CAES Working Paper Series

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WP-2023-002

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February 22, 2023

Abstract

This paper analyzes a rent seeking monopoly price, quantity and profit by using variable elasticity of substitution (VES) utility function. The paper provides a general equilibrium model (two industries and one factor market) in order to describe the behavior of a rent seeking monopoly in the case where one industry is a rent seeking monopoly while the other is competitive. The rent seeking monopoly quantity and price can be determined through general equilibrium model. This paper adopts the VES utility function to examine how the elasticity of substitution affects a rent seeking monopoly price, quantity, profit and employment.

Keywords: Monopoly, Rent-seeking, General equilibrium model, Variable elasticity of substitution.

JEL code: D10, D42, D50, D72.

Introduction

Cobb-Douglas (1928) and constant elasticity of substitution (CES) (Arrow et al., 1967) functions are the widespread utility functions in economics. Most importantly, CES model of monopolistic competition (Dixit and Stiglitz, 1977) is widely applied to theoretical and empirical analysis. Furthermore, Yin (2002) introduced a monopoly in multi-market production economy. He considers one industry is a monopoly and the others are monopolistically competitive. His model may not be complete as a general equilibrium model in the sense that there is no account for factor market equilibrium. Then the representative household (nominal) income is constant and does not depend on the parameters of the utility function.

This paper has two main contributions. At first this paper introduces the Variable Elasticity Substitution (VES) function as the utility function. The VES function has been developed by Revankar (1971) as the production function and has never been used as the utility function in the general equilibrium models to the best of authors' knowledge.

Section I discusses the rent seeking monopoly model under general equilibrium and the properties of demand function derived from the VES utility maximization problem with the budget constraint. Section II discusses numerical simulation analysis on effects of productivity change.

I. The Rent Seeking Monopoly Model

In this section we discuss the rent seeking monopoly model. The commodity A market is monopolized and the commodity B market is perfectly competitive.

A. *The Household Sector*

We assume that the representative household maximizes its utility given as

$$\mathrm{U} = \gamma x_b^{\alpha(1-\delta\rho)}[x_a + (\rho - 1)x_b]^{\alpha\delta\rho}, \tag{1}$$

subject to its budget constraint given as

$$P_a x_a + P_b x_b = w(l_a + l_b) + \pi_a + \pi_b, \tag{2}$$

where x_a is the quantity of commodity A,

x_b is the quantity of commodity B,

P_a is the price of commodity A,

P_b is the price of commodity B,

w is the wage rate,

l_a is the labor hours devoted to produce commodity A,

l_b is the labor hours devoted to produce commodity B,

π_a is the profits of commodity A industry,

π_b is the profits of commodity A industry,

and α, γ, δ and ρ are the positive parameters.

The profits of commodity A industry is rewritten as

$$\pi_a = P_a x_a - w l_a. \tag{3}$$

The profits of commodity B industry is zero because the commodity B market is perfectly competitive.

The elasticity of substitution σ for VES is given as

$$\sigma = 1 + \frac{\rho-1}{1-\delta\rho}\frac{x_b}{x_a}. \tag{4}$$

Revankar (1971) shows that $0 < \delta < 1,\ 0 \le \delta\rho \le 1$ and $\frac{x_a}{x_b} > \frac{1-\rho}{1-\delta\rho}$ if $\sigma > 0$.

The first order condition for this utility maximization is given as

$$\frac{\delta\rho x_b}{x_a - x_b + \rho x_b - \delta\rho x_a} = \frac{P_a}{P_b}, \tag{5}$$

where this means the equality between the marginal rate of substitution and the relative price.

B. The firm sector

The production function of the commodity A industry is given as

$$x_a = Al_a, \tag{6}$$

where A is the parameter and represents the technological level.

The production function of the commodity B industry is given as

$$x_b = l_b. \tag{7}$$

The first order condition for the profit maximization of the commodity B industry is given as

$$P_b = w. \tag{8}$$

This implies that the commodity B industry is under perfect competition.

C. The Labor Market Equilibrium condition

The labor market equilibrium condition is given as

$$l_a + l_b + l_c = \bar{l}, \tag{9}$$

where l_c is the labor inputs spent for rent seeking activity in the commodity A industry, and $\bar{l}$ is the total labor supply in the economy.

We assume that the labor inputs spent for rent seeking activity is obtained by

$$l_c = \frac{\pi_a}{w}. \tag{10}$$

D. The Demand functions for the commodity A and the commodity B

From Equations (2), (5), (6), (7), (9) and (10), we get the demand functions for the commodity A and the commodity B given as

$$x_a = \frac{Alw(P_a - \rho P_a + w\delta\rho)}{P_a(AP_a + Aw - w\rho)}, \tag{11}$$

and

$$x_b = \frac{Aw + A\rho P_a - w\rho - Aw\delta\rho}{AP_a + Aw - w\rho}. \tag{12}$$

E. The Profit Maximization of the Rent Seeking Monopoly

The profit of the monopoly firm is given in Equation (3) as

$$\pi_a = P_a x_a - wl_a.$$

Rewriting this from Equation (6), we get

$$\pi_a = P_a x_a - \frac{w}{A} x_a, \tag{13}$$

where $x_a = \frac{Alw(P_a - \rho P_a + w\delta\rho)}{P_a(AP_a + Aw - w\rho)}$.

We assume that the monopoly firm maximizes its profit with respect to the output price. Then the first order condition is obtained by taking the derivative of Equation (13) with respect to the price of the commodity A and can be written as

$$\frac{\{A(1-\rho)^2 + A^2(1-\delta\rho-\rho)\}P_a^2 + 2Aw\delta\rho P_a + w^2\delta\rho(A-\rho)}{P_a^2(AP_a - w\rho + Aw)^2} = 0. \tag{14}$$

The numerator of Equation (14) is the quadratic equation of P_a and we solve for this quadratic equation for P_a. The solutions for P_a are given as

$$P_a = \frac{w}{A} \frac{\sqrt{A\delta\rho(A-\rho+1)(-A+\rho+A\rho-\rho^2+\delta\rho)} - A\delta\rho}{A - 2\rho - A\rho + \rho^2 - A\delta\rho + 1}, \tag{15}$$

and

$$P_a = -\frac{w}{A} \frac{\sqrt{A\delta\rho(A-\rho+1)(-A+\rho+A\rho-\rho^2+\delta\rho)} + A\delta\rho}{A - 2\rho - A\rho + \rho^2 - A\delta\rho + 1}. \tag{16}$$

F. The Existence of Unique Solution of the Model

We have two solutions given Equations (15) and (16) for the model. In this section we prove that the existence of unique solution in the model by showing the either Equation (15) or Equation (16) gives a positive price value of P_a.

Proposition 1.

The two solutions given in Equations (15) and (16) are all real numbers if $\rho > 1$.

Proof: $\sqrt{A\delta\rho(A-\rho+1)(-A+\rho+A\rho-\rho^2+\delta\rho)}$ is rewritten as

$\sqrt{A^2\delta^2\rho^2+\delta\rho\{A^3(\delta\rho+\rho-1)+A^2\rho(2-\delta\rho+2\rho)+A^2(\rho-1)+A\rho(\rho-1)^2\}}$.

Since $0 \leq \delta\rho \leq 1$ and $\rho > 1$, the above value is positive. Q.E.D.

Proposition 2.

If the denominator of Equation (15) is positive and $\rho > 1$, then Equation (15) gives a positive solution but Equation (16) gives a negative solution.

Proof: The numerator of Equation (15) can be written as

$\sqrt{A\delta\rho(A-\rho+1)(-A+\rho+A\rho-\rho^2+\delta\rho)} - A\delta\rho.$

From Proposition 1 this is rewritten as

$\sqrt{A^2\delta^2\rho^2+\delta\rho\{A^3(\delta\rho+\rho-1)+A^2\rho(2-\delta\rho+2\rho)+A^2(\rho-1)+A\rho(\rho-1)^2\}} - A\delta\rho.$

This is positive if $0 \leq \delta\rho \leq 1$ and $\rho > 1$. Equation (16) gives a negative value because the denominator is assumed to be positive. Q.E.D.

Proposition 3.

If the denominator of Equation (15) is negative and $\rho > 1$, then Equation (15) gives a negative solution but Equation (16) gives a positive solution.

Proof: From Proposition 2 the numerator of Equation (15) is positive and the denominator is assumed to be negative, then Equation (15) gives a negative solution. The numerator is positive, the denominator is negative and the fraction has a negative sign in Equation (16), then Equation (16) gives a positive solution. Q.E.D.

Proposition 4

If $0 \leq \delta\rho \leq 1$,$\rho > 1$, and $A - 2\rho - A\rho + \rho^2 - A\delta\rho + 1 \neq 0$, then Equation (14) has a unique positive solution regardless of the sign of the denominator of Equation (15).

Proof: From Proposition 2 Equation (15) gives a positive solution if $A - 2\rho - A\rho + \rho^2 - A\delta\rho + 1 > 0$, $0 \leq \delta\rho \leq 1$ and $\rho > 1$. From Proposition 3 Equation (16) gives a positive solution if $A - 2\rho - A\rho + \rho^2 - A\delta\rho + 1 < 0$, $0 \leq \delta\rho \leq 1$ and $\rho > 1$. Hence, Equation (14) has a unique positive solution. Q.E.D.

II. Numerical Simulation Analysis on Effects of Productivity Change

In Part I, we have proved the existence of unique equilibrium of the model. In this part we discuss the effects of an increase of productivity on the economy using the numerical simulation method. We employ Microsoft Excel for the simulation analysis.

The simulation program is designed to show the numerical solutions of the model given the parameter values. The parameters are $A, \bar{l}, \alpha, \gamma, \delta$ and ρ. The initial values of these parameters are as follows: $A = 3$, $\bar{l} = 2$, $\alpha = 1$, $\gamma = 1$, $\delta = 0.25$, and $\rho = 2$. The commodity B is the numerator and its price is set to one. $(P_b = 1)$. Table 1 shows the results of simulation for various values of the parameter A.

Table 1.

Variables	A=3	A=4	A=5	A=6	A=7	A=100
P_a	0.403677	0.344949	0.305114	0.275978	0.25354	0.0608
X_a	0.43507	0.726797	0.951627	1.138037	1.299385	5.961651
X_b	1.824372	1.749292	1.709645	1.685927	1.670554	1.637531
L_a	0.145023	0.181699	0.190325	0.189673	0.185626	0.059617
L_b	1.824372	1.749292	1.709645	1.685927	1.670554	1.637531
π_a	0.030605	0.069009	0.100029	0.124401	0.143819	0.302852
U	2.030286	2.081202	2.133034	2.181971	2.22743	3.527591
L_c	0.030605	0.069009	0.100029	0.124401	0.143819	0.302852
real GDP	2	2.042684	2.093796	2.145327	2.195087	4.044115
σ	9.386568	5.813703	4.5931	3.962867	3.571301	1.549355

Table 1 shows the effects of productivity change in the model. An increase of productivity is shown as an increase of variable A. In the rent seeking model, an increase of productivity (A) reduces the monopoly price (P_a), the output of non-monopoly commodity (X_b), the employment of commodity B industry (L_b), and the elasticity of substitution between the commodity A and the commodity B (σ). On the other hand, an increase of productivity (A) increases the output of monopoly (X_a), the profits of monopolist (π_a), the utility level of households (U), the labor inputs spent for rent-seeking (L_c) and real GDP. Moreover, the employment of commodity A industry (L_a) increases until the productivity (A) reaches 5, and after that point it starts decreasing. This is related to the profits of monopolist (π_a) because after mentioned productivity point (A=5) the profit increases with decreasing rate. Similarly, the labor inputs spent for rent-seeking

(L_c) also increases with decreasing rate after the above-mentioned productivity point. The reduction of elasticity of substitution between the commodity A and the commodity B (σ) also has a decreasing rate after that productivity point (A=5). As the productivity (A) in the industry A increases, the elasticity of substitution (σ) decreases and approaches the value of one. Then the labor movement from the industry B to the industry A slows down.

Conclusion

This paper proposes a rent seeking monopoly model by using variable elasticity of substitution (VES) utility function. VES utility function could be used as an alternative to Cobb-Douglas utility or constant elasticity of substitution (CES) utility for a general equilibrium analysis where one industry is a rent seeking monopoly while the other is competitive. VES function has been developed as a production function. This paper will be the first paper which employs VES function as a utility function to the best of authors' knowledge.

If the elasticity of substitution is greater than one and the commodity A industry is the natural resource industry, then this model may explain the "Dutch disease" phenomenon. The phenomenon consists of the "resource movement effect" and the "spending effect". The resource movement effect is shown by the labor movement from the non-natural resource industry to the natural resource industry when the natural resource industry increases its productivity. The spending effect is shown by an increase of consumption expenditure with the technological advancement.

In this paper we assume that ρ is greater than one and then the elasticity of substitution becomes greater than one. If the ρ is assumed to be less than one and the elasticity of substitution is allowed to be less one, then the demand for the commodity B may increase and the direction of “resource movement effect” could be the other way around. In other words, the resource movement effect in the Dutch disease phenomenon may depend on the elasticity of substitution associated with the utility function. However, the analysis of this effect will be left for further research.

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