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Backward-bending Investment-Saving Curve

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Highlights

1. The real per capita GDP and the real Japanese interest rate trended backward from 1994 to 2021.
2. The negative interest elasticity of saving is revealed in a two-period model, in which an agent maximizes the Leontief, the power, and the constant elasticity of substitution (CES) utility. The effect is also apparent when the agent engages in speculative saving.
3. The interest elasticity of the money demand runs to infinity when the interest rate falls to zero.
4. Speculative saving is incorporated into the investment savings-liquidity preference money supply (IS-LM) model. The IS curve then trends backward, and the LM curve is horizontal around the zero interest rate. Thus, shifting the LM curve to the right and down would downsize the equilibrium GDP.

Abstract

First, the per capita real GDP and real Japanese interest rate trended backward from 1994 to 2021. Second, we use a two-period model to show a negative effect of interest on saving when an agent maximizes the Leontief, the power, and the constant elasticity of substitution (CES) utility. This is also true when the motive for saving is speculative. Third, the interest elasticity of the money demand increases to infinity when the interest rate falls to zero; this would be a micro foundation of the "Liquidity Trap". Fourth, when speculative saving is incorporated into the investment savings-liquidity preference money supply (IS-LM) model, the IS curve trends backward and the LM curve is horizontal around the zero interest rate. Thus, a low-interest policy implemented by shifting the LM curve to the right and down would downsize the equilibrium GDP.

JEL classifications: E00; E10; E12; E21

Keywords: Backward-bending IS Curve, Speculative Saving, Negative Interest Elasticity of Saving, Horizontal LM Curve, Interest Rate

1. Introduction

Japan seems to have experienced three decades of recession for reasons that are unclear despite the comprehensive analyses such as Hayashi and Prescott (2000), Ono (2001), Horioka (2006), Ogawa and Wan (2007), and Ogawa (2022), etc. The past 30 years have been characterized by a low interest policy; such quantitative easing is supposed to bolster the gross domestic product (GDP), allowing the economy to emerge from a long recession. It would be ironic if the low interest policy did not in fact stimulate the economy but, rather, deepened the recession. As shown in Figure 1, the Japanese per capita real GDP and real interest rate (the average saving deposit rate of commercial banks per fiscal year) trended backward from 1994 to 2021. Thus, when the interest rate is below a certain level, the GDP follows the interest rate.

The Japanese recession was triggered by the asset bubble burst in the early 1990s (Ogawa and Wan 2007, Ogawa 2022, Wan 2018b). Japan has taught the world to avoid asset bubbles (Wan 2018a). However, a condominium bubble occurred recently in Japan (Kitasaka 2019). Thus, an economy can experience simultaneous recession, an asset bubble, and a low interest rate. It is difficult to explain why 30 years of recession can feature both a low interest rate policy and a new asset bubble. Here, we link the

recession to the asset bubble,[3] and model the effects of easing monetary policy.

The rest of this paper is organized as follows. Section 2 discusses consumption and the speculative motive identified by Keynes (1936). Section 3 sets out the model. Section 4 presents conclusions and policy implications.

2. Consumption and the Speculative Motive as Viewed by Keynes (1936)

Chapter 8. The Propensity to Consume (Keynes 1936, p. 101): '*Great Britain (1935)…housing-building…require sinking funds sufficient to write off the initial cost some time before replacement will actually fall due…dissociated from any corresponding new investment…repayment of mortgages…and today the repayments are probably still higher…*''

Chapter 9. The Propensity to Consume: The Subjective Factors (Keynes 1936, p. 108): "*Eight motives or objects lead individuals to refrain from spending out of their incomes: (VI) To secure a* masse de manoeuvre *to carry out speculative or business projects…*''

Chapter 12. Long-Term Expectation (Keynes 1936, pp. 158-160): "*…speculation predominates over enterprise…*(p. 158)''; "*… capital appreciation as to a favorable*

[3] The transmission of the effects of a housing bubble to the industrial sector is analyzed by Wan (2021a).

change in the conventional basis of valuation…when the capital development of a country becomes a by-product of activities of a casino, the job is likely to be ill-done (p. 159).'' "*So long as it is open to the individual to employ his wealth in hoarding or lending money, the alternative of purchasing actual capital assets cannot be rendered sufficiently attractive (especially to the man who does not manage the capital assets and knows very little about them)* (p. 160)''

Chapter 13. The General Theory of the Rate of Interest in Keynes (1936, p. 170, p. 173): "…*speculative motive*…'' "Footnote 1: … *He has sufficient reason for holding cash or debts in preference to equities; but the purchase of debts will be preferable alternative to holding cash, unless he also believes that the future rate of interest will prove to be higher that the market is supposing* (p. 170).'' "… *whilst a decline in the rate of interest may be expected, cet. par., to increase the volume of investment, this will not happen…if the propensity to consume is falling off* (p. 173).''

Chapter 16. Sundry Observations on the Nature of Capital (Keynes 1936, p. 220): "… *to dig holes in the ground…negative net yield…*'' "…*the eagerness to obtain a yield from a doubtful investment might be such that they would show in aggregate net yield*.''

Chapter 23. Notes on Mercantilism, etc. (Keynes 1936, p. 370): "…*we cannot*

suppose that when the average rate of incomes is falling, individuals will be induced to increase their rate of consumption by the fact that the premium upon thrift is correspondingly diminished…"

In summary, Keynes (1936) made a major effort to link consumption to speculative asset holding. However, he failed to find a clear relationship.

3. An Explanation of the Backward-bending Investment-Saving (IS) Curve

3.1 The Negative Interest Elasticity of Saving in a Two-period Model

Following Friedman (1957) and Ando and Modigliani (1963), assume that a representative economic agent lives during two periods. The incomes in periods 1 and 2 are Y and Y' respectively. The interest rate is r $(-1 < r < 1)$. The agent needs to choose consumption C for period 1 and consumption C' for period 2 using a Leontief-type utility function lacking a bequest motive. The agent maximizes the following objective function, subject to budget constraints:

$$\text{Max } U(C, C') = \text{Max } \{\text{Min}[U(C), U(\alpha C')]\} \tag{1}$$

$$\text{s.t. } C+C'/(1+r) = Y + Y'/(1+r), \tag{2}$$

where > 0 captures the ratio of commitment to consumption in period 1 to that in period 2. We obtain:

Property 1: The interest elasticity of saving is negative iff $Y > \alpha Y'$.

Proof: The optimal consumption in period 1 is:

$$C^*=[\alpha(1+r)Y+\alpha Y'] / [\alpha(1+r)+1], \tag{3}$$

and the saving is defined by $S \equiv Y-C^*$.

$$\frac{\partial S}{\partial r} = -\frac{\partial C^*}{\partial r} = -\frac{\gamma}{[\alpha(1+r)+1]^2}(Y-\alpha Y'), \tag{4}$$

$$\lesseqgtr 0 \text{ iff } Y \gtreqless \alpha Y'. \tag{5}$$

Q.E.D

We can alternatively use a power utility function with a time discounting rate that equals the interest rate [$Max\ U(C, C')=lnC+lnC'/(1+r)$]. This also yields ***Property 1*** above. Furthermore, assume that the representative economic agent employs a utility function of the Constant Elasticity of Substitution (CES) type. Then, the problem becomes:

$$\text{Max } U(C, C') = (C^{\rho} + \beta C'^{\rho})^{1/\rho} \tag{6}$$

$$\text{s.t. } C+C'/(1+r) = Y + Y'/(1+r),$$

where the parameter $\beta > 0$ captures the relative consumption in the two periods. We

obtain:

Property 2: The interest elasticity of saving is negative iff $(1-\sigma)Y >$ $[\sigma(1+r)^{-1}+\beta^{-\sigma}(1+r)^{\sigma}]Y'$.

Proof: The optimal consumption in period 1 is:

$$C^* = \frac{(1+r)^{-\sigma}}{(1+r)^{1-\sigma}+\beta^{\sigma}}[(1+r)Y+Y'], \tag{7}$$

where $\sigma = \frac{1}{1-\rho}$. The saving is $S \equiv Y - C^*$:

$$\frac{\partial S}{\partial r} = -\frac{\partial C^*}{\partial r}$$

$$= \frac{\sigma(1+r)^{-1-\sigma}[(1+r)^{1-\sigma}+\beta^{\sigma}]+(1-\sigma)(1+r)^{-2\sigma}}{[(1+r)^{1-\sigma}+\beta^{\sigma}]^2}[(1+r)Y+Y'] - \frac{(1+r)^{-\sigma}}{(1+r)^{1-\sigma}+\beta^{\sigma}}Y \tag{8}$$

$$\lesseqgtr 0 \text{ iff } (1-\sigma)Y \gtreqless [\sigma(1+r)^{-1}+\beta^{-\sigma}(1+r)^{\sigma}]Y' \tag{9}$$

$$\lesseqgtr 0 \text{ iff } Y \gtreqless \{\sigma[(1-\sigma)(1+r)]^{-1}+\beta^{-\sigma}(1-\sigma)^{-1}(1+r)^{\sigma}\}Y'$$

$$\text{for } 0 \le \sigma \le 1. \tag{10}$$

Q.E.D

3.2 Saving by a Speculative Motive as Revealed by a Two-Period Model

Following Wan (2015, 2021b), we assume that the agent has an opportunity to invest in a bubble asset L with a return rate γ ($>r>0$) in period 1, and that $Y'=(1+r)Y$.

The agent encounters the following problem:

$$\text{Max } U(C, C') = \text{Max } \ln C + \ln C' \tag{11}$$

$$\text{s.t.} \qquad C+S'+rL = Y \tag{12}$$

$$C'=(1+r)S'+Y'+(1+\gamma)rL, \tag{13}$$

where the saving deposit in period 1 is S'. We combine Eqs. (12) and (13) to obtain:

$$C+C'/(1+r) = Y +Y'/(1+r)+ rL(\gamma-r)/(1+r). \tag{14}$$

We further assume that the ratio of the bubble asset L to the saving deposit S' is θ, and thus obtain $L = \theta S'$ for $0 \leq \theta \leq \infty$. No speculation, some speculation, and complete speculation correspond to $\theta = 0, 0 < \theta < \infty$, and $\theta = \infty$ respectively. We obtain:

Proposition 1: The interest elasticity of saving is negative, and the demand for a bubble asset increases with income but decreases with a rising interest rate; a near-zero interest rate induces an infinite demand for the bubble asset.

Proof: We solve Eq. (11) using Eqs. (12) and (14) to choose the optimal consumptions in periods 1 and 2, and obtain:

$$C^*= Y (2+r+\gamma)/(2+2 \gamma), \tag{15}$$

$$< Y \text{ for } r < \gamma. \tag{16}$$

Note that $C^*=Y$ for $r = \gamma$ if there is no speculation ($L=0$), and that both the speculative motive and speculative opportunity reduce current consumption. The saving is defined by $S \equiv Y - C^*$:

$$\frac{\partial C^*}{\partial r} = \frac{1}{2+2\gamma} Y > 0, \tag{17}$$

$$\frac{\partial C^*}{\partial \gamma} = -\frac{2r+1+\gamma}{(1+\gamma)^2} Y < 0, \tag{18}$$

$$\frac{\partial S}{\partial r} = -\frac{\partial C^*}{\partial r} = -\frac{1}{2+2\gamma} Y < 0, \tag{19}$$

$$\frac{\partial S}{\partial \gamma} = -\frac{\partial C^*}{\partial \gamma} = \frac{2r+1+\gamma}{(1+\gamma)^2} Y > 0, \tag{20}$$

and we thus obtain the optimal demand for the bubble asset (L) when a speculative motive is in play:

$$L = \frac{\gamma - r}{r(2+\gamma)} Y, \tag{21}$$

$$\frac{\partial L}{\partial Y} = \frac{\gamma - r}{r(2+\gamma)} > 0, \text{ for } \gamma > r, \tag{22}$$

$$\frac{\partial L}{\partial r} = -\frac{\gamma}{[r(2+2\gamma)]^2} < 0. \tag{23}$$

Unlike the cash transaction demand studied by Baumol (1952), Tobin (1956), and Baumol and Tobin (1989), the bubble asset (here, L) can be viewed as a form of money demand with a speculative motive. By Eq. (21), we obtain the special case:

$$L = \frac{\gamma - r}{r(2+\gamma)} Y \to \infty \; for \; r \to 0 \text{ and } \gamma \gg 0, \tag{24}$$

The infinite speculative demand for a bubble asset or money shown by Eq. (24) is similar to the “Liquidity Trap” of Keynes (1936) and Hicks (1937).

Q.E.D.

3.3 *rL* as a Form of Speculative Saving

Assume that the total debt of the banking sector is L where $L = rL+(1-r)L$. The central bank issues money to the amount of $(1-r)L$ backed by its share of the banking sector. That sector places a total financial resource L into a bubble sector with a marginal rate of return γ $(> r)$, such as that of a national bond or a real estate bubble. The higher return afforded by the bubble sector is the bubble premium of Wan (2018a); the no-arbitrage condition is satisfied because a loss will develop after the bubble bursts. A feature of speculative saving is that such behavior cannot occur absent encouragement by both the central bank and the banking sector

3.4 Saving Demand Associated with a Speculative Motive as Revealed by the Investment Savings-Liquidity Preference Money Supply (IS-LM) Model

When the zero lower bound of the interest rate is relaxed, that rate may be negative in the consumption function as argued by Palley (2018). Following Hicks (1937) and Tobin (1969), we now explore how an interest rate affects saving using the IS-LM model. The only change (from the above) is that the interest rate is added to the consumption function. We obtain:

Proposition 2: The IS curve trends backward.

Proof: Assume that there are only two sectors, thus household and corporate sectors. The domestic income *Y* is the sum of consumption *C* and investment *I*, as follows:

$$Y = C(Y, r)+I(r), \tag{25}$$

$$\frac{\partial Y}{\partial r} \lesseqgtr 0 \text{ iff r} \gtreqless \bar{r}, \tag{26}$$

and the maximum value of *Y* is $\bar{Y}$ for $r = \bar{r}$.

Q.E.D

Note that the interest rate $\bar{r}$ can here be negative, thus ranging from –1 to 1 ($-1 < \bar{r} < 1$). Following Hicks (1937) further, we assume that the monetary authority both supplies and absolutely controls all real money *M/P* in the market where the nominal money and nominal price level are *M* and *P* respectively. Then, when the monetary market is at equilibrium, we obtain:

Proposition 3: The LM curve increases given the infinite elasticity of a near

zero interest rate.

Proof: We combine the money supply with the speculative demand given by Eq. (21), and obtain:

$$M/P = L(Y, r) = \frac{\gamma - \mathrm{r}}{\mathrm{r}(2+\gamma)} Y, \tag{27}$$

At a constant money supply:

$$\frac{\partial Y}{\partial \mathrm{r}} = \frac{\gamma(2+\gamma)}{r(2+2\gamma)^2} > 0, \tag{28}$$

$$\rightarrow \infty \, for \, r \rightarrow 0 \text{ and } \gamma \gg 0. \tag{29}$$

Q.E.D.

The implications of ***Propositions 2*** and ***3*** are that financial easing by shifting the *LM* curve down and to the right (thus to *L'M'*) downsizes the domestic income from $\bar{Y}$ at equilibrium *E* to *Y'* at equilibrium *E'* when the interest rate *r* is lower than the threshold value $\bar{r}$ of Figure 2.

4. Conclusions and Policy Implications

We have found that both the per capita real GDP and real interest rate of Japan trended backward from 1994-2021, and have built an explanatory model. Negative

interest elasticity of saving occurs in a two-period model in which an agent maximized the Leontief, the power, and the CES utility, and when saving is motivated by speculation. We have also showed that the interest elasticity of the money demand increased to infinity when the interest rate decreases to zero, and that this created a micro foundation that can be termed a “Liquidity Trap”. Finally, when speculative saving is added to the IS-LM model, we have showed that the IS curve trended backward and that the LM curve is horizontal around the zero interest rate. Thus, a low interest policy implemented by shifting the LM curve to the right and down would downsize the equilibrium GDP. The policy implication is that interest cannot be too low if it is sought to elevate the GDP. In a future study, we will explicitly analyze asset markets in which non-bubble assets are accompanied by bubble assets.

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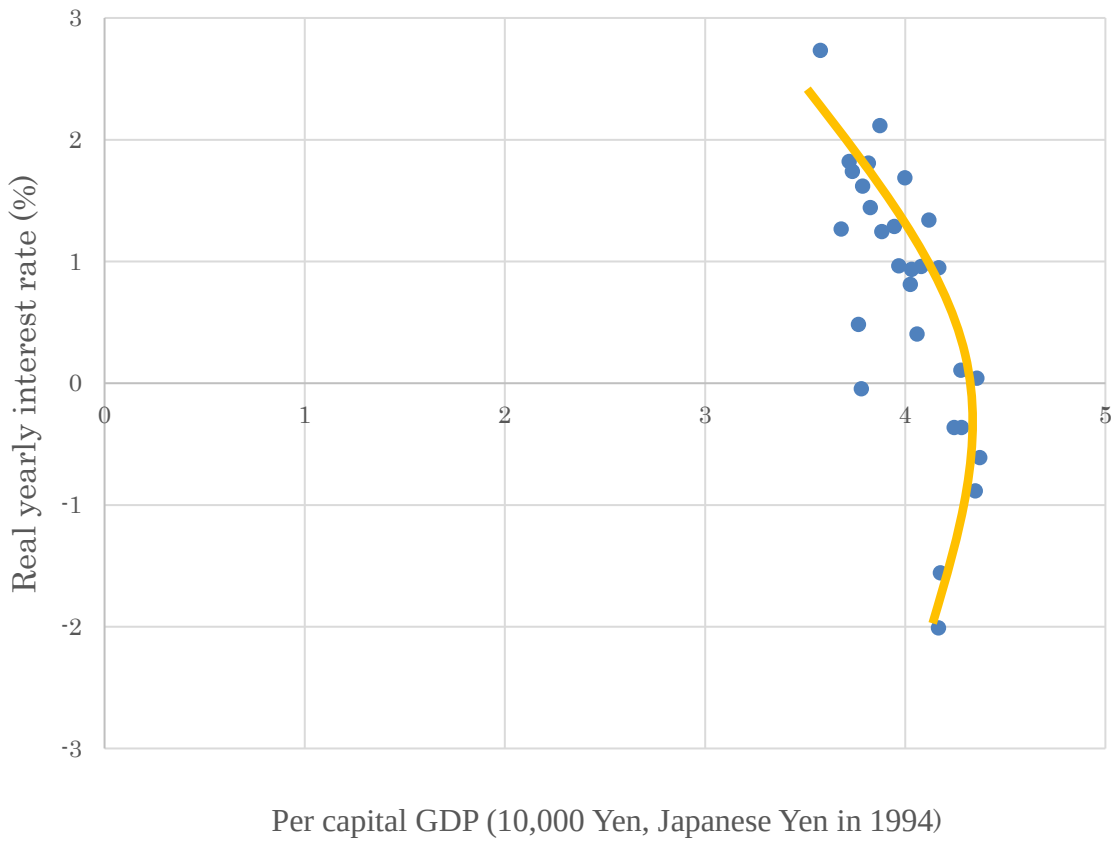
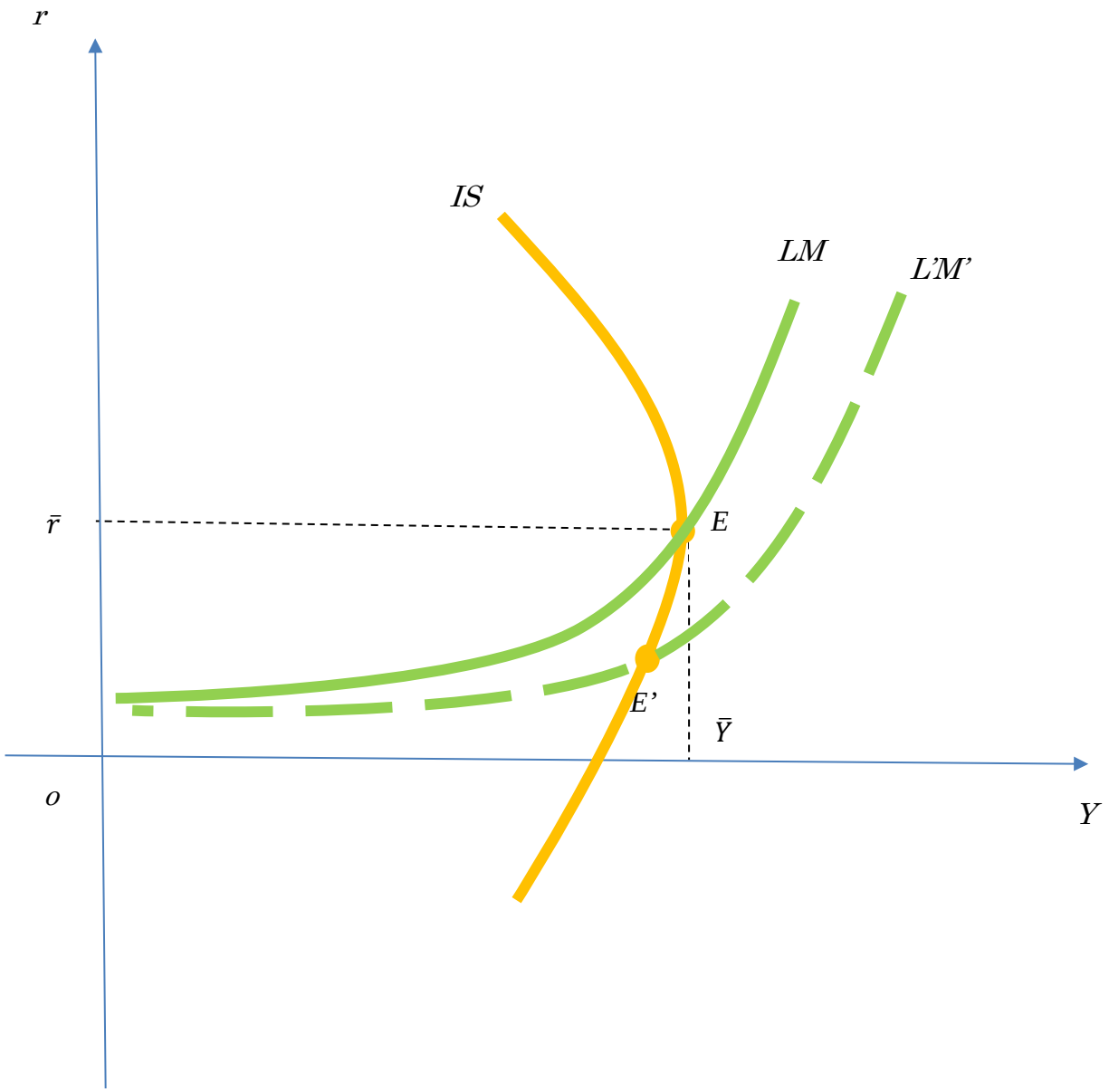
Figure 1: Per-capita Real GDP and Real Interest Rate (Average 1-Year Rate for Saving Deposits in Commercial Banks) in Japan, Fiscal Years 1994-2021.



Source: The calculations are based on data from the Ministry of Internal Affairs and Communications of Japan, and the Bank of Japan.

Figure 2: Saving Demand when the Motive is Speculative as Revealed by the IS-LM Model.



Source: The author.