empirically identifies the bubble premium. This premium is calculated using Hoare's Bank information. The bubble size is roughly equal to the stock price from which the fundamental value (approximately 100 pounds) is subtracted. The bubble premium and size are plotted in Figure 3. A clear positive relationship is evident between the bubble premium and bubble size.

[Insert Figure 3 here.]

Next, I formally tested the bubble premium hypothesis. The null hypothesis was that the bubble premium should not increase as the bubble grows. I summarize the required statistics in Table 2; I then engaged in linear estimation using the ordinary least squares method with robust standard errors. The empirical results are shown in Table 3. The bubble coefficient is clearly positive; the null hypothesis is rejected. This implies that the alternative hypothesis (the bubble premium increases as the bubble grows) is supported.

[Insert Table 2 here.]

[Insert Table 3 here.]

3 Necessary Conditions for a Timeless Rational Bubble

3.1 The Bubble Premium and a Timeless Rational Bubble

Blanchard and Watson (1982) outlined the basic framework of a rational bubble. Assume that there are two assets in an economy; one is safe and the other is risky (examples: land and stock). Assume that arbitrage is lacking; all assets are privately owned. An investor or asset owner considers the following risky asset-trading problem:

r : the interest rate of a risk-free asset;

d_t : the dividend of the risky asset at time t;

p_t : the market price of the risky asset at time t.

Additionally, assume that r is consistently positive over the time horizon. Unlike the risk-averse investor described by Merton (1973), a risk-neutral investor freely chooses to invest in a risk-free or risky asset [consistent with Wan (2018a)]. Furthermore, at the initial time t, a large or infinite number of investors are assumed to be in the market (Figure 4). The risk is created by both the infinite time horizon and the infinite number of investors; these are "*certain uncertainties*." "Uncertainty" arises because it is unclear when time will end or the investors will disappear; the uncertainty is "*certain*" because every investor understands these risks.

Because infinity thus has two dimensions, I first consider the infinite number of investors. For simplicity, I assume that the time span is strictly zero (or the scenario is timeless). In this scenario, the interest rate of a risk-free asset should be strictly zero, and no bubble should occur from the perspective used by Blanchard and Watson (1982). Hence, I introduce the empirically supported South Sea bubble premium when modeling a timeless rational bubble [consistent with Wan (2018a) and pure speculation in Weil (1987)]. I make a new assumption concerning the timeless bubble premium:

Assumption 1:

The bubble premium, γ_0, is strictly positive over strictly zero time and timelessness.

The bubble premium over strictly zero time γ_0 is attributable to trading; there is no default or crash if there is no trade. Under an intra-temporal (or inter-investor) no-arbitrage condition, the following equation should be satisfied when the market is at equilibrium and the investor is a sequential trader:

$$1 = \frac{E_n[p_{n+1}]}{p_n} - \gamma_0 E_n\left[\frac{p_n - f}{p_n}\right], \tag{1}$$

where E_n is the expectation operator of the *n-th* investor, $1 \leq n \ll \infty$, and *f=d/r*. For simplicity, assume that $E_t[d_{t+1}] = E_t[d_{t+j}]$ for any $j \in [1, \infty)$. The forward-looking solution of p_n in Equation (1) is:

$$p_n = \frac{E_n[p_{n+1}]}{1+\gamma_0} + \frac{\gamma_0}{1+\gamma_0}\mathrm{f}, \tag{2}$$

where $1 \leq m < \infty$ as shown in Figure 4; when $m \to \infty$, the result becomes:

$$p_n = f + \lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right]. \tag{3}$$

The first and second terms on the right of Equation (3) are the fundamental value of the income gain and the bubble term of the risky asset, respectively. The bubble term is also known as the transversality condition; a rational bubble develops if and only if:

$$\lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right] > 0. \tag{4}$$

In the literature [Miller and Modigliani (1961) and Montrucchio (2004)], the following condition must be satisfied to violate a rational bubble:

$$\lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right] = 0. \tag{5}$$

However, some cases either satisfy or violate Equation (5). I assume that the growth rate of the p_{n+m} expectation is g_0, where g_0 embraces the speculative aspects of an investor (i.e., aspects independent of the dividend d). The speculative investor believes that the asset will be bought by a new speculator, although the bubble is well-known by all investors. I then have:

$$p_{n+m} = (1+g_0)^m, \tag{6}$$

where:

$$\lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right] \to 0, for\ g_0 < \gamma_0\ , \tag{7}$$

$$\lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right] \to 1, for\ g_0 = \gamma_0\ , \tag{8}$$

$$\lim_{m\to\infty} E_n\left[\frac{p_{n+m}}{(1+\gamma_0)^m}\right] \to \infty, for\ g_0 > \gamma_0. \tag{9}$$

Equations (7), (8), and (9) are applicable when there is no bubble, a concurrent (co-existing) bubble, and an explosive bubble, respectively. An explosive bubble violates the no-arbitrage condition defined by Equation (1). This yields:

Theorem 1:

A rational bubble occurs over strictly zero time or in a timeless manner if and only if a strictly positive bubble premium is levied and a sufficient number of investors are in the market in a no-arbitrage environment.

Proof:

See Equations (7), (8) and (9). Q.E.D.

Most residential property in the U.K., Singapore, Hong Kong, and mainland China is leasehold (i.e., of finite maturity). However, bubbles have occurred in these locations. According to ***Theorem 1***, a fixed period of land usage (such as a lease) does not violate the bubble condition. This explains why Giglio, Maggiori, and Stroebel (2016; 2020) and Domej and Ellingsen (2020) found no significant differences in leasehold and freehold property prices. Both such properties feature bubbles.

Furthermore, high-frequency trading in an asset market [as described by Brogaard, Hendershott, and Riordan (2014) and O'Hara (2015)] can be presumed to proceed over strictly zero time or be timeless. My model will usefully analyze a situation in which a trader or a machine-like artificial intelligence is seeking capital gain alone, rather than income gain (which requires time).

3.2 Rational Bubbles over Multiple Periods

I extend the above model over strictly zero time to multiple periods in Figure 4. Assume that m ($1 \leqq m << \infty$) incidences of asset turnover occur at the initial point of each period. Equation (6) shows that the bubble within each period is:

$$b_m = (1 + g_0)^m. \tag{10}$$

To capture the credit risk imposed by a bubble, I make another assumption concerning the risk of a bubble crash.

Assumption 2:

The bubble premium, γ, is strictly positive within any period.

For any period, the bubble premium is a function of time if ***Assumption 1*** is correct; $\gamma = f(\gamma_{0,} time)$. For a single period, $\gamma = f(\gamma_{0,} 1)$. Additionally, assume that a bubble satisfying the intra-temporal non-arbitrage condition of Equation (10) also satisfies the following inter-temporal non-arbitrage condition described by Wan (2018a):

$$1 + r = \frac{E_t[p_{t+1} + d_{t+1}]}{p_t} - \gamma\, E_t\left[\frac{p_t - d_{t+1}/r}{p_t}\right]. \tag{11}$$

If $E_t[d_{t+1}] = E_t[d_{t+j}]$ for any $j \in [1, \infty)$, then the forward-looking solution of Equation (11) becomes:

$$p_t = E_t\left[\frac{d_{t+1}}{r}\right] + \underset{T\to\infty}{E_t}\left[\frac{p_{t+T}}{(1 + r + \gamma)^T}\right], \tag{12}$$

and the no-bubble condition of Equation (11) is now:

$$\underset{T\to\infty}{E_t}\left[\frac{p_{t+T}}{(1 + r + \gamma)^T}\right] = 0\,. \tag{13}$$

The bubble premium parameter γ does not impact the fundamental term, although it impacts the bubble term. I assume that the bubble grows at a (positive) constant rate over time:

$$p_{t+T} = (1 + g_b)^T, \tag{14}$$

where $1 + g_b = b_m = (1 + g_0)^m$; then, $g_b = b_m - 1 = (1 + g_0)^m - 1$. This yields:

$$\underset{T\to\infty}{E_t}\left[\frac{p_{t+T}}{(1+r+\gamma)^T}\right] \to 0, \text{ for } g_b < r+\gamma, \tag{15}$$

$$\underset{T\to\infty}{E_t}\left[\frac{p_{t+T}}{(1+r+\gamma)^T}\right] = 1, \text{ for } g_b = r+\gamma, \tag{16}$$

$$\underset{T\to\infty}{E_t}\left[\frac{p_{t+T}}{(1+r+\gamma)^T}\right] \to \infty, \text{ for } g_b > r+\gamma. \tag{17}$$

Equations (15), (16), and (17) are the newly applicable conditions when there is no bubble, a concurrent (co-existing) bubble, and an explosive bubble, respectively. Clearly, the growth rate of the bubble is higher than the growth rate of the risk-free asset; the ratio of the growth rate of the bubble to the growth rate of the risk-free asset increases over time. Thus, the bubble term increases with time when a bubble is present. In the standard bubble model, as well as in a model of a risk-averse investor who seeks a risk premium (Wan 2018a), the bubble portion is unchanged. The above exercise plausibly explains why bubbles grow faster than do risk-free assets at both the country and city levels [Wan (2018b)]. Because Equation (17) violates the inter-temporal no-arbitrage condition of Eq. (11), the following bubble is generated that simultaneously satisfies the intra- and inter-temporal non-arbitrage conditions and a new property:

Proposition 1:

The bubble grows at a rate of $(1 + \gamma_0)^m$ within one period; after growth over T

periods to time *T*, the bubble will be $(1+\gamma_0)^{mT}$ if and only if $\gamma = (1+\gamma_0)^m - (1+r)$.

Proof:

By linking Equation (14) to Equation (16), the following equation is produced: $g_b = b_m - 1 = (1+g_0)^m - 1 = r + \gamma;$ this yields the bubble at time *T*.

Q.E.D.

The *m* parameter is the number of risky asset turnovers and thus reflects the extent of speculation. If the bubble premium γ is not sufficiently high, the no-bubble condition of Equation (15) is easily violated, and a bubble occurs. To prevent such a bubble, I now discuss whether the tax tools described by Wan (2018a) might be effective.

3.3 Prevention of a Timeless Rational Bubble

Over strictly zero time, what tax regime might prevent a bubble? Consider the following:

Proposition 2:

An asset-holding tax cannot prevent a timeless rational bubble, whereas a capital gains tax and/or a Tobin tax can.

Proof:

Because an asset-holding tax increases with time, my strict zero-time scenario means that no tax can be levied. If a capital gains tax of rate of $\kappa \in [0, 1)$ is included in Equation (1), the following intra-temporal no-arbitrage condition is generated, combined with that tax:

$$1 = \frac{E_n[p_{n+1}]}{p_n} - \gamma_0 E_n\left[\frac{p_n - f}{p_n}\right] - \kappa\frac{E_n[p_{n+1} - p_n]}{p_n}. \tag{18}$$

The forward-looking solution of p_n of Equation (18) is:

$$p_n = f + \lim_{m\to\infty} E_n\left[\frac{(1+g_0)(1-\kappa)}{(1+\gamma_0)-\kappa}\right]^m, \tag{19}$$

$$\lim_{m\to\infty} E_n\left[\frac{(1+\gamma_0)(1-\kappa)}{(1+\gamma_0)-\kappa}\right]^m \to 0, for\ \kappa \in \left(\frac{g_0-\gamma_0}{g_0}, 1\right). \tag{20}$$

Next, consider a transaction or Tobin (1974) tax of rate $\tau = \in (0,1)$; this yields:

$$1 = \frac{E_n[p_{n+1}]}{p_n} - \gamma_0 E_n\left[\frac{p_n - f}{p_n}\right] - \tau\frac{E_n[p_{n+1}]}{p_n}\ , \tag{21}$$

and the solution of p_n of Equation (21) is:

$$p_n = \frac{\gamma_0}{\gamma_0+\tau} f + \lim_{m\to\infty} E_n\left[\frac{(1+g_0)(1-\tau)}{(1+\gamma_0)}\right]^m, \tag{22}$$

$$\lim_{m\to\infty} E_n\left[\frac{(1+g_0)(1-\tau)}{(1+\gamma_0)}\right]^m \to 0, for\ \tau \in \left(\frac{g_0-\gamma_0}{1+g_0}, 1\right). \tag{23}$$

Q.E.D.

Asset-holding taxes such as a property tax will not prevent an intra-temporal bubble; a capital gains tax and a Tobin tax will prevent a bubble, consistent with the findings by Tobin (1974) and Stiglitz (1989). Note that the transaction tax rate can be lower than the capital gains tax rate; the Tobin tax decreases the fundamental value, whereas a capital gains tax does not.

3.4 Bubble Prevention over Multiple Periods

For multiple periods, consider the following:

Proposition 3:

Capital gains and asset holding taxes can prevent bubble occurrence.

Proof:

Assume that *m* ($1 \leqq m << \infty$) turnovers occur at the beginning of each period and that *f=0* for simplicity. According to Eq. (19), the bubble at the initial point of one period (*t*) when a capital gains tax of rate $\kappa \in [0, 1)$ is present is:

$$p_{(n)(t)} = E_n \left[\frac{(1-\kappa)}{(1+\gamma_0)-\kappa}\right]^m p_{(n+m)(t)}, \tag{24}$$

equivalent to:

$$p_{(n+m)(t)} = \left[\frac{(1+\gamma_0)-\kappa}{1-\kappa}\right]^m p_{(n)(t)}. \tag{25}$$

To capture the bubble credit risk, add the bubble premium to the model and then obtain:

$$1 + r = \frac{E_t[p_{t+1}]}{p_t} - \gamma \frac{E_t[p_t - d_{t+1}/r]}{p_t} - \kappa \frac{E_t[p_{t+1} - p_t]}{p_t} \ . \tag{26}$$

p_t is:

$$p_t = \lim_{T\to\infty} E_t \left[\left(\frac{1-\kappa}{1+r+\gamma-\kappa}\right)^T p_{t+T}\right], \tag{27}$$

for $\kappa \in \left(\frac{g_b-(r+\gamma)}{g_b}, 1\right)$ and $g_b = \left[\frac{1+\gamma_0-\kappa}{1-\kappa}\right]^m - 1$, then $p_t = \lim_{T\to\infty} E_t \left[\left(\frac{1-\kappa}{1+r+\gamma-\kappa}\right)^T p_{t+T}\right] \to 0$.

Next, consider a transaction tax of rate $\tau \in (0,1)$ and obtain:

$$1 + r = \frac{E_t[p_{t+1}]}{p_t} - \gamma \frac{E_t[p_t - d_{t+1}/r]}{p_t} - \tau \frac{E_t[p_{t+1}]}{p_t} \ . \tag{28}$$

p_t is:

$$p_t = \lim_{T\to\infty} E_t\left[\left(\frac{(1-\tau)}{1+r+\gamma}\right)^T p_{t+T}\right], \tag{29}$$

for $\tau \in \left(\frac{g_b-(r+\gamma)}{1+g_b}, 1\right)$ and $g_b = \left[\frac{1+\gamma_0}{1-\tau}\right]^m - 1$ by Eq. (22), then $p_t = \lim_{T\to\infty} E_t\left[\frac{(1-\tau)(1+g_b)}{1+r+\gamma}\right]^T \to 0$.

For an asset-holding tax of rate $\mu \in (0,1)$, obtain:

$$1 + r = \frac{E_t[p_{t+1}]}{p_t} - \gamma\frac{E_t[p_t - d_{t+1}/r]}{p_t} - \mu\frac{E_t[p_{t+1}]}{p_t}\ . \tag{30}$$

p_t is:

$$p_t = \lim_{T\to\infty} E_t\left[\left(\frac{(1-\mu)}{1+r+\gamma}\right)^T p_{t+T}\right], \tag{31}$$

for $\mu \in \left(\frac{g_b-(r+\gamma)}{1+g_b}, 1\right)$ and $g_b = [1+\gamma_0]^m - 1$ by Eq. (3), then $p_t = \lim_{T\to\infty} E_t\left[\frac{(1-\mu)(1+g_b)}{1+r+\gamma}\right]^T \to 0$.

Q.E.D.

3.5 Hard and Soft Landings in Strictly Zero Time and within Multiple Periods

To hard-land a bubble that exists wholly within a single period, the Tobin tax and the capital gains tax described by Wan (2018a) are effective, but an asset-holding tax is ineffective because the holding time is limited or strictly zero. For a bubble that exists within multiple periods, the hard and soft landing policies differ from the policies described by Wan (2018a). To ensure a soft landing within a single period, the approach also differs from the approach used by Wan (2018a); if a soft landing is required within multiple periods, the approach is similar to the approach used by Wan (2018a).

4 The Bubble Premium and the Equity Premium Puzzle

4.1 The Bubble Premium as an Equity Premium

Rewrite Equation (12):

$$r = E_t\left[\frac{d_{t+1}}{p_t}\right] + \lim_{T\to\infty} E_t\left[\frac{rp_{t+T}}{p_t\,(1+r+\gamma)^T}\right]. \tag{32}$$

The first and second terms on the right of Equation (32) are the bubble yield curve and gain, respectively, and the risk-free interest rate is r. Wan (2018c) used information from the U.S.A. and Japan to test Equation (32). Wan (2018c) identified bubbles in the asset markets of China, Greece, Japan, and the U.S.A., as revealed by:

$$r - E_t\left[\frac{d_{t+1}}{p_t}\right] = \lim_{T\to\infty} E_t\left[\frac{rp_{t+T}}{p_t\,(1+r+\gamma)^T}\right] > 0. \tag{33}$$

Equation (33) was empirically supported by Wan (2018c). This implies:

Lemma 1:

The return of a risk-free asset can be higher than the dividend of a risky asset, and the difference is the bubble premium of a risky asset held by a risk-neutral investor.

Proof:

$$\lim_{T\to\infty} E_t\left[\frac{rp_{t+T}}{p_t\,(1+r+\gamma)^T}\right] = r\,\frac{bubble}{p_t} > 0$$

$$\text{if bubble} \equiv \lim_{T\to\infty} E_t\,[\frac{p_{t+T}}{(1+r+\gamma)^T}]>0. \tag{34}$$

Q.E.D.

The yearly dividend stock market yields and the yearly interest rates in the U.S.A. and Japan were described by Wan (2018c, Figures 4 and 5); the null hypothesis (dividend yield equals the interest rate) was rejected at $p = 0.003$. For Japan, the same null hypothesis was rejected at $p = 0.000$ by Wan (2018c). For both the U.S.A. and Japan, the alternative hypotheses (the dividend yield is significantly lower than the interest rate) were accepted. Furthermore, the bubble test used by Phillips, Shi and Yu (2015) and the empirical results described by Wan (2018c, Table 2c) revealed bubbles in the stock prices of both the U.S.A. and Japan. The empirical bubbles in the U.S. stock market were consistent with bubbles reported by White and Rappoport (1993, 1994).

For the bubble of Equation (3), which does not depend on time, the risk-free interest rate should be sufficiently small to be negligible; Equation (3) can then be rewritten:

$$1 = E_n\left[\frac{f}{p_n}\right] + \lim_{m\to\infty} E_t\left[\frac{p_{n+m}}{p_n\,(1+\gamma_0)^m}\right], \tag{35}$$

and Equation (19) shows the conditions required for a rational bubble. Wan (2018c) tested this approach using the daily Bitcoin prices from April 28 2013 to April 24 2018; a bubble was present. Because the Bitcoin dividend is always zero, the price increase was entirely attributable to the bubble; Equation (33) indicates the precise risk-free interest rate.

Because stock prices have increased over 100-fold during the sampled periods in the U.S.A. and Japan, the summed dividend yields, and the ratio of the yearly capital gain to the stock price, became higher than the interest rate used by Wan (2018c). The capital gain is attributable (at least in part) to the stock bubble, implying that the higher return of stock (dividends plus capital gain) includes the bubble premium. In an extreme case of the Miller and Modigliani (1961) economy, corporate firms do not distribute dividends forever; the returns of corporate shares are thus exclusively the capital gains in the stock prices (as in the Bitcoin bubble), as argued by Cheah and Fry (2015). A share price may include a bubble, as argued by Shiller (1981); the bubble imposes a bubble premium that can be viewed as an equity premium for a risk-neutral investor. The equity premium puzzle identified by Mehra and Prescott (1985) is then solved because the risk-aversion of a risk-neutral investor is strictly zero.

5 Conclusions

I found bubbles in the monthly stock prices of the South Sea Company from 1718 to 1722. The bank lending rate to South Sea investors significantly increased with the stock price and was significantly higher than the lending rate imposed on merchants. The South Sea Bubble served as a quasi-natural experiment; the difference between the lending rates is the bubble premium. I built a model showing the necessary and sufficient conditions for rational bubble occurrence over strictly zero time and within multiple periods; I found that a bubble premium is necessary if a rational bubble is to exist in finite time or in a timeless manner. A rational bubble grows faster than does a risk-free asset because a bubble premium is required by a risk-neutral investor. An asset-holding tax cannot prevent bubble occurrence over a finite time or in a timeless manner; a capital gains tax and a Tobin tax can. Specific policies facilitate hard- and soft-landing of existing bubbles. Finally, the bubble premium solves the equity premium puzzle raised by a risk-neutral investor.

The empirical and theoretical findings indicate that rational bubbles readily arise, but these bubbles can be prevented by specific taxes and policies. In the future, the bubble premium and rational bubble occurrence will be analyzed using a general equilibrium framework.

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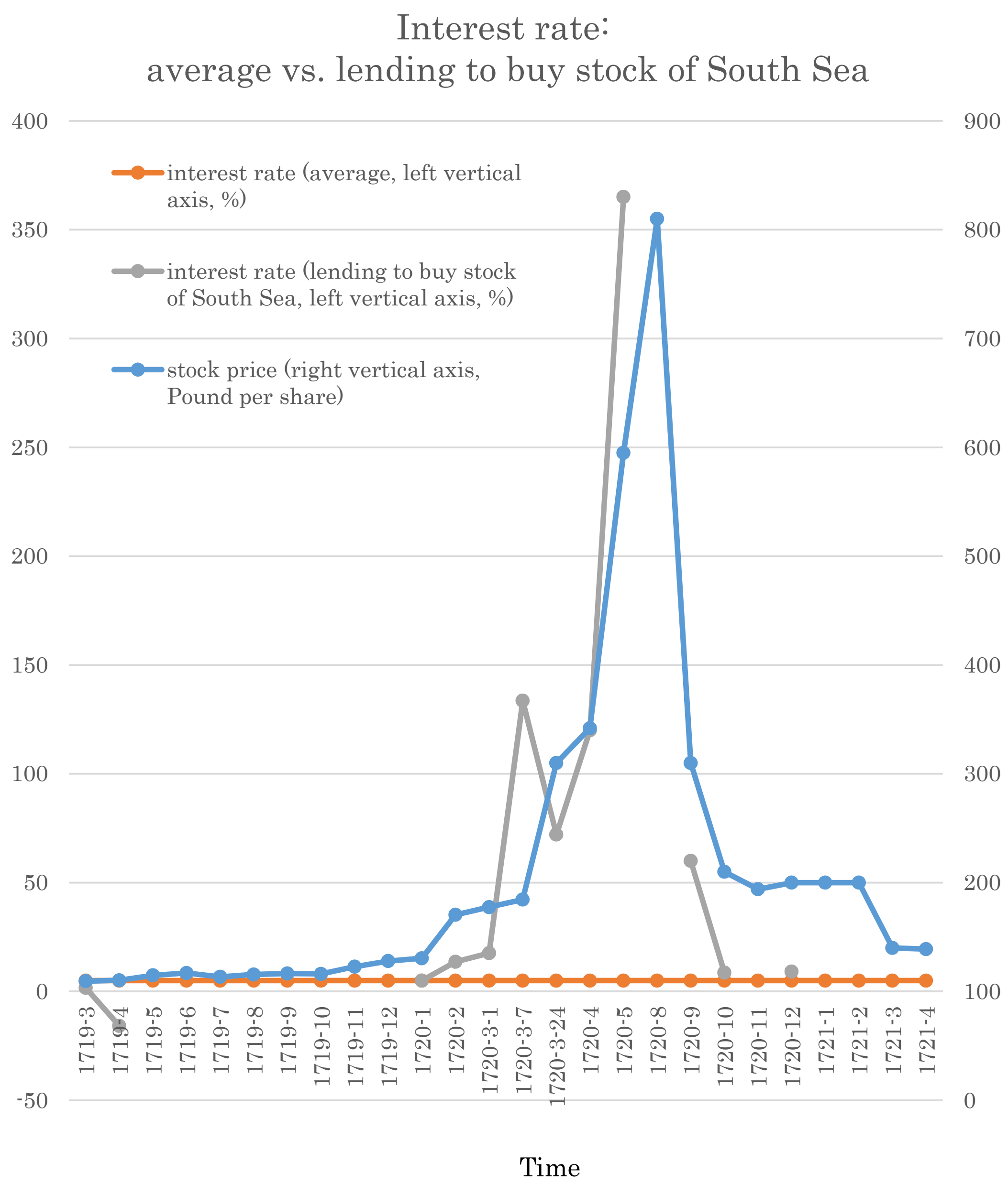
Figure 1: Monthly stock price of the South Sea Company, 1718–1722.

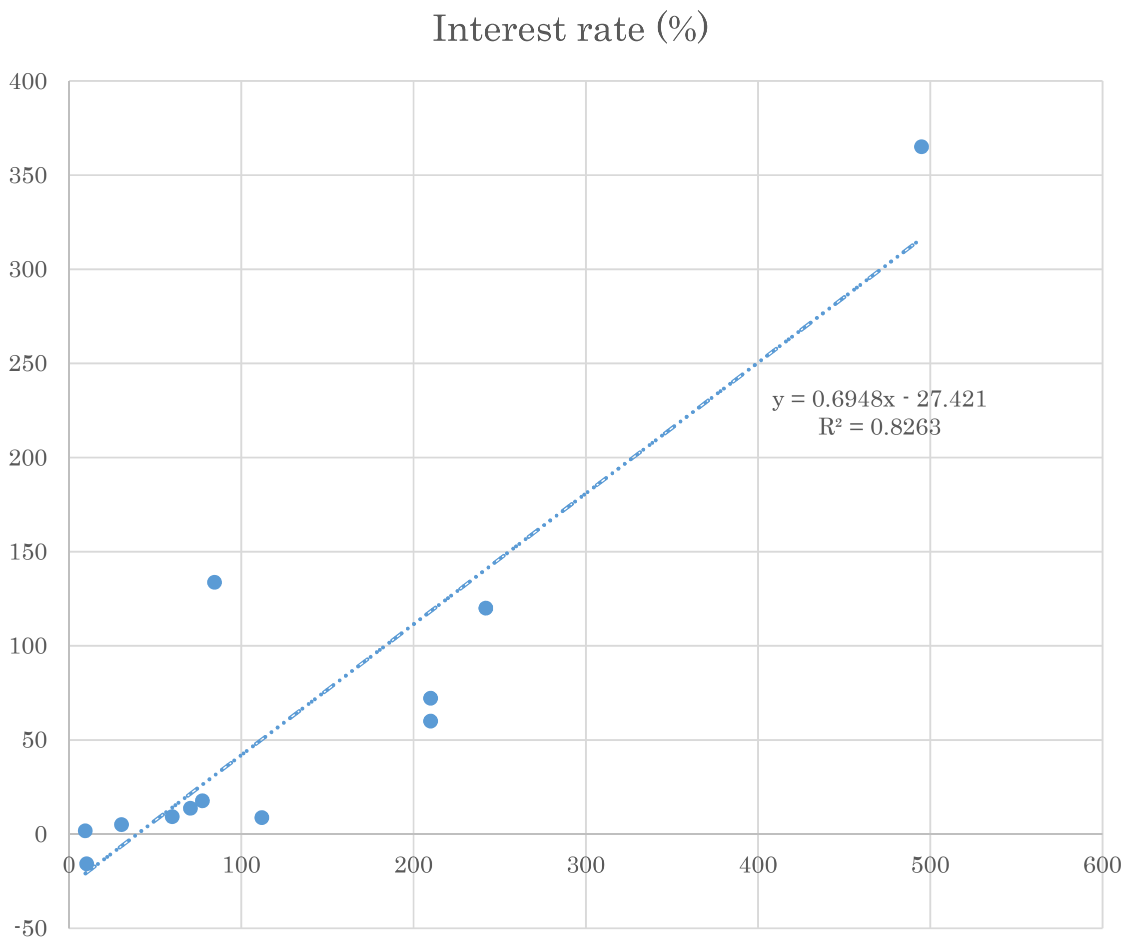
Source: Drawn by the author based on Neal (1990, pp. 234-235).

Figure 2: Mean lending rates to purchasers of South Sea Company stock, 1719–1720.



Source: See the text.

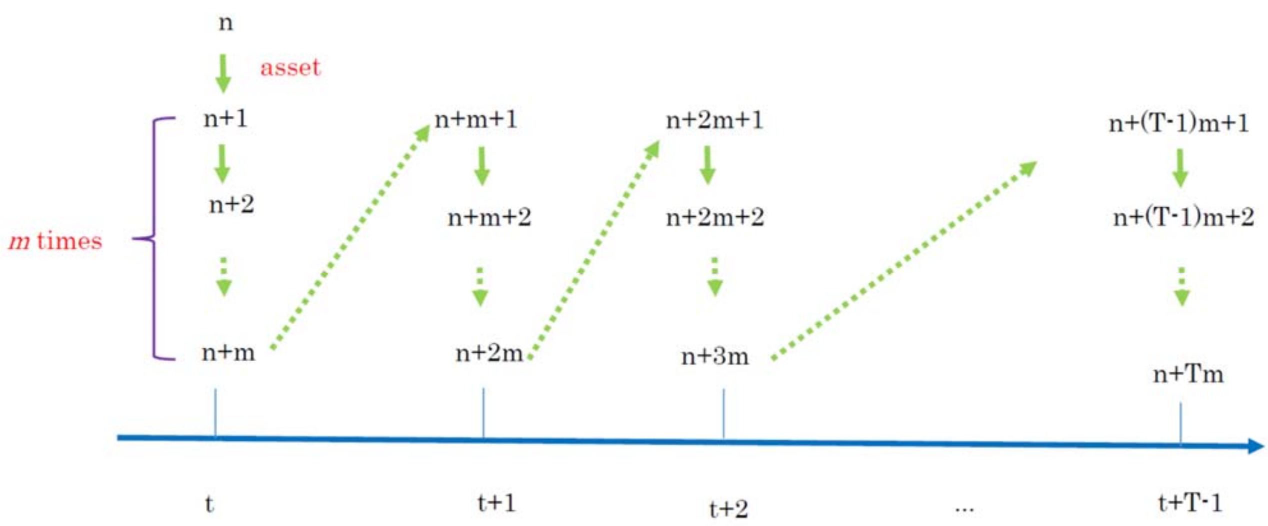
Figure 3: The bubble premium and the bubble, 1719–1720.



Bubble = stock price – fundamental price (unit: £).

Source: See the text.

Figure 4: Turnover of an asset over time.



Source: Drawn by the author.

Table 1: Bubble test of the monthly stock price of the South Sea Company, 1718–1722.

A: Unit root test
Monthly price (Sept. 1718–May 1722, 45 observations)

Null Hypothesis: The series has a unit root.

	t-Statistic	Prob.*
Augmented Dickey–Fuller test statistic	–1.759	0.396

*MacKinnon (1996) one-sided p-values.

B: SADF and GSADF tests of the price.
Null hypothesis: The series has a unit root.

Monthly price (Aug. 1718–May 1722, 46 observations)

	SADF	GSADF
Test statistics	8.919	8.919
p-value	0.000	0.000

Right-tailed test.

Note: Critical values were obtained via Monte Carlo simulation (1,000 replications). The smallest window featured 20 observations. All calculations were performed by the author.

Table 2: Summary statistics of bubble premiums and sizes.

Variable	Obs	Mean	Std. Dev.	Min	Median	Max
Interest rate	12	65.901	105.790	–15.825	15.642	365.000
Bubble	12	134.313	138.399	9.500	81.000	495.000

Table 3: Determinants of bubble premiums (ordinary least squares estimations with robust standard errors).

	(Dependent variable =) Bubble premium
(Independent variables)	
Bubble	0.695
	(0.101)***
Constant	–27.421
	(16.690)
R^2	0.83
N	12

Robust standard errors in parentheses.

* p < 0.1; ** p < 0.05; *** p < 0.01.